### Personification as a lens into relationships with mathematics

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Personification is the attribution of human qualities to non-human entities (Inagaki & Hatano, 1987). Eliciting personification as a research method takes advantage of a naturally occurring means through which (some) people discuss the nuanced emotional relationships they have with those entities. In this paper, we introduce the eliciting personification method for exploring individuals' images of mathematics, as well as discuss an initial set of approaches for analyzing the resulting data. Data from both pre-service teachers and research mathematicians are discussed in order to illustrate the method.

Key words: [personification, relationship with mathematics, conceptual blending]

The available research on people's relationships with mathematics predominantly relies either on assessment instruments (e.g., Likert scale surveys, concept mapping, responses to vignettes or videotapes, and linguistic analyses) or case studies (see Philipp, 2007). Although these are reliable research tools, as we argue later in this manuscript, the results they generate are not conducive to empathizing with others' mathematical experiences. The method presented in this article, *eliciting personification*, comes closer to this goal. We suggest this method complements existing approaches, adding new dimensions to our understanding of individuals' perceptions of, and experiences with mathematics.

### Method and participants

Although personification may occur naturally (e.g., Hill, 1930; Inagaki, & Hatano, 1987; Piaget, 2007), it is also possible to elicit personification data from participants. The personification data discussed in this work come from two sources. The first is an assignment given to 36 pre-service elementary/middle school teachers that invited participants to personify mathematics and describe this imaginary character and their relationship. The second is a series of interviews with a convenient sample of 9 research mathematicians that elicited similar data. Below is the prompt given to the pre-service teachers:

Your assignment is to personify Math. Write a paragraph about who Math is. This paragraph should address things such as: How long have you known each other? What is he/she/it look like? What does he/she/it act like? How has your relationship with Math changed over time? These questions are intended to help you get started. They should not constrain what you choose to write about.

We present the story of our research in two parts: we begin with a consideration of specific data, which we use to frame our introduction of analytic methods appropriate for such an approach to research; we then analyze this data and that of research mathematicians; we conclude with a comparative discussion that attends to the benefits and possibilities of the eliciting personification method.

# Pre-service teachers' personification of mathematics

Below is an excerpt from one of the pre-service teacher participants, which we use as a launching point to discuss approaches to analyzing elicited personification data:

Mathonious was a very sensible young boy from Athens, Greece. Not many people liked him but at age 6 he became the best of friends with a young girl named Kukla.

Every day they would hang out together and while Mathonious was a sensible young boy, Kukla began to notice that over the years he was becoming more and more complex. Kukla had noticed this and suggested that they see the oracle in order to find a solution. The oracle was known for simplifying and clarifying things for people in order to better their lives and though the oracle did great things, there were always consequences for those who do not listen to her advice. Mathonious met with the oracle and she told him that though he thought his complexity was a good thing it was confusing and hurting those closest to him; she warned him that if he did not revert to his more sensible simple self soon he would lose those closest to him and become a terrible beast; feared by many. He returned to Athens to tell Kukla his prophecy and when he did he was not very serious about it. In fact he did not seem to care about the oracles' advice or warning at all. Because he did not simplify himself to those around him the consequences of the prophecy came true and a horrid exiled beast he did become. He was indeed feared by many. The people feared him so much that they dehumanized him and called him MATH, which stood for mental abuse to humans. Despite his awful new nature, Kukla wanted to try to understand him desperately so that maybe he could return to the boy he once was and they could be friends. *However, every time she attempted he would cast her away.* 

The above paragraph describes the relationship between Kukla, the character the author attributes to herself, and Mathonious, who is a personification of mathematics. It paints a rich picture of the author's relationship with mathematics. However, since personification data is novel in mathematics education, there is no well-defined set of approaches for performing an analysis of a set of such elicited personification data.

# Analyzing data using character summaries

The initial approach to analyzing personification discussed in this article is to use open coding (Strauss & Corbin, 1990) to summarize each participant's Math-character. The compilation of a list of all characters produced by a particular group then serves to summarize the types of relationships with mathematics present in that group. For example, in the above excerpt, the writer describes Mathonious in terms of two characters. The first character is a young boy that Kukla befriends. However, this relationship deteriorates and Mathonious becomes a former friend with whom Kukla is trying to rekindle a friendship. The second character is a terrible beast, feared by many. Both characterizations, the terrible beast and the former friend, concisely encapsulate the writer's relationship with mathematics.

