Design research on inquiry-based multivariable calculus: Focusing on students' argumentation and instructional design

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In this study, researchers design and implement an inquiry-based multivariable calculus course as well as derive the characteristic of instructional interventions for enhancing students' argumentation in proof construction activities. Over the course of 14 weeks, 18 freshmen mathematics education majors participated in this study. Multiple sources of data were collected, students' reasoning in the classroom discussions were analyzed within the Toulmin's argumentation structure, and the instructional interventions were gradually revised according to the iterative cyclic process of the design research. The students' argumentation structures presented in the classroom gradually developed into more complicated forms as the study progressed, and the researchers conclude that the interventions were effective at improving students' arguments.

Key words: Design research, Multivariable calculus, Inquiry based learning, Argumentation, Flipped classroom

One of challenges in undergraduate mathematics classrooms is the shift from traditional teacher-centered and textbook-dominated approaches to new instructional approaches that are student-centered and inquiry-based (Holton, 2001). However, there is a shortage of studies that go beyond basic topics of calculus into areas such as multivariable calculus and differential equations (Rasmussen, 2014). Also, there is a lack of instructional tasks developed for inquiry-based learning (IBL) and a lack of research dealing with classroom interactions and the instructor's role in multivariable calculus teaching/learning. This study attempts to develop an inquiry-based multivariable calculus course and derive the characteristic of instructional interventions for enhancing students' argumentation.

In the fall semester of 2013, a multivariable calculus course for first year students majoring in mathematics education was organized as a flipped classroom at a university in Seoul, Korea. In the flipped classroom, instructors' explanatory lectures can be replaced by online video clips in order to assign more time to student inquiry during the face-to-face in-class sessions. The researchers applied the flipped classroom model to development of inquiry-based multivariable calculus course in order to provide students with opportunities for mathematical inquiry in the classroom as well as instructors' lecture in the online courseware.

In this paper, we focus on the design research methodology based on systematic qualitative analysis that the researchers applied to the development of the course in order to 1) understand the characteristics of students' argumentations in the proof construction activities in the inquiry-based multivariable calculus course, and 2) derive the characteristics of three sites of intervention for enhancing students' arguments: instructional design, classroom interaction, and the instructor's role.

Theoretical Background

Analysis on argumentation structure

Toulmin (1958, 2003) describes argumentation structures using six components for discourse analysis: claim, data, warrant, backing, qualifier, and rebuttal. In the meantime, van

Eemeren and Grootendorst (1992) suggest four types of patterns for argumentation structures: single argumentation, multiple argumentation, coordinate compound argumentation and subordinated argumentation, and Kwon et al. (2013) use these patterns to analyze the argumentation structure of the mathematics classroom, but they combine coordinate compound and subordinated argumentation into compound argumentation. A single argumentation structure includes only one claim and warrant, and a multiple argumentation structure contains a claim supported by more than one warrant. The compound argumentation structure includes a variety of warrants for supporting a claim that induces a new claim. In this study, the researchers adopt the framework consisting of these three argumentation structures to analyze the complexity of students' arguments.

Argumentation in general is understood as a process in which one's opinions are justified, or a discourse in which one convinces others of his/her opinion (Krummheuer, 2007; Wood, 1999). Argumentation can become more complicated when the antagonist reveals an unconvincing part of the given arguments, and the protagonist brings forward more arguments to meet this criticism. Consequently, some arguments may have a single argumentation structure while others have a multiple or compound argumentation structure (van Eemeren et al., 2007). Therefore, a more complicated argumentation structure shows that the students participate in more diverse discursive activities such as suggesting arguments, providing counterarguments, giving additional arguments or refuting counterarguments than a less complicated argumentation structure. In this study, the researchers consider the change in students' argumentation structures from single to compound as an evidence of the improvement of students' argumentation and justification.

Argumentation in mathematical inquiry

Inquiry-Based Learning (IBL) has been implemented in mathematics education in the form of problem-solving, the theory of didactical situations, realistic mathematics education, modeling perspectives, anthropological theory of didactics, and dialogical and critical approaches (Artigue, & Blomhøj, 2013). Since justification or persuasion in argumentation is recognized as being similar to theoretical demonstration in mathematics or mathematical proof, argumentation is considered to be an important part of mathematical learning (Krummheuer, 2007; Staples et al., 2012). According to Richards (1991), inquiry in mathematics is characterized by learning to speak and act mathematically through engaging in mathematical discussions, suggesting reasons, and following the process of solving new and unfamiliar problems. In this sense, Goos (2004) consider learning from an IBL perspective as participation in communities of mathematical inquiry, and Rasmussen et al. (2008) argue that inquiry enables students to learn new mathematics through taking part in genuine argumentation.

