Inquiry-oriented instruction: A conceptualization of the instructional the components and practices

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In this paper we provide a characterization of inquiry-oriented instruction. We begin with a description of the roles of the tasks, the students, and the teacher in advancing the mathematical agenda. We then shift our focus to four main instructional components that are central to carrying out these roles: Generating student ways of reasoning, Building on student contributions, Developing a shared understanding, and Connecting to standard mathematical language and notation. Each of these four components is further delineated into a total of eight practices. These practices are defined and exemplified by drawing on the K-16 research literature. As a result, this conceptualization of inquiry-oriented instruction makes connections across research communities and provides a characterization that is not limited to undergraduate, secondary, or elementary mathematics education. The ultimate goal for this work is to serve as a theoretical foundation for a measure of inquiry-oriented instruction.

Key words: Inquiry-oriented, instructional practices, K-16

Rasmussen and Kwon (2007) refer to an inquiry-oriented approach to instruction as one in which "important mathematical ideas and methods emerged from students' problem-solving activities and discussions about their mathematical thinking" (p. 190). Importantly, they state that the students are not the only ones that engage in inquiry. Instead, in inquiry-oriented instruction students inquire into the mathematics and the instructor inquires into student mathematical thinking and reasoning. In this type of instruction the tasks, the students and the teacher work to support the classroom participants in advancing the mathematical agenda. The carefully designed tasks engage students in meaningful mathematical activity that generates student thinking which is then leveraged by the instructor to support the development of more sophisticated mathematics.

In the following section we provide a description of inquiry-oriented instruction by explicating the role of the tasks, the students, and the teacher. We then shift our focus to the components of inquiry-oriented instruction that support the tasks, the students, and the teacher in carrying out their roles. These components will be discussed in relation to relevant K-16 literature, allowing us to draw connections between the RUME and K-12 research communities.

Roles in Inquiry-Oriented Instruction

In inquiry-oriented instruction, the students, task sequence, and the teacher each have an important and interactive role for advancing the mathematical agenda. Here we discuss each of these roles.

Role of the Tasks

Meaningfully designed instructional tasks, regardless the form of instruction, provide a medium through which student mathematical ideas and reasoning can be generated. In inquiry-oriented instruction, tasks are specifically designed to *evoke* informal student strategies and ways of reasoning that can then be leveraged (in subsequent tasks or whole class discussion) to support the development of more formal mathematics (Gravemeijer, 1999; Larsen, 2013; Speer & Wagner, 2009; Rasmussen & Marrongelle, 2006). Instructional activities provide students with an opportunity to re-invent important mathematical ideas by supporting the students in mathematizing both the problem context and their own mathematical activity.

Role of the Student

In an inquiry-oriented classroom students, "learn new mathematics through *inquiry* by engaging in mathematical discussions, posing and following up on conjectures, explaining and justifying their thinking, and solving novel problems" (Rasmussen and Kwon, p. 190). These activities promote the emergence of many important student generated ideas and solution methods which one can think of as providing the mathematical "fodder" available to the teacher for the progression of the mathematical agenda (Speer and Wagner, 2009; Stein et. al., 2008). This fodder is generated through engaging with the mathematical activities that comprise the instructional sequence and by participating in argumentation and justification as students explain their own ways of reasoning and make sense of the reasoning of others. Importantly, by engaging in inquiry and supplying the mathematical fodder for the mathematical agenda, the students assume responsibility for the classroom's mathematics. Indeed, an important goal of inquiry-oriented instruction is for the "learners to come to regard the knowledge they acquire as their own private knowledge, knowledge for which they themselves are responsible" (Gravemeijer & Doorman, 1999, p. 116).