After the pre-service teachers' personification excerpts were sorted into similar character categories, three themes emerged from the scripts. The first and most common theme that emerged was that of a monster or other evil creature. The terrible beast from the excerpt was subsumed under this category along with other goblins ghouls, and nasty things. This theme depicts mathematics as a cruel, unattractive, and unforgiving entity that often takes pleasure from the suffering of others. The second common theme was that of a former friend. Mathematics was described as someone with whom the pre-service teacher once had a healthy and sometimes even happy relationship, but at some point the relationship had soured. This theme also occasionally involves descriptions of repeated attempts on the part of the pre-service teacher to mend the relationship, while mathematics 'resists'. The last type of character, which only occurred once, involved a lover who is loathed by friends, family and even strangers. The lover character might resonate with readers who have encountered deleterious comments when discussing their profession (for a detailed discussion of the lover excerpt see Zazkis, 2015).

We interpreted the Kukla excerpt as drawing upon both the monster and former friend themes. However, we sought a more detailed analysis of participants' relationship with mathematics than that which is afforded simply by identifying common character themes. We develop such analysis below.

#### Personification as a conceptual blend

Conceptual blending involves taking the elements of two (internal or external) input spaces and blending them together to form new inferences which are said to exist in a newly formed (internal) output space (Turner & Fauconnier, 2002). It has been used to analyze a number of mathematics education related phenomena, including proof construction (Zandieh, Roh & Knapp, 2014), task design (Mamolo, Ruttenberg-Rozen, & Whiteley, 2015), and the concept of infinity (Núñez, 2005). Conceptual blending is a crucial way in which people make-sense of, and communicate, complicated and multi-faceted phenomena (Fauconnier & Turner, 2008). For example, in order to make sense of the statement, "My karma ran over my dogma," one needs to blend a road-kill input space in which a car runs over a dog and a theology input space in which the words karma and dogma are defined. The resulting blend allows for the interpretation of the sentence's meaning—my karma overcame my dogma.

Personification can be conceptualized as a kind of conceptual blend (Fauconnier & Turner, 2008). In the case of eliciting personification of mathematics, a mathematics space and a human relationship space are blended to form a space that allows for the communication of one's complex emotional relationship with mathematics. The rich experiences and vocabulary associated with the human relationship space serve as a platform for participants to discuss emotional relationships with mathematics, and as a lens for the researcher to interpret the complexities of individuals' affective experiences with mathematics. This relationship would otherwise be difficult to discuss or interpret with the same level of depth and detail since vocabulary and images associated with emotion are primarily housed in the human relationship space, not the mathematical space.

#### Analyzing pre-service teachers' elicited personification using conceptual blending

As mentioned earlier, the Kukla excerpt describes mathematics as both a (former) best friend and a terrible beast. These two characterizations are quite different and coincide with different categories in the character summary analysis. So we use separate blending diagrams to describe each. First, the best friend: this characterization in the human relationship space maps to comfort with, and enjoyment of, mathematics in the mathematics space. A best friend characterization does not imbue the same level of passion for mathematics that a lover characterization would. However, it still portrays the author as someone who likes to spend time with mathematics and portrays mathematics provides a level of detail in regard to how much, and in what ways, the author enjoyed mathematics. For example, a relationship with a lover would have a closer degree of intimacy, than that with a friend. One would spend more time with the former, compromise differently for him or her, and feel more deeply emotions of elation, frustration, or despair.

Some of the details about the best friend are also revealing. Mathonious the best friend is sensible, a human trait that can be interpreted to map to the logical coherence and understandability of mathematics, since a reasonable definition of sensible is "having sound judgment" and "readily perceived". Additionally, Mathonious is presented as male, while Kukla is female. This is in line with research that points to mathematics being perceived as a male dominated discipline (e.g., Keller, 2001; Picker & Berry, 2000), and may also be indicative of perceived power structures between the author and her "friend". Lastly, there is

a timeline of Kukla's relationship with Mathonious. This timeline can be assumed to coincide with the timeline of the writer's relationship with mathematics – as Kukla learned more about Mathonious, he seemed to become more inaccessible, less friendly, hurtful and indifferent to his effect on his former friends. Details of this blending analysis are summarized in Figure 1.

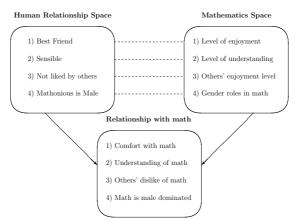


Figure 1. Best friend conceptual blending diagram.

