Graduate students' pedagogical changes using iterative lesson study

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Abstract

Researchers at two universities implemented an iterative lesson study process with ten graduate student instructors (GSIs), five from each university's mathematics department. Over the span of two weeks, each group of GSIs met with a facilitator to collaboratively plan an undergraduate mathematics lesson, implement the lesson, revise their lesson plan, reteach the lesson to another class of students, and complete a final reflection. Using a multiple case study qualitative methodology, we thematically coded GSI consistencies and revisions to lesson planning during the iterative process according to the Principles to Actions national mathematical teaching practices. At both universities there were specific teaching practices that GSIs used throughout the iterative lesson study and specific teaching practices that GSIs revised. Identifying these teaching practices offers insight into the utility and value of iterative lesson study with graduate student instructors.

Keywords: *Lesson Study, Graduate Student Instructors, Multiple Case Study, Measurable Goals, Classroom Tasks*

Given that graduate student instructors (GSIs) serve as instructors of record for hundreds of thousands of undergraduate mathematics students each semester (Belnap & Allred, 2009; Lutzer, Rodi, Kirkman, & Maxwell, 2007), they can significantly impact the quality of mathematics instruction for freshmen and sophomores. Although many mathematics departments acknowledge the need to support mathematics GSIs' learning to teach (Belnap & Allred, 2009; Latulippe, 2009; Speer, Gutmann, & Murphy, 2005; Speer & Murphy, 2009), research on classroom practices of GSIs is severely limited (Speer, Smith III, & Horvath, 2010), and there are only a few studies that examine GSIs' classroom practices (e.g., Gutmann, 2009; Rogers & Steele, in press; Speer, 2008).

Lesson study is a well-established Japanese systematic inquiry into teaching practices where teachers collaboratively create, teach, revise, and reteach lessons to continually grow as educators (Fernandez, 2002). In the last decade, collegiate instructors have begun implementing lesson study (Cerbin & Kopp, 2011; Kaplan, Cervello, & Corcoran, 2009), noting its collaborative format helps instructors develop rich lesson plans and reflect on teaching practices with others. To aid GSIs in developing similarly valued teaching practices at the collegiate level, this study implemented an iterative lesson study with novice[1] GSIs where GSIs had to revise and reflect on their teaching. We posit that the reflections and revisions will actively engage GSIs with teaching theories, offering GSIs an experiential and collaborative foundation for understanding how to address pedagogical concerns that arise during teaching. To bolster current research in GSI pedagogy, this study identifies mathematical teaching practices (NCTM, 2014)

[1] In this paper, we reference *novice* to mean first-year graduate student instructor.

that GSIs noticed and changed when revising their lesson during the iterative lesson study process. Thus this research addresses the following question: *During the iterative lesson study process, how did novice GSIs revise their lesson design and what mathematical teaching practices did they use?*

Framework

Lesson Study Logistics

Lesson study encourages teachers to methodically examine and improve their effectiveness in the classroom (Fernandez, 2002). To this end, a group of teachers will collaboratively create a lesson plan, teach the lesson plan while collecting observable data from the class, and discuss and revise the lesson plan from the observations. These lessons typically focus on instruction (curricular and classroom management issues), students (prior knowledge, student engagement, and anticipatory reasoning), goals (measurable long and short term goals), and content (key concepts and tasks; Stepanek et al., 2007). Usually one member of the group will teach the class, while the rest observe; the group is encouraged to iterate this cycle.

A crucial difficulty associated with implementing lesson study stems from the group logistic dynamics: necessitating a common space and time to collaborate, a common time and setting to teach and observe the lesson, and an asynchronous lesson to allow for iterations (Stepanek et al., 2007). Typically, preservice secondary teachers cannot iterate the process due to logistical issues (Fernandez, 2002; Perry & Lewis, 2009). Working with mathematics GSIs helps alleviate these logistic difficulties because (1) they teach and learn on the same campus, (2) their availability is more predictable (many first-year GSIs take similar mathematics classes), and (3) mathematics educators facilitating lesson study can reserve a space and time for GSIs to meet. Thus lesson study is a viable professional development option for GSIs.

