Why students cannot solve problems: An exploration of college students' problem solving processes by studying their organization and execution behaviors

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This qualitative study investigates undergraduate students' mathematical problem solving processes by analyzing their global plans for solving the problems. The students in three undergraduate courses were asked to write their global plans before they started to solve problems in their in-class quizzes and exams. The execution behaviors of their global plans and their success or failure in problem solving were explored by analyzing their solutions. Only student work that used clear and valid plans was analyzed, using qualitative techniques to determine the success (or failure) of students' problem solving, and also to identify the factors that were hindering students' efforts to solve problems successfully. Many categories of student errors were identified, and how those errors affected students' problem solving efforts will be discussed. This study is based partly on Garofalo and Lester's (1985), and also Schoenfeld's (2010) frameworks, which consist of some categories of activities or behaviors that are involved while performing a mathematical task.

Key words: [Problem solving, Metacognitive processes, Mathematics teaching]

One of the most common ways that instructors assess students' mathematical understanding in colleges is grading their written work. Written work such as homework, quizzes, and exams mostly determine students' overall course grades in most instructors' grading schemes. Students' success or failure in undergraduate mathematics courses is usually determined by the weighted average of grades they obtain in a course by the end of the semester. Based on my own experience as a college mathematics instructor, many students' classroom interaction with the instructor and their peers indicate that they clearly seem to have grasped the understanding of the subject matter. But many of them cannot solve mathematical problems successfully on quizzes and tests, and fail to earn the higher course grades that they actually deserve. Their failure to solve mathematical problems, and getting lower than expected grades could be a source of frustration among students and instructors alike. It is also reasonable to assume that students' poor performance in written assignments could be one of the contributing factors to higher dropped, failed or withdrawn (DFW) rates in undergraduate mathematics courses. There is not enough information as to what factors are hindering undergraduate students' ability to solve mathematical problems successfully in time-constrained in-class assignments, even if they have the required understanding of the subject matter. It is, therefore, a good idea to look for reasons that might be hindering their effort to solve mathematical problems successfully.

Students' writing is a source of information for instructors to assess how their students' think and learn mathematics. Writing can be considered as thinking aloud on paper, and therefore it provides rich data and a means of observing important processes that are difficult to identify using other methods (Flower & Hayes, 1981). The purpose of this qualitative study is to study the connection between undergraduate students' global plans for solving mathematical problems and their success in actually being able to solve those problems in time restraint situations, by analyzing their written work. More specifically, the study attempts to answer the following research questions:

1. How are students' global plans related to their success in problem solving?
2. What are some primary factors that lead to unsuccessful problem solving, even when students have a valid global plan for solving mathematical problems?

This study is based partly on Garofalo and Lester's (1985), and on Schoenfeld's (2011) framework for mathematical problem solving.

Literature Review

Garofalo and Lester (1985) identified four categories of metacognitive activities involved in performing a mathematical task. The categories were orientation, organization, execution, and verification. According to them, the orientation phase pertains to strategic behavior to assess and understand the problem. The organization phase pertains to planning of behavior and choice of actions. Metacognitive behaviors during this phase include identification of goals and subgoals, global planning, and local planning (to implement global plans). The execution phase is related to regulation of behavior to conform to plans. This phase involves metacognitive behaviors such as performance of local actions, monitoring of progress of local and global plans, and trade-off decisions (such as speed vs. accuracy, degree of elegance). The verification phase is related to evaluations of decisions made and of outcomes of executed plans. Schoenfeld (2011) claimed that people's decision making and their success (or failure) in problem solving is a function of knowledge and resources, and beliefs and orientations. He also added that students' metacognitive activities or behaviors during problem solving also play a role in determining their success or failure in problem solving.

