A case study of a mathematic teacher educator's use of technology

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The use of technology in mathematics classrooms remains an important focus in mathematics education due to the proliferation of technology in society and a lag in the implementation of technology in classrooms. In this paper, I present data from clinical interviews with a mathematics teacher educator (MTE) and observations from that MTE's class in order to discuss his use of technology. Specifically, I describe three themes that emerged from the MTE's technology use and how they relate to his epistemological stance. These themes are: (a) his developing a classroom environment around the use of technology, (b) technology providing a precise and dynamic environment, and (c) his using technology to help engender students' mental imagery. Finally, I discuss how the ideas emerging from this paper can be helpful for the mathematics education community.

Key words: Technology, Mathematics Teacher Educators, Epistemology

"The use of technology to study mathematics has changed the very nature of the mathematics we are studying … [T]eaching and learning methods will need to be regularly reconceptualized to take advantage of the power of modern technology to improve mathematics education" (Leung, 2013, p. 523). In other words, technology provides us a way to transform the content we teach and the ways of thinking students construct. Instructional methods must be updated to include the tools we have available to us in ways that help students construct meaningful and sophisticated mathematics. In this paper, I expand upon Leung's notion: teaching and learning methods need to be regularly reconceptualized to take advantage of powerful technological tools. Specifically, I describe one mathematics teacher educator's (MTE) conceptualization and use of technology in a pre- and in-service education course focused on middle and high school mathematics content. I first provide relevant background information to this study and describe the methods used to collect and analyze data. I then provide background on the epistemological stance of the MTE and describe some of the main themes emerging from the MTE's use of technology. Finally, I close the proposal by discussing some productive takeaways from investigating the MTE.

Background

Over the past two decades, the proliferation of technology has been expansive and the use of technology in the classroom has been increasing as well (Kaput, 1992; Zbiek et al., 2007). This societal change raises several questions about how emerging technology can be implemented to support student learning. Pea (1985) argued that technology should be used as more than just an amplifier (i.e., making things faster and/or easier); technology should be used as a tool for reorganizing thinking. He posited, "I take as axiomatic that intelligence is not a quality of the mind alone, but a product of the relation between mental structures and the tools of the intellect provided by the culture" (p. 168). He also pointed out that much could be missed if we confine ourselves to defining technology as just a cognitive amplifier. Echoed by Leung's quote above, Pea argued that not only does the use of technology allow us to do things "faster" and more efficiently, it also changes the way that we engage in tasks.

Speaking to specific ways in which technology can influence the way we engage in tasks, Kaput (1992) differentiated extensively between traditional and dynamic computer media. Dynamic media offer a situation in which variation is easy to achieve. Geometer's Sketchpad

[GSP] (Jackiw, 2006) provides an example of a software program where variation is easy to achieve. Using GSP, a user can create sketches and then drag/modify parts of the sketch around the screen to see a variety of possibilities. This is much more tedious in a pen-and-paper drawing of the situation, as that medium requires the user to draw each instantiation. A program like GSP provides students opportunities to "see" such concepts as variation because instead of being constrained to static media and instantiations of variation, they can "experience" variation in experiential time. Repeated experiences with such activities can help students construct variation cognitively (Thompson, Byerley, & Hatfield, 2013).

Several researchers have conducted studies on how technology can help engender *student* learning in various content areas (e.g., Tall, 1986; Sinclair & Robutti, 2013; Geiger et al., 2012; Heid, Thomas, & Zbiek, 2013). However, there is still a need for studies that focus on how *teachers* implement technology and how their epistemological stance effects that implementation. Some researchers (e.g., Grossman, 1991; Margerum-Leys & Marx, 2002; Mishra & Koehler, 2006) have studied different variations of this issue. Specifically, Niess (2005) discussed the construct of technology pedagogical content knowledge (TPCK), in which teachers must develop an overarching conception of their subject matter with respect to technology and what it means to teach with technology. There is still much work to be done with respect understanding how teachers, including mathematics teacher educators, are using technology in their classrooms. This paper is an attempt to fill part of this void.