Lesson Study Collaboration

Lesson study also has the potential to develop GSIs' collaborative teaching practices. That is, researchers have shown that, after participating in a lesson study process, prospective and practicing secondary mathematics teachers were more likely to collaborate in the future concerning pedagogical issues (McMahon & Hines, 2008). Lesson study allows cooperation and collaboration to become part of the teaching process, which opens up avenues to creating a community of practice amongst teachers (Stepanek et al., 2007). This is a valuable tool for GSIs because often their learning of mathematics is evaluated by individual homework, course exams, and qualifying/comprehensive exams. Helping GSIs understand that their teaching can be more collaborative than assessments of their learning in their graduate mathematics courses can help develop a *community of practice* (Hart, Alston, & Murata, 2011) in mathematics departments.

Lesson Study with Graduate Students

Despite lesson study being initially adapted for K-12 classrooms, recent studies have included lesson study in many STEM graduate programs. In natural science labs, for example, the use of lesson study processes developed biology GSIs' inquiry-based teaching practices (Miller, Brickman, & Oliver, 2014) and chemistry GSIs' pedagogical content knowledge (Barry & Dotger, 2011). Using lesson study had a positive impact on undergraduate mathematics education by giving mathematics GSIs experience similar to a teaching practicum (Alvine, Judson, Schein, & Yoshida, 2014) and allowing GSIs to critically reflect on teaching (Deshler, 2015). However, in these studies, the lesson study process stopped without iterating the teaching process—without a *revise and reteach* opportunity. To address this significant lack of opportunity in the literature, this study offers the field a lesson study design (Table 1) that includes the iterative process to focus on GSIs' revisions as a means to improve pedagogy.

Lesson Study Measurable Goals

The use of clear instructional objectives is an important feature of the lesson study process and one that the facilitators emphasized because GSIs are asked to collaboratively plan a lesson when they often have little experience planning or teaching college mathematics. First-year mathematics graduate students also typically have limited prior experiences taking pedagogical courses (Speer, Gutmann, & Murphy, 2005). Hiebert, Morris, and Glass (2003) emphasize learning to teach by treating lessons as experiments, suggesting novice teachers need to have clear and measurable goals. A primary feature of this lesson study process revolved around GSIs defining goals they would observe and measure during their lesson to require GSIs to actively engage with student learning and not focus solely on their teaching presentation.

Method

Mathematics educators from two universities designed and facilitated this lesson study process, implementing the same process and analysis for both of the cases.

Participants

Ten GSIs from two different universities volunteered to participate. These GSIs formed two groups of five, and all participants and names for the groups are pseudonyms. From one university, five novice mathematics graduate students, who were also recitation instructors, volunteered to participate to help guide their transition to instructors of record the following semester. This group is called the *Calc* lesson study group.

The other group consisted of five novice mathematics and statistics graduate students from another university who participated as part of a mathematics pedagogy course. Although the completion of the lesson study process was required for course credit, participation in the research study was voluntary, and all graduate students participated. These GSIs were preparing to be instructors of record in the following semester and formed the *Stats* lesson study group.

Lesson study process

Researchers followed the same lesson study design (Table 1) using identical handouts for each session and framework to collect GSI data. Data sources included video, audio, and (undergraduate and graduate) student work. Through regular meetings over the span of two weeks, GSIs met with the facilitator to go through the lesson study process, implement the lesson, revise the lesson plan, reteach the lesson, and complete a final reflection (Table 1).

