Developing mathematical knowledge for teaching in content courses for preservice elementary teachers

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In recent years, much attention in the teacher education literature has been given to ways in which inservice teachers develop facility with the construct known as mathematical knowledge for teaching (MKT). Much less is known about the ability of preservice teachers to construct MKT. To address this, the current preliminary report adds to the research base by investigating two primary questions: (1) Can teachers build MKT in their content courses?, and (2) Can teachers engage in meaningful mathematical discourse as a result of their content courses? The report examines the effects of a semester long course on number and operations designed to allow preservice elementary teachers opportunities to build different aspects of MKT. Very preliminary analysis shows that many students lack this knowledge upon entering the course, but most are able to begin to build a degree of facility in it by course completion.

Key Words: Mathematical Knowledge for Teaching, Mathematical Discourse, Reasoning, Justification

Background and Research Questions

A central tenet of teacher education research has long been identifying the types of knowledge that teachers need to know in order to teach mathematics. Such attempts date back to Shulman's (1986) original proposal of a new type of knowledge that he called *pedagogical content knowledge* (PCK), defined as "the particular form of content knowledge that embodies the aspects most germane to its teachability" (p. 9). Since then, research teams such as Ball and company (2008) and Hauk and her colleagues (2014) have worked to conceptualize PCK. Ball and company have developed typologies for the much broader realm of *mathematical knowledge for teaching* (MKT), shown in figure 1, for which PCK is a subconstruct. Note that the left half of the oval consists of subject matter knowledge (SMK) which they claim requires no knowledge of students, which distinguishes it from the right half which is PCK. It is worth noting that the Ball model is specifically designed for the K-8 setting; this is important because as Speer et al (2014) note, generalizability comes into question when trying to apply the model outside the K-8 context.

Within the Hill, Ball, and Schilling (2008) model, *common content knowledge* (CCK) is defined as "knowledge that is used in the work of teaching in ways in common with how it is used in many other professions or occupations that also use mathematics"(p.6). In contrast, *specialized content knowledge* (SCK) is specialized in the sense that it is specific to the task of teaching. SCK includes various ways to represent mathematical ideas, provide mathematical explanations for rules and procedures, and examine and understand innovative solution strategies(Hill et al, 2008, p.377). As an example, consider fraction division. Most middle school graduates can readily use the invert-and-multiply algorithm to divide fractions. Thus, this piece of knowledge is CCK. Yet, few can explain to a novice learner *why* the algorithm exists in school mathematics nor why it is justified, thereby making this particular piece of knowledge SCK. Within the realm of PCK are *knowledge of content and students* (KCS) and *knowledge of content and teaching* (KCT). KCS is

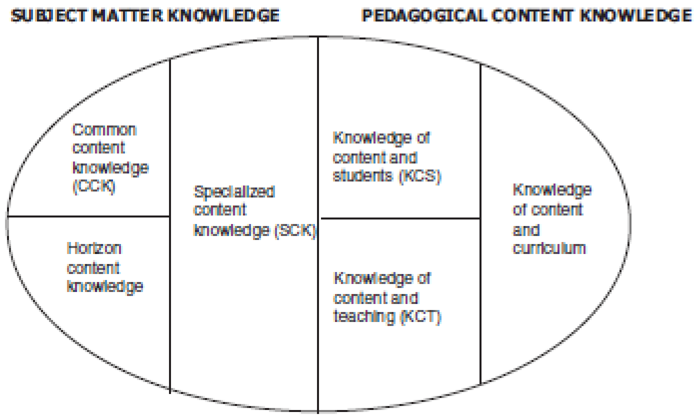


Figure 1. Domain Map for MKT (Hill et al, 2008)

"content knowledge intertwined with knowledge about how students think about, learn, or know this particular content" (p. 375), while they define KCT as a knowledge of teaching moves. So, using our division of fractions example again, a teacher who is aware that students often invert the first fraction instead of the second fraction is demonstrating KCS, and might use fraction diagrams as a way of scaffolding student understanding of division of fractions by using her KCT.

Implicit in the use of KCS and KCT is an awareness of the words, grammar, syntax, and forms of standard mathematical language in use – what Gee (1996) would call the "little d" discourse of mathematics. Also at work in the teaching of mathematics are nuances about what is valued in mathematical discourse in a mathematics class (as opposed to mathematics in a physics or biology class), the socio-mathematical norms for questions and answering, and myriad other interactions that make a mathematics lesson recognizable in an instant (e.g., by someone listening in or looking through the window of a classroom for just a few seconds). This kind of *situated* "little d" discourse is what Gee called "big D" Discourse. Hauk and colleagues (2014) have brought these ideas into a further unpacking of the components of PCK. The extended model, shown in Figure 2, adds a fourth dimension to PCK called *Knowledge of Discourse* (KD). Hauk et alia argue that effective teaching of mathematics includes facilitating student learning of *mathematical discourse* (along with other discourses). Such Discourse is enacted in the classroom when students and teacher engage in mathematically appropriate, accurate, and effective communication situated in the context of reasoning and justification of mathematical ideas. Clearly, a rich and textured Knowledge of Discourse is required for teachers to use and promote the valued mathematical skill of justification: engaging in reasoning about and explaining how one knows something is true (Cioe et al., 2015).