Role of the Teacher

Inquiry serves as an important aspect of the teacher's role. Inquiry-oriented teachers regularly inquire into their students' mathematical thinking and reasoning (Rasmussen & Kwon, 2007). Inquiring into student thinking helps the instructor promote the students' development of a more sophisticated mathematical understanding. In this way, the teacher is a co-participant in the development of the mathematics, in terms of both the mathematics of the moment, and the long and short-term mathematical trajectory intended by the curricula materials (Yackel, Stephan, Rasmussen & Underwood, 2003). Much of the focus on the role of the teacher has emphasized whole class discussions (e.g., Stein et al., 2008, Speer & Wagner, 2009). During these whole class discussions the teacher's aim is to bridge the gap between where the students are and the mathematical goals of the lesson. Specifically, the instructor leverages student ideas to move the students to a more sophisticated mathematical understanding. As noted by Stein et al. (2008) "the role of the teacher during whole-class discussions is to develop and then build on the personal and collective sense-making of students rather than to simply sanction particular approaches as being correct or demonstrate procedures for solving predictable tasks" (p. 315). Given this explication of the roles in inquiry-oriented instruction, we now turn our attention to specific components of instruction that allow these roles to be fulfilled.

Four Instructional Components of Inquiry-Oriented Instruction

Informed by our experiences with inquiry-oriented instruction, a starter-list of components was initially outlined. This list was then refined and explicated through a review of the K-16 research literature. This resulted in the following four components: Generating student ways of reasoning, Building on student contributions, Developing a shared understanding, and Connecting to standard mathematical language and notation. It should be noted that the four components are somewhat artificially separated for the purposes of explication. In actuality, these components are quite intertwined and work together throughout the lesson to support student development of more sophisticated mathematics. To better characterize inquiry-oriented instruction, we delineate each of the components into sets of practices. These practices are grounded in relevant research literature and each set works together to support its respective component. The practices exist at a smaller grain size and provide a high level of detail in terms of how each of the components supports the progression of the mathematical agenda. Some of the practices transcend individual components as they may serve different purposes at different times depending on nature of the component.

Generating Student Ways of Reasoning

In order to utilize student ideas and thinking to move forward the mathematical agenda, the teacher must first have student generated ideas and thinking to work with. Research indicates that eliciting meaningful student contributions requires the teacher to support the production of such contributions (Stein et. al., 2008; Hufferd-Ackles, Fuson & Sherin, 2004). One characteristic of instructors that promote meaningful student contributions is asking questions which drive student investigation of mathematics, support students in explaining their solution strategies, and help the instructor understand students' thinking (Munter, 2014). Questions of this nature require that the students engage in problem solving activity that affords the instructor with opportunities to inquire into student thinking and reasoning.

In inquiry-oriented instruction, purposefully designed tasks are utilized to engage the students in such authentic mathematical activity and lead the students to discover key mathematical ideas (Larsen, 2013; Rasmussen & Marrongelle, 2006; Rasmussen & Kwon, 2007; Speer & Wagner, 2009). The tasks provide a context in which the students engage in mathematical activity, which in turn provides opportunities for the instructor to inquire into student thinking and reasoning. This reasoning can then be used to promote a more sophisticated mathematical understanding. The interaction between the teacher, student and tasks affects the quality of the contributions that can be elicited (Jackson et al., 2013). Jackson et al. (2013) note that the cognitive demand of a task can be lowered depending on how the students are expected to engage with the task or if solution methods are posed before the students begin the task. Their research suggests that, when the cognitive demand of high quality tasks is maintained and when the students are supported in describing the contextual and mathematical features of the task, students are provided with higher quality opportunities to learn.

With this characterization of the practice of *Generating Student Ways of Reasoning,* we have identified three critical components in the literature:

1) *Students are engaged in meaningful tasks and mathematical activity that support the development of important mathematical ideas.* This practice is characterized by student engagement with cognitively demanding tasks, that support students in mathematical activity and are designed to promote ways of thinking about the mathematics that can be leveraged to advance the students' mathematical understanding (Jackson et. al, 2013; Hiebert, 1997; Speer & Wagner, 2009).
2) *Teachers actively inquire into student thinking.* This practice means that instructors purposely and intently inquire into student thinking for the purposes of determining if and how student generated ideas can be utilized to promote a more sophisticated understanding of the mathematics. The questions asked by teachers not only direct student investigations and provide the teacher with insight into student thinking, they also help students refine and reflect on their own thought process (Borko, 2004; Hiebert & Wearne, 1993; Rasmussen & Kwon, 2007). In this way, by inquiring into

student thinking, teachers are able to support students in generating more sophisticated ways of reasoning.

