Perturbing practices: The effects of novel didactic objects on instruction

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Abstract. The advancement of technology has significantly changed the practices of numerous professions, including teaching. When a school first adopts a new technology, established classroom practices are perturbed. These perturbations can have both positive and negative effects on teachers' abilities to teach mathematical concepts with the new technology. Therefore, before new technology can be introduced into mathematics classrooms, we need to better understand how technology affects instruction. Using interviews and classroom observations, I explored perturbations in classroom practice as an instructor implemented novel didactic objects. In particular, the instructor was using didactic objects designed to lay the foundation for developing a conceptual understanding of rational functions through the coordination of relative magnitudes of the numerator and denominator. The results are organized according to a framework that captures leader actions, communication, expectations of technology, roles, timing, student engagement, and mathematical conceptions.
Keywords: Virtual manipulative, Classroom mathematical practice, Didactic objects

The advancement of technology over the past twenty years has significantly altered the practices and routines found in numerous professions. When a company first adopts a new technology, employees experience immediate changes, or perturbations, in their existing practices. These perturbations in practice can have small or large, short- or long-lived, and positive or negative effects on employees' ability to accomplish the work with the new technology. Some examples in which the adoption of new technology has led to perturbations in existing routines are found in the context of labor floors in which new machines are introduced and emergency rooms in which new medical equipment is implemented (Edmondson, Bohmer, & Pisano, 2001; Pickering, 1995).

In mathematics education, new technology is regularly being introduced into instruction (Pope, 2013). This technology comes in many forms such as hardware (e.g., computers or graphing calculators), software (e.g., Geometer's Sketchpad), or educational website licenses (e.g., Nearpod). The goal associated with the implementation of new technology in instruction is to facilitate instruction and improve student achievement and understanding. However, in order to achieve this goal, we need to better understand the process of adopting new technology in instruction. In particular, we need to account for teachers' current mathematical meanings of concepts, the perturbations experienced by teachers when implementing a new technology, and the effect these perturbations have on the instruction of mathematical concepts.

As a step along this path, this paper identifies perturbations that occur in classroom mathematical practices when an instructor uses novel virtual manipulatives to teach a concept for which there are already established instructional practices. In order to connect with the goal of introducing technology to improve student understanding, virtual manipulatives that were purposefully designed to support reflective mathematical discourse were chosen for the study. The observed perturbations in classroom mathematical practices are organized in a framework based on perturbations from industrial contexts when a new technology was adopted (Edmondson et. al., 2001; Pickering, 1995). Additionally, assimilation and accommodation (Piaget, 1967), cognitive conflict (Lee, Kwon, Park, Kim, Kwon, & Park, 2003), and covariational reasoning (Carlson et. al., 2002), were used to tailor the observations to a mathematics classroom.

Virtual manipulatives as didactic objects. Manipulatives are physical objects or concrete models that can be touched and moved around by the learner (Durmus & Karakirik, 2006). In mathematics instruction, manipulatives afford opportunities for learners to interact with abstract mathematical concepts and procedures through visualization and movement. However, we now recognize that the benefits of using manipulatives do not necessarily require the sense of touch, e.g., moving around physical objects. Now, a new class of computer-based manipulatives has been created (Durmus &

Karakirik, 2006; Moyer-Packenham, Salkind, & Bolyard, 2008), where a virtual manipulative is defined as a "web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer, Bolyard, & Spikell, 2002, p.373).

Research on the use of objects (physical or virtual) in mathematics instruction has traditionally focused solely on how the tool itself supports student learning and understanding in terms of cognition (Lee et al., 2003). However, there has been a shift to expand the focus beyond the object itself to include the accompanying discussion (Thompson, 2002). Accordingly, Thompson (2002) defines *didactic objects* as tools or objects that are created with the intent of supporting reflective discourse (p.198) and considers them to have two components: first, the object itself, and, second, the classroom discussion that engages students in constructing mathematical understandings. This study explores didactic objects that are designed to scaffold a conceptual understanding of rational functions.