To measure PCK and MKT more generally, both research teams developed multiple choice assessments designed for administration to inservice teachers receiving professional development for Ball's team and completing a master's degree for mathematics teachers in the case of Hauk's team. It should be noted that in the case of Hauk's team, the focus was on the PCK development of teachers at 7-12 level unlike Ball's team. While items in the instrument developed by Hauk's team measured in large extent the syntactic structure of KD and did attempt to measure the teachers' ability to engage in proof validation, neither their instrument nor the items developed by Ball's team measure the larger components of discourse required to engage in reasoning and justification: i.e. neither team tried to specifically measure mathematical discourse more generally. Hence, the current project is designed to address two key missing

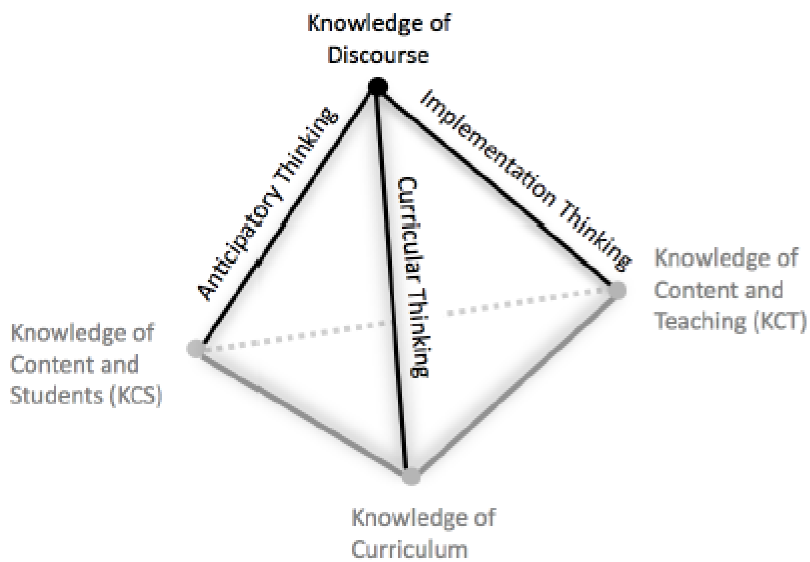


Figure 2. Tetrahedron Model of PCK (Hauk et al., 2014)

ideas in the existing literature: (1) Can *preservice elementary teachers* (PSETs) develop facility in MKT as a result of their learning in the content courses for PSETs?, and (2) Can PSETs learn to engage in meaningful mathematical discourse as a result of their experiences in these courses?

Research Methods

Beginning in the summer of 2011, an instrument was developed to begin measuring different aspects of MKT of preservice elementary teachers, with particular emphasis on items which require some combination of SCK, KCS, and KD. The items requiring a combination of SCK and KD to answer generally require the teachers to engage in mathematical discourse to justify certain mathematical facts or procedures. Two examples of items from the most recent administration of the instrument are given below:

7. John asks you in math class one day why $4^0 = 1$. Give John an explanation that he can understand for why this is true.

8. Nancy, a student in your 5th grade math class, asks you day why she cannot divide 5 by 0. That is, why she cannot do $5 \div 0$. Give Nancy an explanation that she can understand for why she cannot do this.