3) *Teachers elicit student thinking and contributions.* Teacher prompt students to explain their reasoning and justify their solution strategies, with the focus on the reasoning the students utilized during the task as opposed to solely focusing on the procedures used. Research on instructional quality indicates that the type of contributions teachers elicit is directly related to the students' opportunities to learn. Thus it is important that teachers elicit thinking and reasoning that "uncover the mathematical thinking behind the answers" (Hufferd-Ackels, Fuson & Sherin, 2004, p.92).

Building on Student Contributions

Researchers have noted that the practice of building on student thinking is quite complex and difficult to implement (e.g., Ball & Cohen, 1999; Sherin, 2002). Leatham, Peterson, Stockero, and Van Zoest (2015) characterize building on student contributions as engaging the class in student-generated contributions in ways that result in developing students' more sophisticated understanding of important mathematical ideas and relationships. To facilitate such building, teachers must elicit and inquire into student contributions to determine which ideas (correct or incorrect) are important and relevant to the development of the mathematics, which ideas can be leveraged to move the understanding of the class toward the goals of the lesson, and then engage the students in each other's contributions in ways that forward the mathematical agenda (Johnson & Larsen, 2012; Leatham et al., 2015; Speer & Wagner, 2009). Building on student thinking in this way requires that the classroom participants create the "mathematical path as they go," (Yackel et, al, 2003, pg. 117), because student contributions form the trajectory along which the mathematics develops (Johnson, 2013). In this way, teachers need to be sensitive to the ideas students contribute and use them to inform the lesson.

Orchestrating class discussions that build to certain educational goals while allowing the students to retain ownership of the mathematics requires that the instructor "slide between being noninterventionist and assuming greater responsibility" (Rasmussen & Marrongelle, 2006, p. 399). In other words, while the students' own ideas form the basis for the mathematics being developed, it is the instructor's responsibility to guide the development of the mathematics toward the mathematical agenda. Inquiry also plays an important part in how teachers carry out this role during the practice of building. By inquiring into student thinking with an eye towards important mathematical ideas, teachers must determine where to position themselves on the continuum between noninterventionist and interventionist. In either case, "'You are still the teacher. The students might not see your teaching. But you are still in control.' However, the nature and degree of control is different in this setting. Instead of controlling the exact content that gets stated in a lecture, the teacher's responsibility is to monitor, select, and sequence student ideas." (Johnson et al., 2013, p. 13).

With this characterization the practice of *Building on Student Contributions,* we have identified five critical components in the literature:

1) *Teachers elicit student thinking and contributions.* Leatham et al. (2015) note that student contributions can provide opportunities for the class to make sense of each other's thinking as well as opportunities for the teacher to build on student thinking. Hufferd-Ackles, Fuson, and Sherin (2004) echo this idea, stating that the "questioning of students allows their responses to enter the classroom's discourse space to be assessed and built on by others" (p. 92).