Subject, Setting, and Methods

The main participant in this study is a mathematics teacher educator (hereafter referred to as the MTE) employed as a professor in the mathematics education program of a large university in the southeastern U.S. The participant is an experienced member of the faculty at that university, working there for over 40 years. I chose the MTE strategically as someone who would provide me an opportunity to gain insights into his epistemology and how it shapes his teaching. I also wanted to choose an MTE who was teaching a class in which student learning and secondary mathematics content topics would be a main focus.

The study took place around the MTE's class, which met weekly for 2 hours and 45 minutes. There were 15 people in this class and they consisted of both Master and Doctoral students who had a variety of teaching experience (from only student teaching to currently teaching). These students normally formed 4 groups and worked together on many of the tasks during the semester. The majority of these tasks were completed on GSP. The class, which is labeled as a class on curriculum, focuses on student learning of many different mathematical content areas in middle and high school. The syllabus of the class includes the following topics: multiplication, counting/combinatorics, measurement and oriented quantities, rate of change, Pythagorean theorem and square roots, quadratics, parabolas. The observation period for this study consisted of the classes covering the latter four topics.

The study included a pre-interview, lasting about 90 minutes, in which I probed the MTE's views on teaching, the use of technology, and student learning. The crux of the data in the study originates from an eight-week observation (eight class sessions) of the MTE's class. During each of those eight weeks, I: (a) interviewed the MTE before class, (b) observed the MTE during the class, and (c) interviewed the MTE after class. The interview before class, which lasted between 30 and 60 minutes, provided a lens into the MTE's intentions for that class. The interview after class, also lasting between 30 and 60 minutes, allowed the MTE to provide his thoughts on events in the previous class. In some instances, I showed the MTE video clips from my observation to frame my questions. After the observation period completed, I conducted a 90-minute post-interview with the MTE during which I probed the

MTE's use of technology throughout the semester as well as asked some questions on his epistemological stance. All interviews conducted were semi-structured (Roulston, 2010).

I video- and audio-recorded all interviews and observations and digitized them after each event. Field notes were also taken during the observation sessions and screen-recording software was used to capture work done on the computer during those sessions. I analyzed the data following an open and axial coding approach (Corbin & Strauss, 2008). First, I watched the interview and observation videos for any utterances or actions by the MTE that stood out. Such utterances included specific mentions about how and why he would (or did) use technology, or instances in which he described his stance on knowing and learning. Actions targeted included the MTE's implementation of technology and interactions with students when using the technology. Then, I identified any themes emerging from the data to provide insights into the MTE's technology use. As I developed these themes, I repeated the analysis to revise or further clarify a theme if needed.

Results

I focus on three themes that emerged from the interviews and observations with the MTE. These themes are: *MTE developing a classroom environment around the use of technology, technology providing a precise and dynamic environment, and MTE using technology to help engender students' mental imagery*. Although these themes are listed separately, they are connected and each tied to the MTE's epistemological stance. Therefore, before providing an explanation of the themes, I describe aspects of the MTE's theoretical perspective that provide cohesiveness to these themes. First, the MTE identifies as a radical constructivist. As a radical constructivist, he follows von Glasersfeld's (1989) tenets that (a) "knowledge is not passively received, but actively built up by the cognizing subject" (p. 164) and (b) "The function of cognition is adaptive and serves in the organization of the experiential world, not the discovery of ontological reality" (p. 164). Taking this perspective means the MTE believes students cannot be passive receivers of mathematics as he understands it. Instead, they must construct a personally viable mathematics that is idiosyncratic and fundamentally unknowable to another. This stance will be evident throughout the description of the themes.