Table 1

Lesson Study Process with Graduate Student Instructors

Session	Time (hrs)	Description	Outcome of Session
Introduction	1	Introduce lesson study process (Stepanek et al., 2007), sign consent forms, & discuss logistics for teaching.	GSIs determined the course & section for the lesson.
Goal Writing	3	GSIs learn about conceptual and procedural goals, identify measurable goals for their lesson, determine how they will measure those goals, & identify how they will collect data to see if each goal is achieved.	GSIs stated goals, metric for each goal, & data collection methods written clearly.
Mathematical Task	3	Identify high and low level mathematical tasks (Smith & Stein, 1998), create appropriate tasks for each lesson goal, & integrate measurements for goals with tasks.	Task(s) created and aligned with learning goals, metrics for each goal refined in light of mathematical task, & sketched lesson design.
Lesson Plan	2	Integrate goals and tasks with lesson design using four-column technique (Matthews, Hlas, & Finken, 2009).	Four-column lesson plan with activities, anticipated student responses, anticipated teacher responses, & alignment with goals.
Initial Teaching of Lesson	1	Video recorded and observed by other GSIs. GSIs also walk around and take notes of student work.	GSI notes on lesson & measures of goals via observation form.

Revision	2	Discuss what went well, reviewed sections of initial lesson looking for changes, & identify what goals were met, why or why not, making necessary changes.	Identify if goals were met by measurable data. Modified lesson plan intended to meet all desired goals for second iteration.
Second Teaching of Lesson	1	Video recorded and observed by other GSIs. GSIs also walk around and take notes of student work.	GSI notes on lesson & measures of goals via observation form.
Reflection	1	Discuss what went well, what goals were met, & if changes were effective or not and why. Also, some reflection on the entire process.	Identify if goals were met by measurable data. Reflect on the value of the lesson study and what was learned through the process.

Both universities used the same lesson study sessions, handouts, and observation forms to compare GSIs' lesson-plan revisions and mathematical teaching practices. To focus on the research question, researchers kept track of how GSIs changed the lesson plan according to pedagogical issues through the iterative lesson study, as follows: Using a multiple case study qualitative methodology and *naturalistic inquiry* (Lincoln & Guba, 1985), researchers themed GSI revisions relative to their goals and tasks. When theming revisions, researchers referenced the eight mathematical teaching practices (MTPs) as described in the *Principles to Actions* (NCTM, 2014) because these nationally recognized practices are designed to "provide a framework for strengthening the teaching and learning of mathematics" (p. 9).

Results

For structure, each case study includes a brief description of the teaching setting, a table summarizing the lesson goals, revisions, and observations, and a description of each group's maintenances and changes in mathematics teaching practices. A commonality across both groups is that they did not change their goal statements, but modified the lesson design and planned questions to try to better meet their objectives.

Calc Case Study

Teaching Setting. Due to scheduling demands, Calc decided to teach a lesson on area between curves to students in Calculus I because 60% of the GSIs would be running recitation for this content in two weeks. First, Alfonzo taught the lesson to a lecture-sized class of 64, using group work with 16 groups of four. Second, Aaron taught the revised lesson to his recitation class of 32 students. Table 2 describes the three goals related to mathematical tasks the GSIs designed their lesson to measure.

Thematic Revisions. Calc decided that to observe and measure students' work during class, a group structure would be the most efficient. After the initial lesson, Calc chose not to modify the goals or group structure, only the examples and tasks, to address two pedagogical issues. Abe stated, "In the second example, Alfonzo gave them the intersection point which limited their understanding of how to find an intersection." Anna agreed saying, "If he hadn't given them the intersection points, they would have struggled in a good way." Since students struggled when finding intersection points, they ran out of time to evaluate their integral to find the area between the two curves (Goal 3).