The excerpt describes Mathonious getting progressively more complex and confusing, causing Kukla and Mathonious to grow apart. Complexity and confusion can be assumed to be a part of the mathematics space that stands in opposition to the previously mentioned sensibility from the best friend part of the human relationship space. Complexity, a mathematical trait, is not generally associated with emotions. However, describing this complexity in association with a personification of mathematics allows the excerpt's author to describe the complexity as "confusing and hurting." Mathonious' pride in his complexity (he refuses to follow the oracle's advice, despite 'consequences'), and his indifference towards its effects on others, are described as the root causes of the deterioration of Kukla's relationship with Mathonious.

After the falling-out, Mathonious is re-characterized as a terrible beast who exiles himself. This replaces the positive emotions associated with a best friend with the fear and repulsion associated with a beast. This characterization, much like the best friend characterization that preceded it, provides a level of detail in regard to the emotions involved. The excerpt's author could have chosen to describe simply growing apart, which would entail a level of indifference toward mathematics. However, the author instead chose to use a "terrible exiled beast" and draw upon the fear and repulsion that this characterization entails.

Interestingly, the excerpt's author describes repeated attempts to rekindle the friendship with the 'old' Mathonious, which can be mapped onto attempts to return to a state of understanding mathematics. However, notice that Mathonious casts her away repeatedly, placing the blame for the poor relationship with mathematics on Mathonious, not Kukla. What this means in terms of the mathematics space, is that the excerpt's author seems to attribute blame to mathematics, an abstract entity, for her lack of understanding and enjoyment of the subject. It is Mathonious who should, in the eyes of Kukla, return to his former "sensible" self, and it is only he who values his complexity (as the oracle identifies). Figure 2 summarizes this analysis.

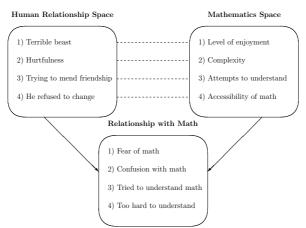


Figure 2. Terrible beast blending diagram.

# Mathematicians personification of mathematics

We now turn to personification data of mathematicians. Unlike the 36 pre-service teachers whose character summaries fit into three categories, the 12 mathematicians we interviewed personified mathematics in more diverse ways. This included descriptions of mathematics in sexual-relationship terms as spouse, ex-spouse, forbidden lover or mistress. Additionally, some mathematicians described mathematics as a person with whom they do not have a personal relationship, but whom they seek to understand. For example, one mathematician described mathematics as a virtuoso whom the mathematician aspired to be more like. Another mathematician described mathematics as a knowledgeable wandering Jew who points out the flaws in peoples thinking and in turn promotes societal progress. We interpret this diversity in mathematicians' descriptions of mathematics as a reflection of the different natures and experiences of mathematicians and pre-service teachers, respectively. The added complexities in the types of relationships forged with personified mathematics, as well as the varied details about the character of mathematics (e.g., religious, sexy, scandalous), are brought to light with our method and add insight into the varied complexities and characteristics that draw or deter career mathematicians.

# Analyzing mathematicians' elicited personification using conceptual blending

We now turn to an excerpt from one mathematician's personification interview. As with the Kukla excerpt, we analyze this excerpt using conceptual blending.

While I'm actually engaged in proving a theorem a lot of the time there is joy. However the time I'm actually proving theorems is very small. So my relationship with mathematics is not just my relationship with proving theorems. It's also my relationship with grading papers, my relationship with going to committee meetings, my relationship with advising students, writing papers, which is kind of tedious and is very different from writing proofs. So all of this sort of comes along with a career in mathematics. So even though there maybe that child's heart that I still have that takes joy in doing it when I do have an hour or two to sit down and do math. I have pleasure in that. But I recognize that that isn't all of mathematics. But after a while you first take a lover and that intense rush you feel. That's fantastic. But after a while you realize that that's not the entirety of a relationship. It's not just that physical rush. There is also a lot of other things that go along with that. So you may still have that feeling. But it's only one piece of a much much large tapestry.... I love the wife, it's not that I don't love the wife. But there's a lot of groceries to buy, and taking out the trash, and stuff like that...it's not all proving theorems. The excerpt describes mathematics as a lover that eventually becomes a wife. The mathematician's love of proving theorems maps to his enjoyment of intimate moments with this lover/wife. This mapping provides a great level of detail with respect to how much this mathematician enjoys proving theorems – it provides a physical rush. However, as he points out there a lot of additional parts to his relationship with this wife; proving theorems is one piece of a much larger tapestry. In particular, for this mathematician, "mathematics" is no longer simply the discipline itself, but now carries with it all of the attributes of an academic career in the discipline. These necessary but not enjoyable parts of his relationship with this wife (e.g., buying groceries, taking out the trash) map to the less enjoyable parts of his (new and broader) relationship with mathematics (grading, meetings, advising students). This mapping provides detail regarding how he feels about other parts of his relationship with mathematics.