The MTE expressed his epistemological stance many times during the study. One example arose during a class post-interview. I showed him a clip from the class observation in which he said, "you can't take anything for granted. When you're teaching math you can't take your mathematics for granted." He clarified this statement in the interview (Excerpt 1).

Excerpt 1. The MTE's describes not taking your mathematics for granted as a teacher

MTE: What I mean by that is, so many teachers take their math- the way their thinking mathematically as that what should be learned … They take as a given what they believe is valued in their culture and sometimes that's- you know- what people say is valued in society. Their representation of that, that's what's taken for granted and I'm saying you can't do that, you shouldn't do that because the students have their own culture. Students have their own ways of thinking and you don't know what those ways of thinking are until you start working with them … Being a teacher, the primary challenge of being a teacher is to learn students' mathematics. To learn how they think and take those into consideration.

The MTE's words provide insights into the importance he places on (a) working closely with students and (b) helping teachers understand the significance of building models of students' mathematics. Although later in the interaction he explained that building models of students' mathematics is something that is very difficult to accomplish, he maintained that all teachers must try to accomplish this goal in order to productively support students' learning. Paying attention to how students learn shapes his teaching and consequently the way in which he implements technology. Dialogue in the post-course interview provides an example of his attention to student learning when planning to use GSP (see Excerpt 2).

Excerpt 2. Using GSP to help students learn.

MTE: Forcing yourself to work in GSP, although it's extremely detailed, I think brings out … the ways that you think and you almost- you have to start being more explicit in how you put things together mathematically in GSP. That's one of the basic reasons I want to use that. That's one of the basic reasons. Another basic reason is, is that when you think about the construction of a mathematical reality … This notion of a mathematical reality with the way you implement your thinking in GSP- that- I think it changes how you think mathematically and it changes your ways of thinking about the nature of mathematics.

His belief that GSP can be used to change how students think mathematically and how they perceive mathematics was pervasive throughout the observations of his teaching and the interviews. I discuss some examples in the description of the three themes below.

MTE Developing a classroom environment around the use of technology

An important part of the MTE's teaching when using technology was the emergence of a welcoming and mathematically productive environment shared by him and the students. To solidify this point during one of the interviews, the MTE said, "I want the students to develop a sense of ownership over the class so it's not [The MTE's] class, it's our class." Through this co-ownership of the class, the MTE hoped students would be actively engaged in tasks and discussing the mathematics with other students and the instructor. In this way, he hoped students would not only engage in their own mathematics, but they would also have to think outside of their own mathematics, considering those around them and the instructor as well.

Throughout the course of the observation period, students were grouped together and asked to work on tasks requiring the use of GSP. In class, the MTE stated, "I believe in my students working independently side-by-side. Does that make sense? So it's interdependent." I asked him to clarify what he meant by having his students working interdependently in a post-class interview. His explanation is found in Excerpt 3.

Excerpt 3. The MTE explains working interdependently.

MTE: Working together in groups of 2 or 3 provides them with uh, ways of- one student has an insight they explain it to the other students, the other students interpret and they may be able to assimilate what the other student is doing but yet by those assimilations be able to modify what- how they're operating and understand how they're operating but it also could provide this person that's there making the explanation- it gives them a way to uh, learn to communicate with other people mathematically and start carrying on mathematical conversations.

Such conversations were the norm, not the exception during my observation of the class. To understand why he feels such communication is important, we can go back to his epistemological stance. In the process of assimilating what another person has done, a student needs to consider another's mathematics and how it might relate to one's own mathematics.

Another aspect of the MTE developing a classroom was the idea of having students present and discuss their work. There were very few instances in which the MTE lectured to the class for an extended period of time. Instead, he let the students take the lead in explaining the mathematics, allowing students to examine their (and other students') thinking more explicitly. An interesting discussion resulted from this strategy. After the MTE mimicked a student's directions on a construction, one student discovered an error in the construction and said, "So the way it's written there shouldn't it be x and y instead of x-prime and *y*-prime?" The MTE replied to the student by saying, "well I'm just doing what you told me. I'm your robot." This exchange provides evidence that he intended the students to take ownership of the class, both in its discourse and in its mathematical focus. He mimicked the actions suggested by the students and allowed them to discover the error on their own.