Practices in and out of the classroom. Practices or routines are ways of doing things that are known and shared by a group of people as they engage in some activity. They are established over time and emerge as a group works together repeatedly to accomplish an activity. Changing the tools that are used in an activity, therefore, changes the associated practices, both in the long and short term. In the long term, tools can transform practices and significantly change the very nature of an activity. In the short term, the introduction of a new tool or technology can perturb established practices and lead to the adoption of new practices. For example, Pickering (1995) noted multiple disruptions in established practices on the labor floor and within management due to the adoption of numerically controlled machine tools by General Electric's (GE) Aero Engine Group in the early 1960's. Similarly, Edmondson et al. (2001) discovered disruptions in routines that occurred when minimally invasive cardiac surgery equipment was introduced to cardiac surgery in an emergency room. Table 1 contains a framework categorizing, summarizing, and providing examples of the perturbations in practice informed by research in industry.

Table 1. *Framework summarizing perturbations in practice in industrial contexts*

Aspects of practice	Description	Example
Leader Actions	Leader's interpretation of the technology and how the leader implements the technology	Edmondson et al. (2001) demonstrated how the surgeon's beliefs in the technology were correlated with how the ER team adapted to the technology.
Communication	The discourse and environment	In Edmondson et al. (2001), the discourse in the ER changed from the surgeon being the only speaker to every member of the team needing to communicate.
Expectations of Technology	Predicted outcomes for the implementation process	In Pickering (1995), prior to implementation GE management expected the technology to increase production.
Roles and Responsibilities	The individual's original responsibilities are altered during the implementation process	In Pickering (1995), the role of workers evolved from button pushers to integral members in the success of the machines.

If we consider a mathematics classroom, then the teacher and students together represent a team of individuals with a shared, collective goal of learning, and with the teacher as the team leader. The teaching practices that have been established over time in the context of the classroom by the teacher and her or his students in the course of their ongoing interactions (Cobb, Stephan, McClain, & Gravemeijer, 2001) and that may be disrupted by the implementation of new technology include pacing, student engagement, communicative norms, etc. For mathematics classrooms, such practices also include the emergent mathematical conceptions of the students as well as the mathematical understandings the teacher plans to cultivate within students (Thompson, 2013). Classroom mathematical practices, once established, are maintained through reflection and consistency. When a teacher reflects on the effectiveness of a practice, he or she is assessing whether the practice is effective in helping attain the goal of learning. Maintaining established classroom mathematical practices relies on the consistency of the teacher. Teachers continually reinforce classroom mathematical practices (for example through selecting tasks and activities) that engage students in productive ways and help them to build mathematical understandings.

Establishing and maintaining classroom practices is arduous work and takes a fair amount of time, effort, and coordination to accomplish, which makes changing practices difficult as well (Thomson, 2004). Humans are creatures of habit and easily fall into a set routine or practice for accomplishing specific tasks. Furthermore, if a practice is disrupted or forced to change, there may be an accompanying experience of discomfort. Teachers, like every working professional, easily fall into a comfort zone that is made up of carrying out established practices. When these established classroom mathematical practices are perturbed by a new technology, such as a virtual manipulative, the teacher may well experience disequilibrium or discomfort. Furthermore, the teacher beliefs of how the mathematics classroom should function, what mathematics concepts are important, and what resources are to be used for instruction can affect how the established practices are changed.

Rational Functions. Students are first introduced to rational functions in Intermediate Algebra, a course that is usually taken in high school. Traditionally, instruction centers on finding the asymptotes of rational functions algebraically. However, this calculational orientation does not provide students with a conceptual understanding of how rational functions behave. In particular, simply setting the denominator equal to zero does not capture the covariational relationship that exists between the two polynomials that make up the rational function. It does not support students' ability to see the relative magnitude of the numerator in terms of the denominator as a single quantity. This issue sets the stage for the adoption of a virtual manipulative that allows students to explore rational functions more dynamically and to construct a covariational understanding of how the functions behave near vertical asymptotes.