2) *Teachers actively inquire into student thinking.* Teacher inquiry serves many functions and roles throughout a lesson (see Hufferd-Ackles, Fuson, & Sherin, 2004; Johnson, 2013; Rasmussen & Kwon, 2007). With regard to building on student contributions, teacher inquiry allows teachers to form models of student thinking and understanding, reconsider important mathematical ideas in light of those models, and formulate questions and tasks which enable the students to build on those ideas (Rasmussen & Kwon, 2007).
3) *Teachers are responsive to student thinking and use student contributions to inform the lesson.* Rasmussen and Marrongelle (2006) state that, "an important part of mathematics teaching is responding to student activity, listening to student activity, notating student activity, learning from student activity, and so on" (p. 414). By doing so, the teacher can generate instructional space where "the nature of student mathematical thinking might compel one to take a particular path because of the opportunity it provides at that moment to build on that thinking to further student mathematical understanding" (Leathan et al., 2015, p. 118).
4) *Teachers guide and manage the development of the mathematical agenda.* Teacher need to actively guide and manage the mathematical agenda and can do so by: identifying and sequencing student solutions to "ensure that the discussion advances his or her instructional agenda" (Jackson et al., 2013, p. 648); utilizing *Pedagogical Content Tools* "to connect to student thinking while moving the mathematical agenda forward" (Rasmussen & Marrongelle, 2006, p. 389); or by refocusing the class towards the use of certain student generated ideas, marking important student contributions, and assigning tasks meant to clarify and build on students' ideas/questions. In these ways, teachers can guide and manage the development of the lesson while building on student contributions, developing mathematical ideas in directions commensurate with the mathematical agenda, and maintaining the student ownership of the mathematics.
5) *Students engage in one another's thinking.* Stein and colleagues (2008) provide several examples of how teachers can support students in making mathematical connections between differing student contributions and important mathematical ideas. Some of these examples include asking students to reflect on the contributions of other students, assisting students in drawing connections between the mathematics present in solution strategies and the various representations that may be utilized, and facilitate mathematical discussions about different student approaches for solving a particular problem. Doing so can prompt students to reflect on other students' ideas while evaluating and revising their own (Brendehur & Frykholm, 2000; Engle & Conant, 2002).

Developing a Shared Understanding

As discussed by Stein et al. (2008), "a key challenge that mathematics teachers face in enacting current reforms is to orchestrate whole-class discussions that use students' responses to instructional tasks in ways that advance the *mathematical learning of the whole class*" (p. 312, emphasis added). Within the inquiry-oriented instruction literature base, many articles make use of and highlight the importance of developing a shared understanding (e.g. Stephan & Rasmussen, 2002; Rasmussen, Kwon & Marrongelle, 2008; Rasmussen, Zandieh & Wawro, 2009). For instance, Stephan and Rasmussen (2002) discuss ways in which important mathematical ideas and ways of reasoning, emerging from ideas originating with individual students or small groups of students, become *taken-as-shared* within a classroom. Elaborating on how this occurs, Tabach, Hershkowitz, Rasmussen and Dreyfus (2014) discuss the reflexive relationship between ideas formulated by individuals or small groups and the normative ways of reasoning evident in whole class discussion. Their research suggests that the development of shared understandings supports student construction of mathematics by allowing ideas to be formulated by individuals or small groups and become normative ways of reasoning during whole class discussions. Further, McClain and Cobb (1998) note that supporting the development of taken-as-shared understandings help students with less sophisticated understandings participate in and benefit from whole-class discussions.

The important distinction between *Building on Student Contributions* and *Developing a Shared Understanding,* is characterized by who is making sense of the evolving mathematical agenda: the teacher and a select group of students who have provided the bulk of the contributions, or the classroom community as they develop and co-construct a taken-as-shared understanding. As described by Fredericks (in Johnson et. al., 2013):

> There is this risk that you can pose the problem and then you can have five groups share how they did it and then you can go to the next problem [without any additional discussion of the groups' ideas]. And you can assume that the students will make the connections, and some of them will and some of them won't. I think to really be effective you have to push yourself further than that. That you have to think about what those connections are and you have to make sure that they explicitly come out. Otherwise you don't know who got it and who didn't. You are right back to where you were when you taught the old way. (p. 13-14)

With this characterization the practice of *Developing a Shared Understanding,* we have identified three critical components in the literature. It should be noted that, while these three practices are also important for building on student thinking, their use and purpose is slightly different for developing a shared understanding.

