How well prepared are preservice elementary teachers to teach early algebra?

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As algebra has gained more attention in the K-12 curriculum, mathematics educators and policy makers have studied ways to support early algebraic thinking (e.g., Carraher, Martinez & Schliemann, 2008; McCallum, 2011). However, algebra in the early grades is sometimes misunderstood and is misrepresented as merely bringing algebra content down to the early grades (Kaput, 2008, p. 6). Instead of adding new content to an already packed curriculum, experts have suggested that elementary school teachers can support their students' algebraic thinking by being more selective and attentive to mathematical content as it is related to algebra during routine classroom discussions (e.g., noting that when you add two numbers, the order of numbers does not change the answer). Teachers can also support this thinking by considering ways to highlight algebraic connections and recognize patterns for generalization (Wu, 2001). This approach contrasts with a more traditional focus on computation and symbolic manipulation which Smith and Thompson (2008) consider a "fundamentally flawed" introduction of algebra, noting that "developing students' abilities to conceptualize and reason about situations in quantitative terms is no less important than developing their abilities to compute" (p.128). Therefore, even beginning elementary school teachers need to be knowledgeable about relevant algebraic content and what pedagogical choices will support their students in developing early algebraic thinking.

Although researchers theorize that early development of algebraic thinking is important for students' later understanding of algebra, the research base is not yet sufficient to identify what teachers know about their students' understanding of basic algebraic concepts (Asquith, Stephens, Knuth, & Alibali, 2007). This study builds knowledge by documenting the responses of a sample of preservice elementary teachers to a set of early algebra items designed to measure their mathematical knowledge for teaching (Ball & Bass, 2002). This will help us understand what knowledge such undergraduates have of the content needed to teach early algebra. We will use these findings to discuss whether and how teacher education programs across the nation are preparing undergraduates to teach early algebraic thinking. For this purpose we asked the following research questions:

- How do undergraduate preservice teachers interpret and respond to common patterns of student thinking in early algebra topics?
- What are the strengths and weaknesses among undergraduate preservice teachers in preparing appropriate materials to support students' early algebra development?

Conceptual Framework

We adopted the conceptualization of teacher knowledge as introduced by Shulman (1986) as pedagogical content knowledge, and later Ball, Thames, and Phelps (2008) elaborated on and operationalized as Mathematical Knowledge for Teaching (MKT). In this particular study we have attended to Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT) from the Ball et al. framework. KCS refers to teachers' knowledge of their students with respect to mathematics (e.g., common misconceptions or students' level of understanding). In KCT the content knowledge is related to teachers' knowledge of teaching (e.g., choosing a mathematically valid representation to use in introducing a concept).

This study aims to investigate undergraduate preservice teachers' knowledge in the domain of early algebra. In building an understanding of early algebra many researchers mention the importance of "Equivalence Statements" and how students are challenged to see the equal sign as indicating equivalence. Rather, students tend to understand the equal sign as indicating an action to carry out (e.g., Nathan & Koellner, 2007). In addition, the transition to algebra is related to gradual "symbolization of computations" (Kaput, 2008). The literature refers to the importance of having students attend to and be able to "use structure in solving problems" (Kaput, 2008). This kind of work can enhance students' algebraic thinking skills. In addition, to develop functional thinking students need to be able to attend to and make sense of variables involved in a problem and try to explain relationship between variables in a problem situation, often referred to as "relational thinking"(Carraher, Martinez & Schliemann, 2008).

Methods

We conducted 90-minute clinical interviews with 15 preservice teachers (PST) in their fourth year of a five-year long teacher preparation program. At the time of the interview participants were enrolled in the Teaching Methods in Mathematics undergraduate course, and three of the 15 were math majors.

These interview sessions collected the PST's responses to a series of 17 assessment[1] items designed to measure their content knowledge for teaching early algebra, with follow up questions probing their content-based reasoning. We also collected self-reported information about their preparation in this content area. Our initial coding of items was designed to separate those that focused on student thinking from those that focused on preparation for instruction. The algebra focus of these items included equivalence statements, symbolization of computations, using mathematical structure in solving problems, and relational thinking. The distribution of items in terms of algebra focus is given in Table 1. Algebra focus categories were not mutually exclusive; therefore categorization of the items reflected primary content focus of the items.