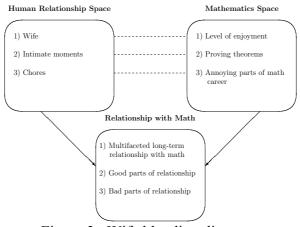


Figure 3. Wife blending diagram.

# Advantages of personification data

Let us compare the Kukla and wife excerpts above in relation to hypothetical responses to a Likert-scale item that asks for one's level of agreement with the statement "Mathematics is enjoyable and stimulating to me". This item is borrowed from Bessant's (1995) factors influencing mathematics anxiety (FIMA) assessment. This is a fairly common instrument used to assess relationships with mathematics. The author of the Mathonious excerpt could have likely answered "strongly disagree", given her horrible beast characterization, whereas the mathematician might have answered "strongly agree", given his lover characterization. However, both responses provide only a snap shot – in the former example, the assessment is a current one, lacking the background illustrating how the relationship came to sour, in the latter example, the assessment is narrowly construed, extracted from the broader context in which the mathematician now engages with mathematics. This points to the Likert-scale question's limitations for capturing the tangible emotions involved, and their connection to the broader experiences and history of the individual. Further, more nuanced details are captured in the picture of a lover who inspires a physical rush or a sensible friend who was once a favorite companion than are afforded by the Likert-scale item. Certainly, eliciting personification is not a replacement for survey methods, but it does have particular advantages in terms of detailing emotional relationships that add an important dimension to research on affect and mathematics education.

We suggest one of these advantages is empathy. When reading both the discussed excerpts, one might notice similarities to his or her own experiences and feelings. The

experience of having a friendship turn sour in spite of repeated attempts to salvage it is common, eliciting empathy for the author. Similarly, settling into a long-term relationship is also a common human experience to which many of us not only relate but also strive for. However, one might also empathize with the personified mathematics – as a friend who has been shunned, a wife who feels neglected, or a 'monster' whom nobody understands. Most people have either undergone these experiences first hand or been exposed to someone who has. By relating these experiences to their relationships with mathematics, participants in this study helped us understand the nature of their relationship with mathematics and how it evolved. In short, the data provides an avenue for our original stated goal of allowing one to empathize with others' relationships with mathematics, and it sheds new insight on what mathematics "looks like" from the eyes of our participants.

We propose the eliciting personification method as an indirect way of examining someone's relationship with mathematics, which can paint a particularly vivid image of this relationship and the characters involved. The conceptual blend of personified mathematics is certainly a dramatized version of participants' relationship with mathematics itself due to the very nature of personification. However, we do not view this dramatization as a detriment. Rather, we view this dramatization as a useful means through which to foster empathy, as well as to elicit and distill the essence of individuals' relationships with mathematics.

#### Discussion

In this article, we introduced *eliciting personification* as a method in which participants describe their relationship with mathematics by describing mathematics as if it were a person. Two personification excerpts, one from a pre-service elementary school teacher and one from a professional mathematician, were used to illustrate the lens into participants' affect provided by the method. Several approaches to analyzing the data were also discussed. These approaches included using character summaries for summarizing the data, and conceptual blending, which was used for deeper analysis. The use of these techniques helped to distill a rich image of two participants' dispositions toward mathematics and how their relationships had evolved over time. Interestingly, both excerpts highlighted the complexities of human relationships that are not necessarily accessible through quantitative approaches to data collection. Both relationships involved issues of trust and caring (friend, loving wife), that grew overtime to include frustrations and even resentment (horrible monster, nagging wife). We note that conceptual blending and character summaries are by no means a complete list of appropriate, applicable approaches to personification analysis, and we encourage other researchers interested in using the eliciting personification method in their studies to experiment with other analysis approaches.

The eliciting personification approach offers a particularly vivid window into study participants' relationships with mathematics, and we view it as an important complement to the case study and assessment instrument methodologies used in the past. We suggest that our approach offers a novel lens through which to research mathematical disposition and the nuances involved in individuals' affective experiences with the discipline. Eliciting personification is proposed as an innovative research tool that affords participants creative ways to describe aspects of their relationship with mathematics that might otherwise remain tacit. Additionally, as discussed in Zazkis (2015), personification can be used in teacher education as a method for facilitating pre-service teachers' self-reflection about their relationship with mathematics and for fostering empathy toward their future students.

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