Methods

I focused on a single instructor (Elaine) and a pair of novel didactic objects for teaching rational functions. Elaine was a graduate student teaching Pathways Pre-calculus (Carlson et. al., 2013) and had taken a technology and visualization course, but was unfamiliar with teaching rational functions with the didactic objects used in this study. Therefore the didactic objects are considered novel to Elaine. The didactic objects (Rat Bar and Rat Graph) were accompanied by a teacher guide, containing display setting and questions to ask students to foster a discussion around four phases, as summarized in Table 2. In order to study the perturbations caused by the novel didactic objects, I used two pre-interviews, classroom observations, and a post-interview with the instructor. In the first pre-interview, which took place one week prior to the classroom observations, I explored the participant's current understandings of rational functions and gathered descriptions of the participant's instructional practices prior to the introduction of the novel didactic object. In the second pre-interview, which took place two days prior to the classroom observations, I guided the participant through an exploration of the didactic objects. The instructor was provided with a journal to record instructional preparations made following the second pre-interview. Two days after the pre-interviews, I conducted three real-time classroom observations covering the instruction on rational functions to identify moments of perturbations from my perspective. Using stimulated recall methodology (Stough, 2001), video clips of the moments I identified during the observations were then shown to Elaine during a post interview two days later so she could give a retrospective analysis of the instances that I had flagged as perturbations.

Table 2. *Phases of didactic objects*

Phase	Didactic Object	Purpose of the Activity	See Figure
1	Rat Bar	Assist students in conceptualizing and representing relative magnitude as a quotient of functions by comparing the relative lengths of the red and blue bars.	1a
2	Rat Bar	Assist students in internalizing relative magnitude as a quantity by no longer just seeing *two separate magnitudes* but instead seeing the relative magnitude of these magnitudes *as its own quantity*.	1b
3	Rat Bar	Assist students in coordinating the change in parameter values with the change in the relative magnitude.	1c
4	Rat Graph	Assist students in graphing the rational function by attending to changes in the relative magnitude of its numerator and denominator.	1d

Figure 1 depicts the implementation of the didactic objects for each phase shown in Table 2. In Phase 1, the teacher displays various lengths of two bars and asks students to provide a numerical guess of their relative magnitude (Figure 1a); in Phase 2, the teacher changes the length of the two bars and asks students to use the distance between their fingers to represent the changing magnitude (Figure 1b); in Phase 3, the teacher changes the length of the two bars and asks students to now use their fingers to coordinate the change of one magnitude relative to the other (Figure 1c); in Phase 4, the teacher shows students a graph of the numerator and denominator of a rational function and asks students to graph the resulting rational function (Figure 1d).