Table 2

Calc Lesson Goals, Revisions, and Observations

Measureable Goal	Did GSIs Conclude the Goal was Met After the Initial Lesson? How did they know?	Revisions	Did GSIs Conclude that the Goal was Met After Second Lesson? How did they know?
Goal 1: Students will identify intersection points and determine which function is on "top" versus "bottom."	Partially. In a task, a majority of students were able to identify and discuss "top" and "bottom" functions, but were not able to identify intersection points because the intersection points of the two curves were given as the endpoints of the interval.	The endpoints for the tasks' intervals changed. One task became an open-ended question where students had to determine the endpoints and the intersection points of the functions.	Yes. The open-ended question forced every group discussions on how to determine intersection points of graphs which lead to meaningful group conversations about determining intersections.

Goal 2: Students will be able to switch the “top” and “bottom” functions to set up the region of integration.	Partially. Polling after a task, a majority of students understood the need to switch the “top” and “bottom,” but to set up the integral, they had to know the points of intersection (Goal 1)	Modified the instruction to illustrate how to more clearly identify intersection points of graphs algebraically.	Partially. Polling indicated half the students still struggled with multiple integrals and multiple points of intersection.
Goal 3: Students will be able to evaluate the desired integral to find the area between curves.	Inconclusive. Limited time had many groups of students not get to the last part of a task addressing this goal; results could not be determined.	More time given for the open-ended task and time was removed from the task with the modifications to the endpoints (Goal 1)	Yes. Seven of the eight groups evaluated the integral they created in the first task appropriately using the power rule.

To resolve these issues, Calc chose to make the first task an open-ended question to provoke meaningful discussions about what interval was appropriate to find the area between curves and how to identify intersection points of curves. Aaron hypothesized this change would better develop students’ understanding of when and how to set equations equal to determine intersection points of curves. Aaron’s hypothesis was proven true when all eight groups were heard discussing how to find the intersection points. As a result, Aaron’s students spent more time on the first task, but then applied their understanding of intersections to move more quickly through the remaining tasks, accomplishing Goals 1 and 3.

Calc held to their measurable goals, the use of mathematical tasks, and observational data to measure their goals throughout the entire lesson study process. Thus Calc maintained established mathematical goals to focus learning (MTP1), implemented tasks that promote reasoning and problem solving (MTP2), and elicited evidence of student thinking (MTP8). Calc’s main revisions stemmed from their observations of student work on tasks. Thematically, their revisions modified the first task to be an open-ended question and more cognitively demanding. The modified task promoted small-group discussion and GSIs saw how modifications of tasks can facilitate meaningful mathematical discourse (MTP4), which helped efficiently facilitate other tasks and goals by building procedural fluency from the conceptual understanding (MTP6) of how to find intersection points of two curves. Although difficult at first, the revised tasks encouraged productive struggles in learning mathematics (MTP7).

Stats Case Study

Lesson Setting. Due to logistics and timing, Stats taught an introductory statistics lesson on linear correlation. Sam was the instructor for the first lesson (50mins with 20 undergraduates) and Steve taught the second lesson (50mins with 21 undergraduates). Their study lesson included four goals and one mathematical task that incorporated four main activities (Table 3).

Thematic Revisions. Stats had students work in a variety of group structures: in pairs, in groups of four (by pairing the pairs together), and as a whole class so students could make sense of content with their classmates and Stat could observe and measure students’ work during class. After Sam’s teaching, Stats also did not change the goals or overall structure of the lesson (Table 3). They focused, instead, on modifying the lesson plan to address two pedagogical issues: student misconceptions and pacing of the lesson. First, Stats realized that students expressed an unanticipated misconception: Sarah observed that “a couple of people were confused that . . . it doesn’t matter what the slope is. Students’ reason for a graph with $r = 0$ was ‘because it’s a horizontal line’ . . . not because the points were really spread out.” Other Stats members agreed that students considered *r*-values as the slope of the line of best fit rather than the descriptor for the strength of the correlation, leading to confusion about how to tell the strength of correlation in a scatterplot (Goal 3). To address this unanticipated misconception, Stats discussed how “it could have been explained better” (Sam). Suzie suggested incorporating an example during the introduction of the activity for Goal 3 where you have “two lines of best fit, both with the same slope but with different *r*-values, and see that the *r*-value is higher for the one with the points closer to the line." Steve incorporated this suggestion in the revised lesson and students seemed to follow along with this explanation. However, Stats did not modify the associated task, nor their way of measuring if Goal 3 was met. Thus students still needed clarification to more thoroughly understand the differences between $r = -0.42$ and $r = -0.72$.

