Secondary teachers confronting mathematical uncertainty: Reactions to a teacher assessment item on exponents

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Teaching is inherently uncertain, and teaching secondary mathematics is no exception. We take the view that uncertainty can present opportunity for teachers to refine their practice, and that undergraduate mathematical preparation for secondary teaching can benefit from engaging pre-service teachers in tasks presenting uncertainty. We examined 13 secondary teachers' reactions to mathematical uncertainty when engaged with concepts about extending the domain for the operation of exponentiation. The data are drawn from an interview-based study of items developed to assess mathematical knowledge for teaching at the secondary level. In our findings, we characterize ways in which teachers either denied or mathematically investigated the uncertainty. Potential implications for instructors include using mathematical uncertainty to provide an opportunity for undergraduates to learn both content and practices of the Common Core State Standards. The proposal concludes with questions addressing how undergraduate mathematics instructors could use uncertainty as a resource when teaching preservice teachers.

Key words: Preservice Secondary Teachers, Uncertainty, Algebra, Teacher Assessment

Overview and Research Questions

This study attends to how cases of mathematical uncertainty can be used to elicit teacher thinking in ways that might help them learn to engage in and model mathematical practices for their students. The research questions are:

- How did teachers respond to a mathematically challenging teacher assessment item?
- How the student work in the item influence the teachers' responses?

In this paper, we discuss an example case of a teacher assessment item that creates mathematical uncertainty, and a set of teacher responses illustrating patterns of thinking that emerged as those teachers reasoned about the situation.

Theoretical Perspective: Uncertainty

"Teaching is evidently and inevitably uncertain" (Floden & Buchmann, 1993, p. 373). Uncertainty in thought, encountered while teaching, can be conceptualized as cognitive "perplexity, confusion, or doubt" (Dewey, 1933). Sources of uncertainty are ubiquitous in teaching and the teaching environment, and range from instructional content or teacher authority (Floden & Buchmann, 1993) to student traits or school culture Labaree (2003).

While the literature reflects clear consensus that uncertainty is inevitable because of the complexity of teaching (Cohen, 1988; Floden & Buchmann, 1993; Helsing, 2007; Labaree, 2003; Zaslavsky, 1995), there is less agreement about whether either the existence of or the ways that teachers respond to that uncertainty should be considered a good thing (Helsing, 2007). Helsing (2007) describes two schools of thought, one of which characterizes uncertainty as a deficiency and another that characterizes it as productive in teacher learning. In other words, uncertainty could be a signal of systemic dysfunction or teacher underpreparation, but uncertainty can also be something that would be productive to celebrate rather than avoid. Denying uncertainty may restrict teachers' opportunities to look for alternative teaching methods and in turn limit students' learning (Cohen, 1988; Helsing, 2007). By pretending everything is certain and under control, teachers potentially lower their standards so to mask potential ineffectiveness, establishing routines that increase predictability, accepting status quo to maintain security, and blaming students, other teachers, parents, or society for students' failures (Helsing, 2007). These attempts decrease the opportunities teachers have to enhance their own practices. When a teacher confronts uncertainties and accepts that teaching is open and fluid, the chance to develop their practices and their subject matter knowledge increases (Floden & Buchmann, 1993; Labaree, 2003). And we might expect this to be as true for more expert teachers than for more novice teachers; Floden and Chang (2007) suggest the metaphor of a jazz score for teaching, where certain frames of reference can be nailed down and others are, of necessity, open to creativity and interpretation, and in fact a sign of expertise is the ability to make use of uncertainty rather than the ability to avoid uncertainty.

In this paper, we focus our attention on teachers' mathematical uncertainty. Most literature on uncertainty in teaching attends to general sources that have less to do with subject matter knowledge and more to do with the complexity of teaching itself (exceptions include Rowland (1995) and Zaslavsky (1995)). This may be due to the perception that teachers can deal with and resolve uncertainty around the subject matter by further studying it and simply learning the subject better. However, as Lakatos (1976) argues persuasively, mathematical knowledge can be under continual revision. In other words, the subject matter itself may be uncertain beyond whatever uncertainty a teacher may have due to not understanding it fully. Therefore, mathematical uncertainty can be as irreducible as other uncertainties. For the purposes of this paper, we conceptualize mathematical uncertainty following Zaszlasky (1995) as any mathematical situation in which competing claims, an unknown path or questionable conclusion, or non-readily verifiable outcome occurs. Uncertainty is both a condition of the situation (that something cannot be known) and the associated emotions. As Mason (1994) posits, "emotion is harnessable". Here we examine one instance of how the experience of uncertainty may be harnessable for learning to attend to a particular mathematical issue, that of domain extension.