Research shows that a metacognitive framework is evident in students' writing about their problem solving processes (Pugalee, 2004). From a review of studies related to metacognition in problem solving, Simon (1987) found that the monitoring, regulation, and orientation processes appear more frequently in the problem solving protocols of successful problem solvers. From a study with middle school students, Lester (1989) found that orientation to the problem actually has the most influential effect on students' successful performance in problem solving. Pugalee (2004) found that the students who construct global plans (stated or implied) are more likely to be successful at the problem solving tasks. In addition, he reported that the execution behaviors comprised the largest number of problem solving actions. It is therefore reasonable to assume that students' likelihood of making errors during the execution phase is somewhat higher, even if they have a valid global plan for solving the problem. Pugalee (2004) also found that most students do not check the accuracy of their final answers. Use of few or no metacognitive behaviors in the verification phase might therefore also hinder students from being able to solve problems successfully, even if they have a clear conceptual understanding needed to solve the problems. Students' conceptual and procedural knowledge, therefore, might increase the likelihood of, but that alone does not guarantee, their success in problem solving. Schoenfeld (1985) found that effective problem solvers engage in self-regulation (or metacognitive) activities more often than others. Other studies have shown that successful problem solvers engage in metacognitive activities, and also have better understanding of mathematical concepts (Pugalee, 2004; Schur, 2002). Representation analysis of students' problem solving contexts in a recent study revealed that students who employed a nonsymbolic representation were more than three times more likely to solve the problems than the ones who employed symbolic representations (Yee & Bostic, 2014). From an analysis of the types of errors made by high school students in Algebra I, Pugalee (2004) found that 66.2% of all errors were procedural, 23% were computational, and 10.8% were algebraic. This suggests that students' lack of prerequisite concepts might also be adding challenges to students' effort to solving problems successfully.

Methodology

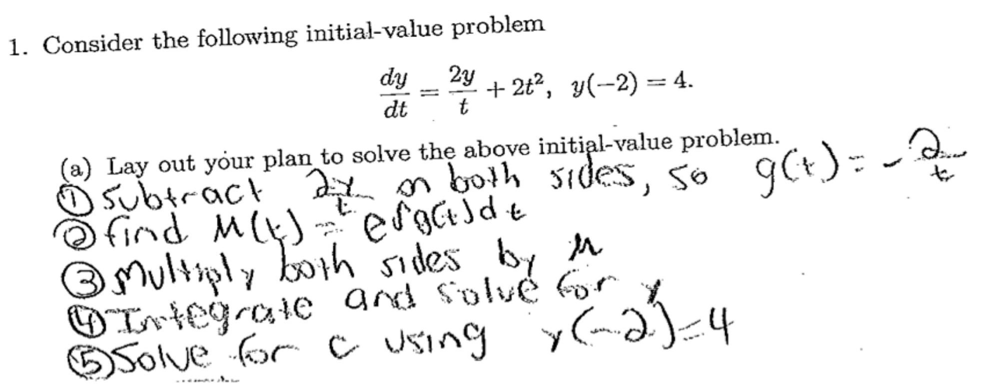
The researcher, who was also the instructor of the courses, collected data over the Spring 2015 and Summer 2015 semesters from three undergraduate mathematics classes:

Introductory Differential Equation (IDE) (three sections), Calculus I (one section), and Calculus II (one section). The participants were traditional undergraduate students from a medium-sized university in the southeastern United States. Data collection will continue until the Spring 2016 semesters as well. The collected data is comprised of students' written work from in-class quizzes and tests. During the first phase of data collection, the researcher required all the students to write their brief global plans and follow their plans to solve the mathematical problems. In this phase, students were given mostly procedural problems that required multiple steps to solve. In other words, most of the problems were not word problems. One of the reasons for asking them to write their global plans was to see if they had proposed a valid global plan (a clear big picture) for solving the problems during the organization phase. Since some of the students did not attend classes regularly or stopped attending, the number of responses collected per student varies. In the second phase, the researcher will also conduct audio or videotaped interviews with a few selected students, asking them to describe their plans for solving the problems (both procedural and word problems). One of the reasons for conducting audio or videotaped interviews is to ask them to provide a detailed plan for solving problems, because it is not easy to describe everything about their plan in writing. Brief global plans might not provide enough evidence to know if the students have clearly understood the underlying concepts and the procedures for solving the problems. The interviews will also allow the researcher the flexibility to ask students to clarify their plans for solving the problems. Although the whole class was required to write their global plans, interviewees will be selected based on a purposeful sampling method. The selection will be based on the analysis of students' written work. A representative sample of students having clear and valid global plans but failing to successfully solve the problems will be selected based on the types of errors they make while solving the problems.