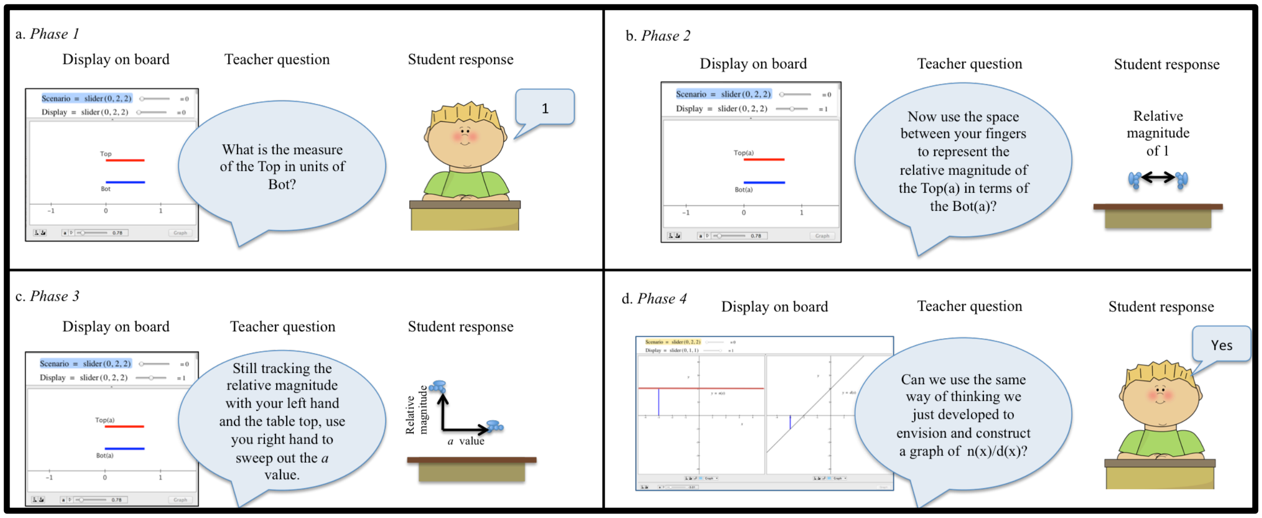

Figure 1. Four phases of using didactic objects for teaching rational functions

Results

The preliminary results of the study provided converging evidence for the aspects of practice that are perturbed when novel technology is introduced in the context of industry, e.g. leader actions, communication, expectations of technology, and roles/responsibilities (Table 1). However, there were also ways in which the novel didactic objects perturbed practices in the classroom that were not observed in industry. These included student engagement and mathematical conceptions, as shown in Table 3 which categorizes, describes, and provides examples of the aspects of practice that were perturbed as a result of the introduction of novel didactic objects.

Table 3. *Framework summarizing perturbations in practice in mathematics classroom*

Aspects of practice	Description	Example
Leader Actions	How instructor perceives novel didactic object and how the instructor uses the technology in the classroom	Elaine's introduction to the didactic object demonstrated her uneasy feeling toward trying something new.
Communication	Classroom discourse surrounding the novel didactic object	Elaine's students no longer relayed exact answers but instead they explained the behavior of the function.
Expectations of Technology	What understandings the teacher expects students to develop	Elaine had expected the novel didactic object to take the exact amount of time as her previous lesson.
Roles and Responsibilities	Responsibility for assimilating conceptual and procedural	Elaine's role was altered from lecturer to discussion facilitator.
Student Engagement	Student participation while the didactic object is being used	Elaine's students became more active in the lesson through the activities that accompanied the virtual manipulatives.
Mathematical Conception	How students perceive the mathematics addressed by the novel didactic object	The novel didactic objects change the emphasis of rational functions to behaviors rather than symbolic manipulation.

A possible reason for these additional perturbations in classroom practice (student engagement and mathematical conceptions) stems from differing expectations of technology. Industry adopts technology with the intent of increasing productivity and efficiency. In contrast, the purpose of using didactic objects in a mathematics classroom is reorganizational (Sherman, 2014) and supports the development of deeper understandings (Thompson, 2002). All of the sources of data collected in this study together point to the difficulties of accomplishing this task. Figure 2 displays two examples drawn from the classroom observations showing how two of the six aspects of practice, namely leader actions and mathematical conceptions, were affected by the implementation of the novel didactic objects.