Table 1
Distribution of Interview Items According to Early Algebra Content

Primary Content Focus	Definition	Number of Items
Equivalence Statements	Items included meaning of the equal sign and equivalency statements, understanding of equivalency, and properties of equivalence relations (reflexive, symmetric, transitive).	6
Symbolization of computations	Items included the use of variables in solving problems, moving from one representation to another, representing verbal information in symbols, and illustrating a story problem by using a graphical representation.	3
Using Structure in Solving Problem	Items included using the mathematical structure of the problem in finding a solution. These items also focused on ways students can make use of properties of operations and identifying flaws in student use of operations (e.g., incorrectly commuting over subtraction).	7
Relational Thinking	Items assessed the ability to move from recursive thinking to general, or to characterize the relationship among variables.	1

[1] These assessment items were developed by researchers at Educational Testing Service.

As demonstrated in Table 1, the majority of the items' main focus was either equivalence statements or using structure in solving problems. An Equivalence Statement item can assess PST's evaluation of student work or can require the PST to consider examples to support their students' view of equal sign as a balance. An example of an Equivalence Statement item (where the PST needs to understand that equivalence is *not* highlighted by the examples provide) is given in Figure 1.

Mr. Keller's sixth-grade class is learning about algebraic equations. In his teachers' edition of the textbook, Mr. Keller finds a page that suggests he ask students to critique the following two solutions to determine if they are valid.

$4x + 2 = 12$ $6x = 12$ $x = 12 \div 6$ $x = 2$	$5 = 2x + 3$ $5 = 5x$ $5 \div 5 = x$ $x = 1$

Which of the following is most clearly highlighted by asking students to critique the invalid strategies?

- Understanding the meaning of the equal sign
- Understanding the importance of combining like terms
- Understanding the use of properties of operations to simplify expressions
- Understanding the use of inverse operations to solve equations

Answer the question above by clicking on the correct response.

Figure 1. Example of a released Early Algebra item.

With respect to assessing PST's use of Structure, an item might ask a test-taker to consider responses to simplifying the expression 7 – 3 +2. A common misconception would lead a student to evaluate the expression as 2 by adding 3 and 2 before subtracting (Hewitt, 2012).We would code an item asking the test-taker to interpret this kind of work as *using structure in solving problems* and having a pedagogical focus on *student thinking*.

Data Analysis

Interviews were recorded and were transcribed to allow for data analysis in NVivo. A team of researchers used a grounded theory approach with open and axial coding techniques (Glaser & Strauss, 1967) followed by constant comparative analysis (Miles & Huberman, 1994). These methods allowed us to arrive at a set of themes describing PST interpretation of common patterns of student thinking, strengths and weaknesses in PST use of algebraic concepts, and refinement of those themes by going back to existing literature and to the data.

Coding Framework

The coding framework was developed in multiple steps. First an initial framework with broader themes was developed and later refined by reviewing the data more closely and by revisiting our research questions. An initial round of item-level coding documented whether items required consideration of the equal sign, presented or asked for a student misconception, and/or presented student thinking. In addition, we worked together as a group to code three items and revised the coding framework before starting the pair-coding process. Analysis was conducted at the item level because items differed in the content they targeted. A sample of our coding framework that distinguishes PST understanding of the equal sign is provided in Figure 2.

Area/ Focus	Detail	Code (Node)	Definition
Mathematical Focus			
Equal Sign	Equal sign as a balance	M_EQ_Balance	This code is used for evidence that the PST understands or uses the equal sign as a balance or considers that both sides of the equation need to be equivalent.
	Equal sign as an action	M_EQ_Action	This code is used for evidence that the PST understands or uses the equal sign as an indication of an action or computation. (i.e., the equal sign is considered as a signal to produce an answer).

Figure 2. Example of the coding framework.

Coding for these two views of equal sign, for example, will help us to characterize PSTs' common view of the equal sign and how they use these views in interpreting students' work or addressing student misconceptions related to the use of the equal sign. For example, a response like the following:

> *... maybe they don't understand the equal sign means that both sides of the equation are going to be the same value so this side of the equation is going to equal be the same value as this side of the equation.*

was coded as M_EQ_Balance because this PST clearly noted a balance view of the equal sign.

Preliminary Results

The presentation focuses on findings that we believe have direct interest for RUME participants: (1) the study participants were least likely to answer correctly on items targeting the meaning and use of operational properties, (2) they struggled in evaluating the appropriate use of the equal sign when presented with different uses in student work; and (3) they reported that they had had few opportunities to learn about early algebra as mathematical content and as a topic to teach.

When solving the assessment items by "thinking aloud," some participants shared their embarrassment at not knowing the definition of the commutative property or the associative property. In other assessment items, for example, they showed that they knew that the order of numbers does not matter when adding numbers. In other words, the participants had the mathematical knowledge of the properties but lacked the knowledge on their names. Some participants also did not recognize that the use of multiple equal signs was problematic. In terms of opportunities to learn about early algebra, a number of participants reported that they did not have many chances to discuss early algebra as content to learn and content to teach in the previous four years. Although it is possible that they studied early algebra topics in their mathematics content courses and in their mathematics teaching method courses, the participants' lack of recall in this area suggests the need for more emphasis on early algebra.