As the MTE continued to call on students to show their work, I asked him to explain how he decided which students to call on. One reason for his decision can be found in Excerpt 4.

Excerpt 4. The MTE explains his decisions on call students to the front.

MTE: One of my goals for the course is that they become explicitly aware of how they think … I want them to become more explicit with the way they are thinking and start explaining how they are thinking. But it's also important that I put people up there who have solutions so- so the people- so the class- oh my classmate's solving this problem. And that's really important that I understand that the classmate's solving the problem that makes it more possible for me and I'll work harder. It's a matter of energizing the students and setting goals so they become goal directed in their mathematical activity.

Like in the other examples in this section, the MTE described the importance of considering others' mathematics when working on a task. In the same interaction, the MTE discussed other ways he decided who would present solutions. One notable way was selecting someone who has not taught but will teach in the future in order to get them comfortable (in his words "building a social confidence") with communicating mathematics in front of a class.

Technology providing a dynamic and precise environment

Over the duration of the observation period, it became apparent that the MTE used GSP for both its dynamic and its precise nature. By dynamic, I mean that one can create a sketch, click on a point, and drag it around in ways determined by how the sketch was constructed. Similarly, aspects of the sketch can be animated. This allows students to see the changes in experiential time rather than requiring students to redraw every possible shape, as is the case in pen-and-paper constructions. Precise refers to the ability to, for example, create a circle and know that the shape created is representative of a circle (and not subject to the error of human hands). Although it is true the limitations of computers (e.g., the screen is made up of pixels) do create some error, such error is visually minimal. One cannot easily, by hand, draw a line with length of the square root of a non-square number. But, GSP allows students to do provide a near exact representation through a construction in which the students are hands-on.

Specifically, the MTE exploited the dynamism of GSP by developing constructions with the intention of helping students develop proving skills. One example occurred when the class was tasked with showing something is true for all cases. For instance, the class was asked to justify that the inscribed shape in figure 1 is a square. The students were then asked to drag the points of the inscribed square along the sides of the outer square. Is the shape still a square? Will it always be a square? The MTE explained the affordances technology provided in proof building, "In GSP, you can move this thing. Now you know that you can see all cases … it seemed to be visually palatable that it was true for all cases." Although he says later that a GSP construction does not represent a complete proof, it can help students generalize. A teacher only showing the base case and the inductive step would be taking his own mathematics for granted and perhaps ignoring the mathematics of students who have not yet constructed a scheme for generalization.

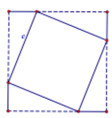

Figure 1. Square inscribed within another square

For similar reasons, the MTE felt the preciseness of the software was important in teaching his class. Specifically he felt that precision was pivotal in the unit covering square roots. I asked him to expand on the reasons GSP was important for that unit and how it compared to previous units. His response can be found in Excerpt 5.

Excerpt 5. The importance of preciseness

MTE: It's more essential to do these constructions because GSP offers you the power of being extremely precise. OK, and it does in the context of the other problem sets as well. … I can't do this easily without GSP. I really can't. Using GSP is much more essential here with the precision it offers you and the opportunities it offers you. So how can you construct a segment whose length is precisely the square root of two long, unless you go to GSP? You can't do it. I can't do it!

The preciseness was important for the MTE in other areas of the course as well. For example, the students were tasked to prove two lines were perpendicular (they were constructed as perpendicular in GSP). To do so, they needed to find the slope of both lines. The MTE said, "That was a difficult problem so GSP was a beautiful medium- I could have written it on the whiteboard but it wouldn't have been quite so dramatic because it wouldn't have been as precise. GSP provides you something that's precise." Through this precision, the MTE hoped that the students would be forced to become more explicit in their thinking and the use of GSP would perhaps change the very nature of the way they think of perpendicular lines.