Data, Method, and Analysis

We drew on a subset of data from a larger study focused on validation of secondary-level items for assessing teachers' mathematical knowledge for teaching (MKT) (Ball & Bass, 2003; Ball, Thames, & Phelps, 2008). Researchers in the larger study conducted retrospective cognitive interviews (Ericsson & Simon, 1985) of more than 20 secondary mathematics teachers on a subset of assessment items, in which the participating teachers were asked to talk aloud about their reasoning processes in responding to the items. The interviews were audio recorded and transcribed.

For this analysis, we focused on 13 teacher interview transcripts responding to a particular item from the set. This item (see Figure 1) asks teachers to examine the validity of two samples of student work where students have been asked to evaluate the expression $((-9)^{1/2})^2$.

Ms. Williams is reviewing a set of homework problems on which students were asked to evaluate exponential expressions, including the expression $\left((-9)^{\frac{1}{2}}\right)^2$. Ms. Williams asks two students to share their work.

For each of the following student's work, indicate whether it demonstrates a valid application of the laws of exponents to solve the problem.

	Valid Application	**Not Valid Application**
Craig said: I used the exponent rule to change the order of the squared and the one half because that way you don't have to take the square root of a negative number. $(-9)^2 = 81$ Then the square root of 81 is 9.		
Katlynn said: I did it an easier way. 2 and $\frac{1}{2}$ cancel, so it's just -9.		

Figure 1. Ms. Williams item. (Copyright @ 2013 Educational Testing Service)

In terms of the underlying mathematics, the item seeks to assess whether the respondent knows that the law of exponents $(x^a)^b = x^{ab}$ does not hold in general, for instance when x is a negative number and a or b is non-integer. One student work sample has reached the 'correct' answer of –9 and the other has reached an incorrect answer of +9, but both have over-generalized the above law of exponents and applied it in a situation where it is not appropriate. In other words, the student whose answer is correct reached a correct answer coincidentally and not by use of a valid method, by overgeneralizing a rule that applies when both the base and the exponent are positive integers. The situation is additionally complex in that, while one student has reached the 'correct' answer of –9, the mathematical justification for this being the 'correct' answer depends on the use of complex numbers, and the item describes the students as not yet familiar with complex numbers. Arguably, if limited to real numbers, it might be more 'correct' to state that the expression is not defined. This example resembles one of Zaslavsky (1995)'s examples of competing claims, taken from Tirosh and Even's (1997) study, which discusses different mathematically-substantiated possibilities for $(-8)^{1/3}$. In the case here regarding squaring the 1/2 power of –9, the student work represents competing claims.

Responses to this particular item, because of the embedded mathematical uncertainty, and because many of the participants reported experiencing uncertainty or showed evidence of uncertainty, presented an ideal situation in which to examine patterns of reasoning in response to uncertainty. Transcripts were coded using grounded theory (Suddaby, 2006) for evidence of uncertainty, to characterize the nature of the response overall, and more specifically, to characterize the nature of the response to uncertainty. We paid particular attention to strategies that teachers engaged in as methods of resolving uncertainty, and to the valance they assigned that uncertainty when they noted it.

Potential Implications for Future Research

In this paper, we examined teachers' experiences of uncertainty, in particular, in the face of competing claims. One interesting observation about the teachers' different experiences was the range of certainty about the verifiability of the claims. Some teachers felt more certain than others about the verifiability of the answer than others. The engagement of those that felt less certain suggests that the perception of verifiability may play a role in how productive a situation of competing claims can be for learning. If there are competing claims, but one is perceived to be automatically more correct to the point of disregarding flawed reasoning to a valid conclusion, then engagement with the claims may be less productive than if claims are seen to be less automatically verifiable. Indeed, the point of this task was to engage in verifying the claims so as to be aware of how students may have overreached on the domain of the laws ostensibly applied.

Feedback and Suggestions for Future Directions for the Research

- One of our conclusions is that perceived verifiability may play a role in the productiveness of a task structured around competing claims. Does this conclusion seem plausible to you? What experiences have you had as a researcher or undergraduate instructor that might support or counter this conclusion? (The purpose of this question is to see whether this conclusion is reasonable enough to dig deeper into the analysis with this idea in mind.)
- If we were to investigate the role of verifiability further, what analytic frameworks or coding strategies would you suggest for looking into the data further?

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