Evidence of Impact

While this study was conducted at one institution and, therefore, is not intended to be representative of all programs, the number and types of mathematics and mathematics methods courses these undergraduate PSTs take are similar among teacher preparation programs nationwide. Our results are likely typical of the types of challenges other undergraduate PSTs would be expected to have. We will detail these challenges in the presentation, such as an appropriate use of the equal sign and a flexible and appropriate use of properties. In addition, we will talk about how it is important for teachers who are teaching undergraduate PSTs to provide a broader view of algebra and help their students to move from a "fundamentally flawed" view of algebra with a focus on computations (Smith & Thompson, 2008).

Research provided evidence that students can learn so-called difficult algebraic concepts and overcome their misconceptions with appropriate pedagogical choices (Hewitt, 2012). Therefore it is important to know what PSTs are in need of the most in preparing to teach early algebra. Such information matters because it can inform the development of teacher-education curricula and support materials. Participants in this presentation will get a summarized list of findings and an opportunity to discuss implications for designing courses for undergraduates who will become teachers.

Organization of the Session

In this session we will present the findings of the study and use excerpts from the interviews to allow teachers of undergraduate preservice teachers to characterize the mathematical knowledge and reasoning revealed in the interviews. In addition, we will have participants discuss what kind of curriculum and course work is needed so that undergraduates who will become teachers will be well prepared to teach this content area. We will present the following questions for consideration:

1. If these findings were indicative of a broader need for increased undergraduate instruction in algebra, where should responsibility for this instruction sit within the undergraduate program?
2. What opportunities are currently given to undergraduate students to use algebraic reasoning in authentic problem solving contexts?

References

Asquith, P., Stephens, A. C., Knuth, E.J., &, Alibali, M.W. (2007). Middle school mathematics teachers' knowledge of students' understanding of core algebraic concepts: equal sign and variable. *Mathematical Thinking and Learning*, *9*(3), 249-272.

Ball, D. L., & Bass, H. (2002, May). Toward a practice-based theory of mathematical knowledge for teaching. In *Proceedings of the 2002 annual meeting of the Canadian Mathematics Education Study Group* (pp. 3-14).

Ball, D.L., Thames, M.H., & Phelps, G. (2008). Content knowledge for teaching: what makes it special? *Journal of Teacher Education*, *59*(5), 389-407.

Carraher, D.W., Martinez, M.V., & Schliemann, A.D. (2008). Early algebra and mathematical generalization. *International Journal on Mathematics Education, 40*, 3-22.

Glaser, B. G., & Strauss, A., L. (1967). *Discovery of grounded theory: Strategies for qualitative research.* Chicago: Aldine.

Hewitt, D. (2012). Young students' learning formal algebraic notation and solving linear equations: are commonly experienced difficulties avoidable? *Educational Studies in Mathematics, 81*(2), 139-159.

Kaput, J.J. (2008). The nature of early algebra. What is algebra and what is algebraic reasoning? In J.J. Kaput, D. W. Carraher & M. L. Blanton (Eds.), *Algebra in the Early Grades* (pp. 5-19). New York: Lawrence Erlbaum Associates/National Council of Teachers of Mathematics.

McCallum, B. (2011, May 29). Progressions for the common core state standards in mathematics. K, counting, and cardinality: operations and algebraic thinking [Blog post]. Retrieved from http://commoncoretools.me/2011/05/29/complete-draft-progression-for-cc-and-oa/

Miles, M. B., & Huberman, M. A. (1994). *Qualitative analysis: An expanded sourcebook* (2nd ed.). Thousand oaks, CA: Sage.

Nathan, M. J., & Koellner, K. (2007). A framework for understanding and cultivating the transition from arithmetic to algebraic reasoning. *Mathematical Thinking and Learning, 9*(3), 179-192.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching, *Educational Researcher, 15*(2), 4–14.

Smith, J., & Thompson, P.W. (2008). Quantitaive reasoning and the development of algebraic reasoning. In J.J. Kaput, D. W. Carraher & M. L. Blanton (Eds.), *Algebra in the Early Grades* (pp. 95-132). New York: Lawrence Erlbaum Associates/National Council of Teachers of Mathematics.

Wu, H. (2001). How to prepare students for algebra. *American Educator, 25*(2), 10-17.