Second, Stats recognized a number of places where "the big thing for the lesson is . . . time management" (Suzie) because Sam's discussions took longer than anticipated, leaving insufficient time for the closure activity (Goal 4); thus, they revised the lesson plan to address the pacing of the lesson (e.g., explicitly announcing when groups needed to wrap up part of the activity, grouping students up to make certain transitions smoother, and polling the class to indicate if it was OK to move on). Steve incorporated many of the group's suggestions (e.g., explicitly pairing and then grouping up students), which encouraged students to interact with their classmates and ask additional questions during the second iteration of teaching. Although pacing of the second study lesson improved, students still had less time than planned (~4mins) for the closure. Realizing they were running behind, Steve modified the participatory structure of the closure on the spot, encouraging students to work with their partner, thereby shortening the final activity in hopes of still addressing the final goal.

Table 3

Stats Lesson Goals, Revisions, and Observations

Measureable Goal	**Did GSIs Conclude that the Goal was Met After Initial Lesson? How did they know?**	**Revisions**	**Did GSIs Conclude that the Goal was Met After Second Lesson? How did they know?**
Goal 1: Students will be able to explain that correlation is not causation.	Partially. Correct responses elicited, but only from a handful of students (volunteered or called on). GSIs unsure if a majority of the class understood confounding variables and appropriate conclusions.	Additions to lesson plan for instructor: (1) ask students to explain their reasoning during whole-class discussion, and (2) poll the class to ask everyone to indicate (thumb up or down) agreement with conclusions or reasoning shared.	Partially. Discussion involved a wider variety of participants; students asked clarifying & contextual questions. Instructor pressed students for more examples, but felt pressed for time forgetting to use the polling strategy. More data needed to conclude how a majority of the class understood.
Goal 2: Students should recognize the correlation must be linear to calculate *r*.	Yes. Instructor called on each group, and each group shared at least one answer and all answers were correct.	Managing group dynamics: have instructor explicitly group pairs of students (from earlier in lesson) into groups of four to facilitate group work time.	Yes. The transition into groups of four went more smoothly, and responses from each group as they were called on were correct.
Goal 3: Students will be able to associate *r*-values with scatterplots	Partially. Groups mostly provided correct answers during class discussion, but expressed confusion about how strongly correlated the graph was and how to tell.	Amended lesson plan to include more explicit instruction about slope of a line being different from the *r*-value before students worked on this question to address the students' apparent source of confusion	Partially. The added explanation seemed to help guide the activity more clearly, and most answers from groups were correct values. Some confusion remained as to how to determine if a given scatterplot had an *r*-value closer to -0.42 or -0.72, for example.
Goal 4: Using provided technology (excel or statcrunch), students will be able to calculate *r* from two lists of numbers.	Inconclusive. Groups ran out of time and many only calculated some of the *r*-values. Many groups did not attempt to answer the questions about what conclusions could be drawn.	Timing recommendations: stressed areas where instructor could give students explicit instructions about how much time was left to move earlier parts of the lesson along faster.	Inconclusive. Time was still a factor so instructor encouraged students to work in pairs (fewer *r*-values to calculate), working through the open-ended closure more quickly. Insufficient data about a majority of students' understanding.