The complexity of an argumentation structure depends on the reactions between the arguments of the protagonist and the critical responses of the antagonists. The complexity of the argumentation structure grows as the discussion becomes more active (van Eemeren et al., 2007). Thus, argumentation structure analyses can serve as a quality criterion for mathematical inquiry through proof construction activities in IBL. Considering that learning in IBL is to learn to act and think like a mathematician, students' change of argumentation structure is a proper criterion for the students' learning in IBL. For this purpose, the researchers adopt an empirical approach to study students' arguments in the classroom, and use Toulmin's argumentation structure (1958, 2003) and the classification of argumentation structures suggested by van Eemeren and Grrotendorst (1992) as the frameworks of analysis.

Design research

Design research is appropriate when researchers develop innovative and complex teaching methods to implement unknown principles and guidelines, to quickly test an early model on site, and to refine such a model (Borgman et al., 2008; Kelly, 2009; Nieveen, & Folmer, 2009). Therefore, we adopted the designed research methodology to move them toward the goal of this study: to discover the characteristics of an instructional design that supports students' argumentation through in-class session discussions.

In design research, intervention involves the use of the curriculum, students' learning and teaching strategies, educational materials, and learning environments to improve students' ability to solve complex problems in a real educational context through repeated experiments. After researchers design and implement the interventions, they examine the educational products (e.g. student achievements) to determine whether they are able to answer their research questions. If the research questions are not answered with the current interventions, the researchers reflect on the educational products and improve the interventions (Plomp, 2007). The researchers consider instructional tasks, classroom interactions, and instructor's role as three elements of interventions and derive the features of these interventions from the literature reviews, which serve as a starting point to the iterative design process in this research.

The study aims to derive characteristics of the interventions for the multivariable calculus IBL classroom that induce the development of argumentation structure. This aim is addressed in the following research questions: 1) how do students present their argumentation in proof construction activities in the inquiry-based multivariable calculus course? 2) what are the characteristics of an intervention that improves students' argumentation?

Methodology

Research process

At the preliminary stage, results from previous studies on teaching/learning of mathematical content related to the subject of multivariable calculus, analysis of existing textbooks, and collegiate math education were analyzed in light of the current educational setting. At the design stage, tasks were developed based on each session's objective and content, and the researchers planned interventions for the in-class sessions based on anticipated characteristics on interventions. At the implementation stage, the researchers, consisting of one instructor and three research assistants, played the role of field participants for these in-class sessions. At the reflection stage, the researchers met to debrief on the implementation of the approach and the observation immediately after each in-class session. This approach enabled the gradual improvement of the interventions, and the cyclical process of the design research contributed to the final proposal of the characteristics of instructional interventions for inquiry-based multivariable calculus.

Settings

Over a total period of 14 weeks, the students observed two or three online video lectures (20–30 minutes each) and participated in one face-to-face in-class session (75 min) every week. The class was composed of 18 freshmen majoring in mathematics education majors who had taken the course "Calculus I" as a prerequisite, and a total of five small groups of three or four students each were set up for learner-centered discussions during the in-class sessions. Depending on the task at hand, laptops or tablet computers were provided for the students to use for discussion or problem-solving purposes.

Data collection and analysis

All in-class sessions were video-recorded and the reflection journals written by students after the session were collected. Additionally, a focus group interview with selected students was conducted at the end of the semester in order to complete the triangulation on the analysis. In this paper, the researchers analyze three in-class sessions that focus on mathematical proof construction activities in order to present a detailed account of students' argumentation structures. Two coders transcribed all the utterances of the students and the instructor, and coded the elements of students' discussions according to the components of Toulmin's argumentation structure. Afterwards, they cross-checked the argumentation structures of these components and reviewed the work sheets and the reflection journals in order to validate the results of the analysis. In order to validate the assumptions of the above questions, the researchers compared the Hypothetical Argumentation Structure (HAS) with the actual implemented argumentation structure and derived the characteristics of interventions by refining them in each cycle.

Result

The students' argumentation structures presented in the in-class sessions gradually developed into more complicated forms as the study progressed, and the researchers conclude that the interventions were effective at improving students' arguments.

Phase 1

The aim of the week's in-class session was to provide students with the opportunity to observe whether the symmetry of partial derivatives holds for two functions *f* and *g* and to examine several aspects of the functions, such as graphs, limits, and continuity, in order to inquire about the conditions that would satisfy the property. In the in-class session, however, the students could not reach the final step, in which they were to suggest their own conjectures about the symmetry of partial derivatives. In some steps, students had difficulties constructing their arguments as the researchers had intended, and the instructor had to directly convey certain mathematical knowledge to students that they were expected to be able to derive themselves. Finally, students could not performe well in the last two steps of the task, and the argumentation structure was also different from what the researchers had expected (Figure 1).