1) *Teachers are responsive to student thinking and use student contributions to inform the lesson.* When teachers are responsive to student contributions they can create new instructional space (Johnson and Larsen, 2012). In regards to this component, the instructional space is created for the purpose of developing a shared understanding within the classroom community.
2) *Students are engaged in one another's thinking.* By engaging with one another's thinking, students are able to deepen their thinking, generate new ideas, and make mathematical connections. As discussed by Jackson et al. (2013), "the teacher plays a crucial role in mediating the communication between students to help them understand each other's explanations" (p. 648).
3) *Teachers guide and manage the development of the mathematical agenda.* Here the focus in on guiding and managing the development of the mathematical agenda *for the whole class.* This involves monitoring and assessing what is taken-as-shared.

Connecting to Standard Mathematical Language and Notation

One of the major tenants of inquiry-oriented instruction is the idea that formal mathematics emerges from students' informal understandings (Gravemeijer, 1999). This is contrasted with more traditional forms of instructions where formal definitions or standards algorithms serve as the starting place for students' mathematical work. However, this does not mean that mathematically standard language and notation have no place in inquiry-oriented instruction. As Stein at el. (2008) discuss, there is an "increasingly recognized dilemma associated with inquiry- and discovery-based approaches to teaching: the challenge of aligning students' developing ideas and methods with the disciplinary ideas that they ultimately are accountable for knowing" (p. 319). One way for a teacher to approach this challenge is to act as a broker "between the entire classroom community and the boarder mathematical community by the insertion of formal convention and terminology" (Rasmussen, Zandieh, Wawro, 2009, p. 201).

With this characterization the practice of *Connecting to Standard Mathematical Language and Notation,* we have identified two critical components in the literature.

1) *Teachers introduce a minimal amount of language and notation prior to students' engagement with a task*. Formal notation is introduced after the students have generated an understanding of what is being notated and a need for it has been established. "In contrast to more traditional teaching in which formal or conventional terminology is often the starting place for students' mathematical work, this teacher [one implementing an inquiry-oriented curriculum] chose to introduce the formal mathematical language only after the underlying idea had essentially been reinvented by the students" (Rasmussen, Zandieh, Wawro, 2009, p. 203)
2) *Teachers support formalizing of student ideas/contributions*. In inquiry-oriented instruction, as the students reinvent the mathematics, their reinventions build to be commensurate with formal mathematical ideas. The instructor must be able to promote the students' ability to connect the their mathematical ideas to more formal mathematics. "The teacher plays a crucial role … in supporting students to link student-generated solution methods to disciplinary methods and important mathematical ideas" (Jackson et al., 2013, p. 648).

Implications

Within the undergraduate mathematics community, the last decade has seen a sharp rise in inquiry-oriented, research based, instructional innovations. Inquiry-oriented instruction is being used in mathematics classes from calculus through abstract algebra. The limited research that does exist on mathematicians teaching practices has shown that these inquiry-oriented curricular materials present a number of challenges for implementation. Such challenges include: developing an understanding of student thinking, planning for and leading whole class discussions, and building on students' solution strategies and contributions (Johnson & Larsen, 2012; Rasmussen & Marrongelle, 2006; Speer & Wagner, 2009; Wagner, Speer, & Rossa, 2007). Given these challenges with the implementation of inquiry-oriented instructional materials, the need for a measure of instructional quality becomes an important way to understand differences in these classrooms.

Before such an instrument can be developed, "inquiry-oriented instruction" first needs to be operationalized in a way that can be observed, measured, and analyzed. The work here contributes to this in two ways: it represents a conceptualization of inquiry-oriented teaching, including the identification of the components and the specification of small-grain practices that support those components, and it can be used to as a theoretical foundation for a measure of inquiry-oriented instruction. Importantly this conceptualization draws on a wide spectrum of literature from the K-16 research base, allowing us to make connections across research communities and provide a characterization that is not limited to undergraduate, secondary, or elementary mathematics education.

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