As was also observed for Calc, Stats maintained established mathematical goals to focus learning (MTP1), implemented tasks that promote reasoning and problem solving (MTP2), and elicited evidence of student thinking (MTP8). Stats' revisions stemmed primarily from their need to address an unanticipated misconception and desire to build in enough time to provide opportunities for students to generate their own understanding of relevant content relevant. The need to elicit and use evidence of student thinking (MTP8) was further reinforced by the fact that it was only through observing students' responses during Sam's teaching that the group was aware of the slope vs. *r*-value misconception. During Steve's teaching, his incorporation of specific examples pertaining to that misconception further reinforced the need for the group to elicit more evidence of students' thinking (MTP8) than what they planned to determine if this change helped address their goal. Through the modifications for the pacing of the lesson, Stats observed how modifications in student participatory structures could support meaningful mathematical discourse (MTP4) because students interacted with their group members more during Steve's lesson and raised additional questions about the material (Table 3, Goal 1, Column 4). Finally, it is important to note that Steve recognized the need to modify the closure activity on the spot because the group stressed the importance of making more time for the closure to help address Goal 4. Steve modified the closure activity in a way to encourage productive struggles in learning mathematics (MTP7) by maintaining the open-ended, cognitively demanding features of the activity, but encouraging students to talk with one classmate (instead of three) to make sense of the statistical ideas.

Discussion

By comparing two universities' iterative lesson study process, this study (1) identified *mathematical* teaching *practices* (NCTM, 2014) GSIs used consistently and revised, and (2) demonstrated the utility of collaboratively reteaching and revising lessons as a GSI professional development tool. The revisions and second teaching iterations have rarely been examined in lesson study literature. This iterative process provided an opportunity for both groups to implement the changes they deemed necessary to more clearly address their goals. Table 4 states the mathematical teaching practices GSIs used and revised during this iterative lesson study process. In Table 4, after each mathematical teaching practice, there is a reference to the GSI choices in Tables 2 and 3 that justify the coding.

Table 4

Iterative Lesson Study's Mathematics Teaching Practices Thematic Revisions

	Calc Lesson Study	Stats Lesson Study
MTPs with GSIs Throughout Both Lessons	(MTP1) Establish mathematics goals to focus learning (MTP2) Implement tasks that promote reasoning and problem solving (MTP8) Elicit and use evidence of student thinking	(MTP1) Establish mathematics goals to focus learning (MTP2) Implement tasks that promote reasoning and problem solving (MTP8) Elicit and use evidence of student thinking
MTPs that Changed with Revisions	(MTP4) Facilitate meaningful discourse, *Table 2, Goal 1 Revisions* (MTP6) Build procedural fluency from conceptual fluency, *Table 2, Goal 3 Revisions* (MTP7) Support productive struggle in learning mathematics, *Table 2, Goal 1 Revisions*	(MTP4) Facilitate meaningful discourse, *Table 3, Goal 1, Column 4* (MTP7) Support productive struggle in learning mathematics, *Table 3, Goal 4, Revisions & Column 4* (MTP8) Elicit and use evidence of student thinking, *Table 3, Goal 3, Revisions & Column 4*

To answer our research question, these results demonstrate this iterative lesson study process encouraged GSIs at both universities to consistently (1) establish mathematical goals to focus learning (MTP1), (2) implement tasks that promote reasoning and problem solving (MTP2), and (3) elicit evidence of student thinking (MTP8). At both universities, GSIs revised their lessons to facilitate meaningful discourse (MTP4) and support productive struggle in learning mathematics (MTP7). Useful future research could more specifically examine how the use of measurable goals, highly cognitive tasks, and the iterative process lead to similar mathematical teaching practices at both universities.

The results of this study indicate how revisions with the iterative lesson study lead to specific teaching practices being addressed. Thus, GSI educators can use a lesson study process to actively involve GSIs in learning about pedagogical concerns in undergraduate mathematics education prior to, or in conjunction with, GSIs learning about specific pedagogical topics and theories; thereby reinforcing or supporting GSIs' understanding of collegiate mathematics pedagogy. This study provides GSI educators with a format for iterative lesson study for GSIs as well as specific teaching practices that GSIs can gain via this teaching practicum.

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