MTE using technology to help engender students' mental imagery

The third theme emerging from the data, which relates to the dynamic affordances of technology, is the MTE's use of technology to help students develop mental imagery. This was an idea that permeated through the observation period. The MTE hoped to have students construct an imagery of the situation through use of the technology. This does not mean just remembering pictures. Instead the MTE wants the student to be able to reconstruct the image of the situation, including its dynamic nature, in his mind. In doing so, the MTE hopes this re-presentation adds constructive building blocks to the students' mathematical reality so the student can operate mathematically on similar situations in the future.

A specific example where helping students create and operate on imagery was an explicit goal occurred during the unit on rate of change. The unit relied heavily on reasoning about quantities and proportions (many tasks involved conversions between monetary units). The MTE discusses engendering students' mental imagery for rate in Excerpt 6.

Excerpt 6. Rates and imagery

MTE: One of the aspects of the rate scheme is the dynamic imagery. That you can regenerate the operations. So when this when-uh- this point moves [*referring to a point on a sketch*]. Well if you think about that as a motion, okay. Which we do. Well if we think about it conceptually if I can regenerate these two, the pounds and the dollars in relationship, then I- we're talking about mental operations. So what the motion becomes if I can regenerate the motion and produce the motion mentally, those are conceptual operations and no longer just a physical motion. And so that transformation is something I can't do for the students but that's what I'm trying to engender is this dynamic imagery.

The MTE's statement fits with the epistemological stance that students cannot passively receive knowledge. The teacher must be cognizant of students' mathematics in order to provide tasks and experiences that engender mathematical reasoning.

There are several other examples of attempts by the MTE to engender dynamic imagery that occur during the class periods (quadratic functions using area model, oriented quantities, etc.). One of the more dramatic examples in the class was the construction of the parabola.

The MTE showed them an animation in GSP that, when put in motion, created the envelope of a parabola. This animation not only provided a stimulating image but also showed the step-by-step construction. The class sounded a collective “oh!” during the first run of this animation. Although the “oh!” is nice, the MTE’s goal was for the students to operate on this imagery and productively re-present in a situation without the technology. In the post-course interview, the MTE expounded upon the importance of mental imagery (see Excerpt 7).

Excerpt 7. The importance of mental imagery.

MTE: By using GSP, you can develop this concept of mental imagery. So, in which I really think is important, you know, in all aspects of mathematics. This is something quantitative reasoning really has going for it because imagery is really- is really foundational. But it’s just not imagery. It’s how you operate on your imagery okay. And so you gotta spend a lot of time developing the imagery and how you operate on imagery.

Later in the interaction, the MTE relates this development of a mental imagery to his epistemological idea that a student creates his own mathematical reality. The construction of various imagery and the operations on that imagery help construct the students’ reality.

Discussion

At the beginning of this paper, I quoted Leung (2013) and agreed with the idea that using technology can transform the very content that is being taught. I contend that the MTE discussed in this paper provides an example case of where his conceptualization of and the students’ engagement with the technology transformed the content being taught and also the way in which it was taught including the environment of the classroom. The MTE made several considerations in his decision to use technology. Like Kaput (1992) described, technology can be used in dynamic ways. The MTE used the dynamic nature of GSP to help students generalize situations with infinite cases. Similarly, in conjunction with Pea (1985), the MTE attempted to use technology as a cognitive reorganizer by using it to help engender students’ mental imagery. The MTE intended for students to construct imagery in their minds and then operate on it in a way they can re-present later in a situation absent the technology.