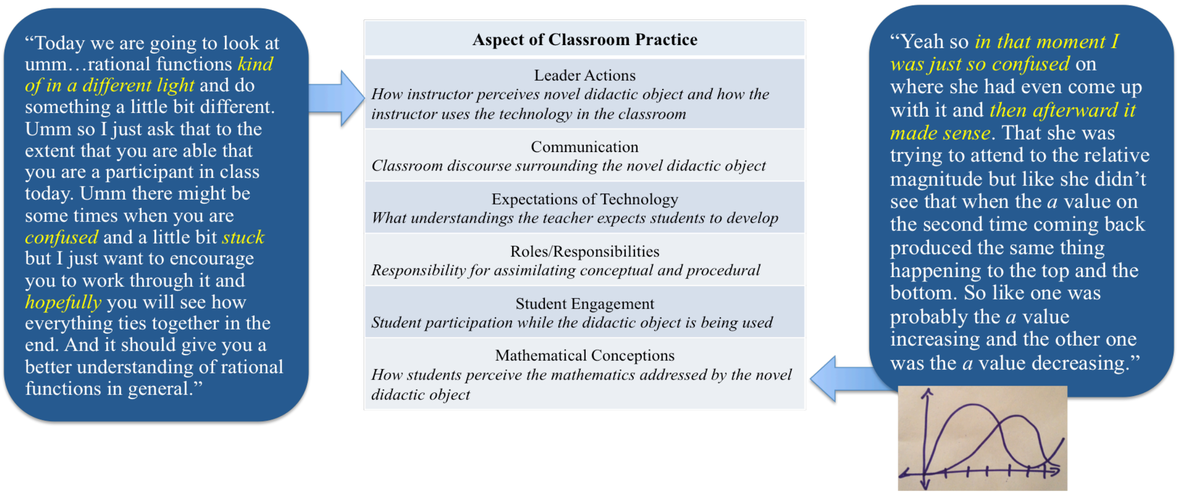

Figure 2. Examples of perturbations in classroom mathematical practices

Leader actions. In industry, it was noted that the introduction of new technology impacts the actions of the leader, which in turn affects how the team operates. This was also true in the observed classroom environment. In this case, the novel didactic objects caused Elaine to adopt a hesitant, foreboding approach to the upcoming lesson on rational functions. As seen in Figure 2 (left), her introduction of the didactic objects to the students sounded much like a parent trying to explain to a child that vegetables may not taste good but that they are good for your health. Thus, as the leader, she gave students plenty of reason to be wary of the upcoming lesson and mathematics, instead of exuding confidence and a belief in the value of conceptual understanding. This is noteworthy because teachers, as classroom leaders and role models, profoundly influence student beliefs in both the short and long term and thus ultimately shape the perception of what it means to understand and do mathematics (Thompson, 2013).

Mathematical conceptions. However, unlike what was observed in industrial contexts, in the classroom the introduction of novel didactic objects also perturbed the conceptions of those involved, causing new conceptions to emerge and unexpected conceptions to surface. Thus, as seen in Figure 2 (right) Elaine was baffled by the mathematical conception of one of her students when working with the class through Phase 3. In this phase, the students are asked to construct a graph of the relative magnitude of the numerator in terms of the denominator. Elaine admitted to being stumped in the moment when the student drew a graph on the board of two functions on the board. It was not until after the end of the class session that Elaine figured out the student's conception and how this was reflected in the presentation on the board. This is an example of how perturbations can lead to adding to key practices, in this case the practice of anticipating student responses (Stein, Engle, Smith, & Hughes, 2008).

Discussion and Conclusions

This preliminary work found converging evidence of perturbations found in industry (Edmondson et al., 2001; Pickering, 1995). Evidence from the study confirmed that novel technology caused perturbations in classroom practice with regard to leader action, communication, expectations of technology, and roles and responsibilities. Additional evidence was found to support the tailoring of the original framework to include perturbations in student engagement and mathematical conception.

Although this study has obvious limitations in scope, it sets the stage for delving more deeply and extensively into the ways in which novel didactic objects perturb classroom practices so that we can find ways to foster productive perturbations (e.g., supporting cognitive conflict and conceptual understanding) and mitigate the effects of less productive perturbations (e.g., transmitting a lack of confidence to students through the instructor's actions). The framework can guide development of interventions to smooth the path of using technology in classrooms. If teachers are more comfortable introducing and making use of new technology, we will be one step closer to improving student achievement and understanding.

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