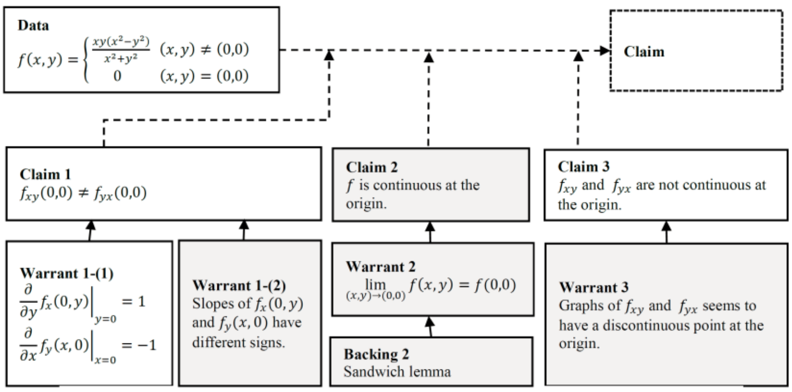

Figure 1: Student's argumentation structure in Phase 1

In the Figure 1, a solid line is used to represent stages of argumentation that students performed well and a dotted line is used to link parts of the students' argumentation that did not occur in the in-class session; shaded regions indicate parts that the researchers did not anticipate in the design stage or had to change spontaneously during the in-class sessions.

Phase 2

At the end of the in-class session in phase 1, the instructor had explicitly presented Young's theorem and the above lemma and asked students to suggest how it could be proved and to complete the proof of Young's theorem in their reflection journals using the MVT. Student S2 proposed an argument using the MVT twice, and the researchers decided to begin the discussion of how to prove Young's theorem in the fourth in-class session by sharing her idea with her peers. The researchers anticipated that during the session, students would point out some of the problems with S2's proof.

Students proposed three different ways, including S2's proof mentioned above. All proposals were based on the same idea, namely exhibiting the difference in terms of the function $D(x,y)$ and to determine when the concept of limit should be used in the proof. During the whole-class discussion, a multiple argumentation structure focusing on showing the validity of each proof and on comparison between them was observed (Figure 2).

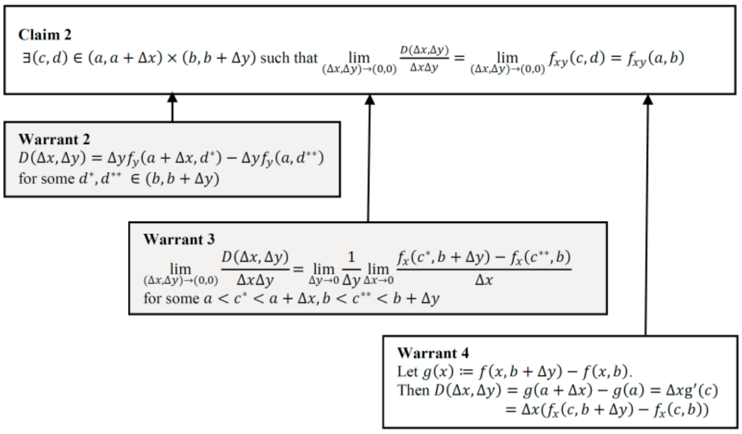

Figure 2: Student's argumentation structure in Phase 2

In this session, the more complicated task of proving Young's theorem was proposed, and a task sequence was implemented beginning with an incomplete solution. It seems that this approach — posing a relatively difficult question by incorporating a suggested idea — was more effective than simply providing students with the idea on its own without a specific starting point. By explicitly revealing the controversial point in the proof, the tasks enabled students to suggest multiple warrants for one claim in each small-group discussion, causing the whole-class discussion to result in a multiple argumentation.

Phase 3

In vector calculus, conservative vector fields can be defined in different ways, and most textbooks introduce the definition with several equivalent statements. The task asked students to prove that a potential function exists if the value of line integration is independent of the curve when the starting point and the terminal points are fixed. Researchers design the sequence of the task to construct a new function and examine the function to ensure that it satisfies the definition of potential functions. Although the instructor showed part of the proof to students in the online session to reduce their burden with this unfamiliar and complex task and to improve their concentration, she didn't provide students with individual steps to the proof. In other words, students need to find strategies to develop proofs by themselves.