Data Analysis

The first phase of data analysis involved the analysis of students' global plans and their solutions to mathematical problems from the first phase of data collection. The data was analyzed using two qualitative techniques, the constant comparative method (Strauss & Corbin, 1998) and thematic analysis (Braun, 2008; Creswell, 2012). The data analysis involved both inductive and deductive approaches (Braun, 2008). Students' work was categorized into many predetermined categories, based on their solution plans and the actual solutions. The solution plans were categorized as *clear and valid* or *unclear and/or invalid*. Figure 1 shows an example of a valid solution plan provided by a student.

Figure 1. *An example of a valid solution plan*



1. Consider the following initial-value problem

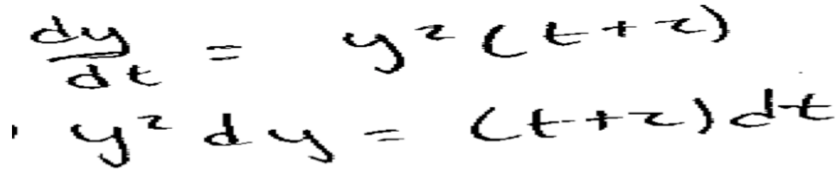
$$\frac{dy}{dt} = \frac{2y}{t} + 2t^2, \quad y(-2) = 4.$$

(a) Lay out your plan to solve the above initial-value problem.

① subtract $\frac{2y}{t}$ on both sides, so $g(t) = -\frac{2}{t}$
② find $M(t) = e^{\int g(t)\,dt}$
③ Multiply both sides by M
④ Integrate and solve for y
⑤ Solve for c using $y(-2)=4$

Students' errors in the solutions were also categorized into the predetermined categories *algebraic, computational,* and *procedural* (Pugalee, 2004). See figure 2 for an example of an algebraic error found in one of the responses. Other error categories emerged during data analysis: *calculus, carelessness, conceptual, other prerequisite,* and *plan not followed.* Some of the students' work had multiple occurrences of errors that fell into the same category, but they were recorded collectively as 1 (detected) or 0 (not detected). Students' final answers were categorized as *correct* or *incorrect*. Solutions with correct answers supported by valid work were considered as solved successfully. The percentage of points that was taken off from each problem (out of the maximum possible score) due to one or a combination of all errors was also recorded. Solutions with correct answers not backed up by valid work did not receive full credit. This preliminary analysis includes data from the first phase of data collection in the form of students' written work from one section of Calculus II and one section of IDE, both from the Spring 2015 semester. A few colleagues have agreed to cooperate in establishing the validity of the analysis through intercoder reliability.

Figure 2. *An example of algebraic error in student work*



Results and Discussion

There were 30 students in the Calculus II class and 23 students in IDE. Of the 272 responses collected from Calculus II, global plans of 199 responses (73%) were coded as clear and valid. This means that 199 of the plans convinced the researcher that the students could solve the problems successfully if they followed their plans and did not make any execution errors. Of those 199 responses, solutions in only 97 responses (48.7%) were correct and in 102 responses were incorrect. Similarly, 118 written samples collected from IDE were analyzed. Of these, 98 responses (83%) had clear and valid global plans, and solutions in only 43 of these 98 responses were correct. This section will focus primarily on those responses that had correct and valid plans but did not solve problems successfully.