Another important takeaway from this study is the importance of a teacher’s epistemological stance on the way in which he or she implements technology. If the teacher does not hold that student knowledge is actively built, that person is unlikely to see the importance in using technology as a way to engender dynamic imagery (for example). Nor are they likely to consider student interactions essential for student learning, as they may not have considered the importance of others’ mathematics. Although this may seem like a fairly obvious contention, I believe that one must consider a teacher’s stance before making any pedagogical recommendations. The writers of standards and recommended practices may tell teachers that using technology is important but if the teachers have not considered how students learn at any depth, then it would be hard to expect them to implement the practices in the way the writers intend. Thus, attention to how students learn should be of utmost importance in teacher preparation classes (as it was in this MTE’s classroom).

Finally, the study has implications on the community of MTEs, a group of people that has been studied minimally compared to other populations in the mathematics education field. MTEs are responsible for preparing PSTs and thus determining how to provide them with experiences that focus on ways of thinking important to the teaching and learning of mathematics is a pressing area of need. An MTE’s use of technology is a model for the PSTs he teaches. This paper provides examples of the different considerations MTEs must make when using technology.

References

Corbin, J. M., & Strauss, A. (2008). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (3rd ed.). Thousand Oaks, CA: Sage.

Geiger, V., Forgasz, H., Tan, H., Calder, N., & Hill, J. (2012). Technology in mathematics education. In *Research in Mathematics Education in Australasia* 2008–2011 (pp. 111-141). SensePublishers.

Grossman, P. L. (1991). Overcoming the apprenticeship of observation in teacher education coursework. *Teaching and Teacher Education*, *7*, 245-257.

Heid, M.K., Thomas, M., & Zbiek, R. (2013). How Might Computer Algebra Systems Change the Role of Algebra in the School Curriculum? In *Third International Handbook of Mathematics Education* (pp. 597-641). Springer New York.

Jackiw, N. (2006). The Geometer's Sketchpad. Berkley, CA: Key Curriculum Press.

Kaput, J. (1992). Technology and mathematics education. In D. A. Grouws (ed.), *Handbook of research on mathematics teaching and learning* (pp. 515-556). New York: Macmillan.

Leung, F. (2013). Introduction to Section C: Technology in the mathematics curriculum. In *Third International Handbook of Mathematics Education* (pp. 517-524). Springer New York.

Margerum-Leys, J. & Marx, R. W. (2002). Teacher knowledge of educational technology: A study of student teacher/mentor teacher pairs. *Journal of Educational Computing Research, 26*(4), 427-462.

Mishra, P. & Koehler, M. J. (2006). Designing learning from day one: A first day activity to foster design thinking about educational technology. *Teacher's College Record.*

Niess, M. L. (2005). Preparing teachers to teach science and mathematics with technology: Developing a technology pedagogical content knowledge, *Teaching and Teacher Education, 21*, 509-523.

Pea, R. (1985). Beyond amplification: Using the computer to reorganize mental functioning. *Educational Psychologist. 20*(4), 167-182.

Roulston, K. (2010). *Reflective interviewing: A guide to theory and practice*. London: Sage.

Silverman, J., & Thompson, P. W. (2008). Toward a framework for the development of mathematical knowledge for teaching. *Journal of Mathematics Teacher Education*, *11*, 499-511.

Sinclair, N., & Robutti, O. (2013). Technology and the Role of Proof: The Case of Dynamic Geometry. In *Third International Handbook of Mathematics Education* (pp. 571-596). Springer New York.

Thompson, P. W., Byerley, C., & Hatfield, N. (2013). A conceptual approach to calculus made possible by technology. *Computers in the Schools, 30*, 124-147.

von Glasersfeld, E. (1989). Constructivism in education. In T. Husen & N. Postlethwaite (Eds.), *International encyclopedia of education* (pp. 162-63). Oxford: Pergamon

Zbiek, R. M., Heid, M. K., Blume, G., & Dick, T. P. (2007). Research on technology in mathematics education: The perspective of constructs. In F. K. Lester (ed.) *Second handbook of research on mathematics teaching and learning* (pp. 1169-1207). Charlotte, NC: Information Age Publishing.