In this session, the students' proof construction activity was implemented as expected in the HAS, but the instructor had to provide students with scaffolds to help them reach certain sub-claims. Therefore, the students' argumentation structure appeared in the form of the compound argumentation, but showed a slight difference in the shaded regions of the HAS. The shaded regions indicate the instructor's active engagement in the discussion (Figure 3).

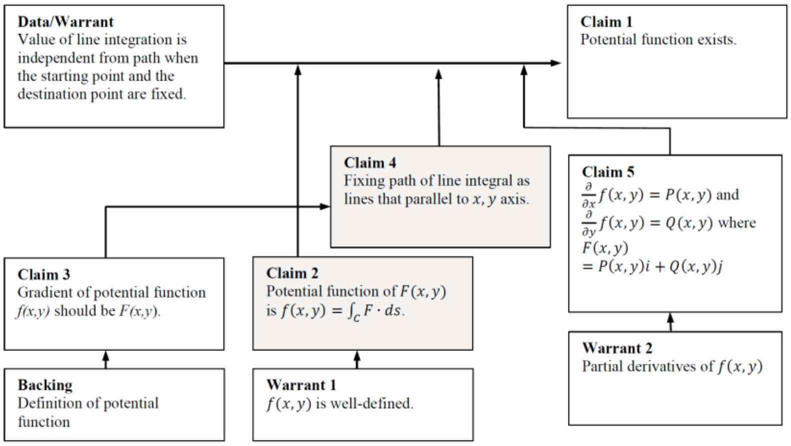

Figure 3: Student's argumentation structure in Phase 3

The main goal of the task in this session was to find and specify new ideas to accurately advance and complete the proof. While the task was described relatively clearly, it was difficult for students as it demanded several complex sub-claims and warrants, and promoted more elaborated arguments. Also, it led to active small-group discussions and required the instructor's engagements and discussions between small-groups. Therefore, the task contributed to the appropriate environment for IBL so that the students can construct the desired compound argumentation.

Conclusion

The students' argumentation structures presented in the sessions gradually developed into more complicated forms as the study progressed. That is, the structures transformed from single argumentation to multiple argumentation and compound argumentation structures as the interventions changed. The revised interventions employed in Phase 3 can exemplify the characteristics of interventions that are effective at changing argumentation structures.

Instructional tasks consist of sub-claim-based questions that can be used to provide students with room for inquiry to solve each question and to motivate them to take ownership of the entire proof construction process. Each question should be set at an appropriate level of difficulty in order to promote students' mathematical inquiry with discussion, and it should also provide the necessary prior knowledge, skills, and crucial idea required to help them find a valid orientation to their inquiry. Incomplete, but improvable solutions suggested by students can induce active student participation.

Classroom interaction should have a flexible structure consisting of within-small-group discussions, between-small-group discussions, and whole-class discussions. Students are encouraged to participate in whole-class discussions after sharing opinions with each other in small groups and reaching a similar degree of understanding.

The instructor should encourage students to argue for their ideas even when they could not definitively convince their peers of the validity of those ideas. The instructor should consistently monitor the discussions and take appropriate actions to indirectly guide students in the right direction in constructing their argumentations. In the environment with flexible interaction structures and open-ended questions, the instructor should re-organize classroom interactions and the tasks according to the students' progress observed in the discussions.

The iterative application and improvement of the interventions acquired in this study provided students with a structure in which they could participate more actively in whole-class discussions, while the instructor, who directed them to productively construct knowledge, played the role of facilitator of discourse. In addition, given a lack of explanatory lectures, the students were able to solve inquiry-based tasks in small groups and draw conclusions regarding the solutions in whole-class discussions during in-class sessions. Overall, the students participated responsibly and productively in knowledge construction and learning, as confirmed by the gradual development of their argumentation into more complex structures.

Discussion

In this study, the researchers focus on the design products and the design principles in the inquiry based multivariable calculus course, which are derived from the systematic qualitative analysis on students' reasoning in argumentation. The complexity of the students' argumentation structure serves as the quality criterion for optimizing the interventions in the IBL multivariable calculus course. The systematic qualitative analysis based on the well-established theoretical framework contributes to the methodology of this research, which assures the effectiveness of the design products from the empirical data.

The design research methodology thus made a clear contribution to the development of a multivariable calculus course based on the flipped classroom model and the pursuit of IBL. Using a cyclic process of design research, researchers design, implement, and reflect on the curriculum and instruction in order to validate their assumptions about three instructional interventions based on evidence from practice. This implies that design research can be beneficial to the many instructors who have troubles designing effective instructions without sacrificing the quality of education due to a lack of well-established design principles or practical guidelines.

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