Calculus II

There were 204 errors recorded in 102 responses with clear and valid plan but incorrect solutions; if any error type occurred more than once in a response, they were counted as one. The table below summarizes the number of errors detected in those written samples that had clear and valid plans but incorrect solutions. The sum total of all procedural and conceptual errors was 49, which is 24% of all errors. The sum total of all errors related to required prerequisite knowledge was 96, which accounts for 47% of all errors. Of the 102 responses with clear and valid plan but incorrect solutions, 43 (42.16%) responses lost more than 20% of the maximum possible score. This means that they received a grade of C or lower.

Differential Equations

There were 85 errors recorded altogether from the 98 responses with a correct plan but incorrect solutions. The total number of errors related to all types of prerequisite knowledge was 45, which accounts for 53% of all errors. Among those 98 responses, 27 (27.5%) responses received less than 80% of available points.

Table1. *Error counts in responses with clear and valid plans but incorrect solutions*

Types of Errors	Calculus II (Total = 102)	IDE (Total = 98)

Algebraic	13 (6.4%)	23 (27.06%)
Carelessness	23 (16.2%)	6 (7.06%)
Calculus	44 (21.6%)	14 (16.47%)
Computation	14 (6.9%)	1 (1.18%)
Conceptual	26 (12.8%)	20 (23.53%)
Others	5 (2.4%)	3 (3.53%)
Other Prerequisite	25 (12.2%)	7 (8.23%)
Plan not Followed	21 (10.3%)	0 (0.00%)
Procedural	23 (11.3%)	11 (12.94%)
Total	204 (100%)	85 (100%)

The preliminary results show that 48.7% of all responses with clear and valid global plans actually solved the problems successfully in Calculus II. In the IDE course, the corresponding figure was 43.9%. This shows that having clear and valid plans does not guarantee students' success in problem solving in writing. If the students had not made required prerequisite knowledge related errors, many more would have performed better in the courses. Even though the responses had clear and valid plans for solving the problems, such errors accounted for 47% of all errors in Calculus II, and 53% of all errors in IDE. This result indicates that lack of required prerequisite knowledge, especially the lack of precalculus concepts, is hindering their efforts to successfully solve problems.

Many students seem to be not performing well because of carelessness and not following their own global plans for solving the problems. In Calculus II, 44 out of 204 errors were due to either carelessness or not following their own plan, which accounts for 21.6% of all errors. But this number is different in IDE course; only 7.06% of all errors were due to carelessness, and none of the responses failed to follow their plans. Although the purpose of the research was not to compare the performances of students in two courses, it is noteworthy that 27.06% of all errors were algebraic in IDE, as compared to only 6.4% in Calculus II. On the other hand, 6.9% of all errors were computational in Calculus II as compared to only 1.18% in IDE. Obviously the problems posed in the courses were different, and a different level of algebraic or computational manipulations might have been necessary to solve the problems. But it was interesting to see that 27.06% of all errors in the IDE course were algebraic. This number could be much higher if we also count the students' responses with incorrect and/or invalid global plans. Almost comparable numbers in both courses (21.6% in Calculus II and 16.47% in IDE) were errors related to simple prerequisite calculus concepts, which the students of both courses were expected to know.

Preliminary results show that less than 50% of the responses could not execute their plans successfully even though their valid global plans clearly indicated that they knew how to solve the problems. The result also confirms the findings from earlier research that most students do not even check the accuracy of their answers (Pugalee, 2004). Students wrote in their global plans that they would check their answers (and did so in their solutions) in less than 1% of the responses. Time restraints could have limited their self-regulation activities during in-class tests and quizzes. It would be interesting to see how often the students would engage themselves in such activities if the given time is increased significantly for solving similar problems. At the present time, when online homework systems are taking the place of traditional homework assignments, these results indicate that students should be encouraged or required to communicate their understanding in writing more often, and also to become more efficient problem solvers in time restraint situations, if we continue to use our conventional in-class exams and quizzes to measure their success in the courses taught.

Questions to the audience: What do you want to hear from the interview data? Are there any frameworks for comparing regulation behaviors in free time and time restraint situations?

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