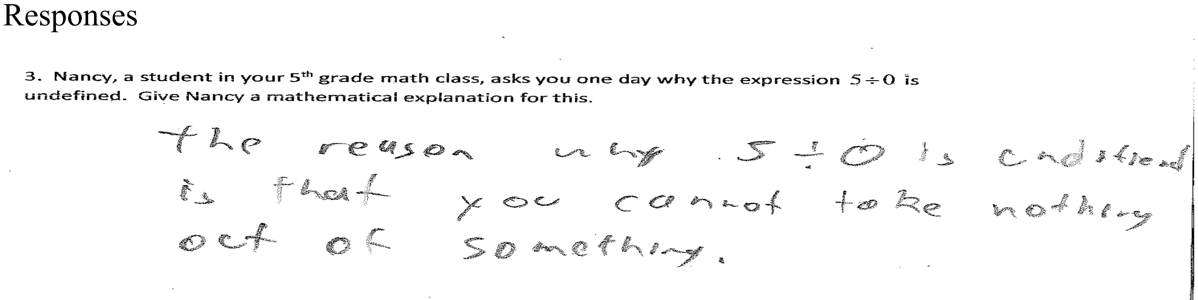
PSETs enrolled in a course on number and operations at a large public state university in the northeastern US were given the instrument upon entering the course as well as upon exiting in a standard pre-post format. During the course, PSETs are expected to engage in mathematical discourse through reasoning and justification consistently as a socio-mathematical norm in class and group discussions, online homework exercises, and on exams. The instrument contains 13 items, and the current report focuses on data collected from 4 sections of the course in the Fall 2014 and Spring 2015 semesters, with N=78 teachers. In addition to administering the instrument, five teachers were interviewed in the spring semester concerning their answers to 3 of the items to discern their ability to communicate effectively orally in addition to written formats. Participants were also presented with novel tasks for them during the interviews that gauged their abilities to engage in reasoning and justification more generally through validation. For instance, one of the items in the instrument asks for a justification of the invert-and-multiply algorithm. During the interviews, the teachers discussed their own justifications for the algorithm and then were presented with justifications that had not been discussed during the course and were asked to discuss the appropriateness of the justification for an elementary classroom.

Preliminary Data Analysis and Results

Data analysis is ongoing and will continue through the end of the Fall 2015 semester. Pre and post responses to the free response items in the MKT instrument are to be scored by researchers with emphasis on interrater reliability based upon predetermined criteria involving mathematical accuracy of the responses, the effectiveness of the responses in reaching the intended audience of elementary students, and the appropriateness of the responses based upon the grade level of student whom the teacher is communicating with. The five interviews are to be transcribed and coded based upon similar criteria. However, in addition, the interviews will also be analyzed to look for evidence of surface validity in instrument items in constructing various components of MKT.

Early analysis shows that a significant proportion of teachers did build some facility in different aspects of MKT, although some teachers were more successful than others. To highlight some these successes or lack thereof, a few sample corresponding pre-post response pairs are given below:

Pre Responses


3. Nancy, a student in your 5th grade math class, asks you one day why the expression $5 \div 0$ is undefined. Give Nancy a mathematical explanation for this.

the reason why $5 \div 0$ is undefined is that you cannot take nothing out of something.

6. John asks you in math class one day why $4^0 = 1$. Give John a mathematical explanation for this.

Don't know

Corresponding Post Responses:

3. Nancy, a student in your 5[th] grade math class, asks you one day why the expression $5 \div 0$ is undefined. Give Nancy a mathematical explanation for this.

$5 \div 0$ is undefined since $5 \div 0$ is really asking what times 0 is going to give you five. As we know any number times 0 is 0 so there is no number times 0 that gives us five. Therefore 0 is undefined

6. John asks you in math class one day why $4^0 = 1$. Give John a mathematical explanation for this.

$4^4 = 256$ $4^3 = 64$ $4^2 = 16$ $4^1 = 4$ $4^0 = 1$

As the exponent goes down by 1 the answers are divided by 4 each time, so if $4^1 = 4$ then to figure out 4^0 you would do $4 \div 4$ which equals 1.

As is readily seen, there are dramatic shifts in ability to engage in mathematical discourse in these two PSETs. Upon entering the course, the first teacher gave a very common mathematically inaccurate response among PSETs which makes effectiveness and appropriateness moot, while the second teacher was unable to justify the given statement. Upon leaving the course, both teachers effectively engaged in mathematical discourse that is commonly found in elementary curricula. These outcomes are not unique of course, but are shared throughout the data. However, there is another interesting aspect of the project design feature that the current report does not deal with: the attitudes and beliefs of the PSETs. The data shows a clear shift for some teachers in how they think about mathematics: many participants entered the course with responses that included some mention of a type of rule as a justification for a given statement, whereas their post responses seldom if ever talk about rules in mathematics. Beliefs and attitudes about mathematics and teaching it also surfaced in the interviews as some participants talked about why they felt it was important for teachers to know certain things based upon those beliefs. Again, this is not a focus of the current report, but it is indeed an avenue of exploration for future study.

Questions for the Audience

(1) What kinds of things would you like to see as teacher educators/researchers in the responses to the items? Why?
(2) Are there other ways of measuring the ability to engage in justification which in and of itself requires measuring the ability to engage in mathematical discourse? What are they? What are the advantages and disadvantages to each method?
(3) What are some of the most important topics in the elementary curriculum that you as teacher educators/researchers believe that no PSET should exit their content courses without being able to have a somewhat stable mathematical discourse in those topics?

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