Eliciting mathematicians' pedagogical reasoning

Christine Andrews-Larson
Florida State University

Valerie Peterson
University of Portland

Rachel Keller
Virginia Tech

Given the prevalence of work in the RUME community to examine student thinking and develop instructional materials based on this research, we argue it is important to document the ways in which undergraduate mathematics instructors make sense of this research to inform their own teaching. We draw on Horn's notion of pedagogical reasoning in order to analyze video recorded conversations of over twenty mathematicians who elected to attend a workshop on inquiry-oriented instruction at a large national mathematics conference. In this context, we examine the questions: (1) How do undergraduate mathematics instructors engage in efforts to make sense of inquiry-oriented instruction? (2) How does variation in facilitation relate to instructors' reasoning about these issues? Preliminary findings suggest that differences in facilitation relate to how participants engaged in the mathematics, and that the nature of participants' engagement with the mathematics was related to their subsequent pedagogical reasoning.

Key words: mathematicians, pedagogical reasoning, inquiry-oriented instruction

Given the prevalence of work in the RUME community to examine student thinking and develop instructional materials based on this research (e.g. Wawro et. al., 2013; Larsen, Johnson & Bartlo, 2013; Rasmussen & Kwon, 2007), we argue it is essential that our community consider issues related to the dissemination and use of findings from our research. This work aims to serve that goal by examining our efforts to engage practitioners (instructors of undergraduate mathematics) in thinking about research-based, inquiry-oriented instructional materials for undergraduate mathematics courses. In this preliminary report, we begin to explore two research questions: (1) How do instructors of undergraduate mathematics (who are interested in inquiry-oriented instruction) reason about instructional issues, particularly in the context of inquiry-oriented mathematics instruction? (2) How does variation in facilitation relate to the ways in which instructors engage in reasoning about these instructional issues?

Theoretical Framing and Literature

National organizations have called for instructional change in undergraduate STEM courses, relating poor instructional quality to a lack of student interest and persistence (e.g., Fairweather, 2008; PCAST, 2012; Rasmussen & Ellis, 2013). Researchers from a range of STEM fields have developed and documented student-centered instructional approaches that result in greater conceptual learning gains and student attitudes when compared with classes in which lecture is the dominant form of instruction (e.g. Kogan & Laursen, 2013; Kwon, Rasmussen, & Allen, 2005; Larsen, Johnson, & Bartlo, 2013). While it is well-documented that instructional change is difficult to achieve at scale (Henderson et. al., 2011), there is evidence that suggests a sizable number of faculty in undergraduate STEM fields are making efforts to offer their students the kinds of student-centered learning experiences supported by these studies. Indeed, though 61% of STEM faculty report they use extensive lecturing when they teach, a full 49% of STEM faculty report they incorporate cooperative learning into their courses (Hurtado et. al., 2012). Given the rates at which STEM faculty now report use of cooperative learning in their instruction, we argue that there is a pressing need to document and leverage the pedagogical reasoning of those faculty who are working to implement these kinds of instructional approaches.

In this work, we follow Rasmussen & Kwon's (2007) characterization of inquiry-oriented instruction in which students are actively inquiring into the mathematics (e.g. by developing, justifying, and generalizing their own solution methods to open ended problems) and instructors are actively inquiring into students' thinking about the mathematics so as to build on students informal and intuitive ideas to help them make sense of and engage in more formal and conventional forms of mathematical reasoning.

We take a situated perspective, in which we view knowledge and learning to be evidenced in the interactions among members of a community (Lave & Wenger, 1991) – in this case, the community of instructors of undergraduate mathematics. As such, we look to document mathematicians' pedagogical reasoning by examining their conversations about instruction. We follow Horn's (2007) characterization of pedagogical reasoning, considering it to be instructors' reasoning about issues or questions about teaching "that are accompanied by some elaboration of reasons, explanations, or justifications" (p. 46). Analytically, we draw on the vertices of the instructional triangle (teaching, students, and mathematics) as a conceptual tool for organizing our analysis of these conversations about instructional issues.

Data Sources and Methods of Analysis

The data under consideration in this study were collected from a workshop conducted as part of a national mathematics conference, and these data are part of a broader project that is developing and analyzing a set of instructional supports for undergraduate mathematics instructors interested in implementing inquiry-oriented instruction. The workshop focused on implementing inquiry-oriented instruction, and was organized around research-based curricula that have been developed in the areas of linear algebra, abstract algebra, and differential equations. The workshop lasted a total of four hours, which was split across two 2-hour sessions on consecutive days. On each day, about half of the time was devoted to content-specific work in breakout groups (self-selected by the participants), and the other half of the time was spent discussing issues of inquiry-oriented instruction that cut across all three curricula. On Day 1, facilitators planned to engage participants with an overview of inquiry-oriented instruction, followed by time to engage in mathematical tasks from the curricula in the area of their selected breakout group. On Day 2, the focus was to be on student thinking related to the day 1 tasks and instructional moves designed to help instructors implement inquiry-oriented curricula.

The workshop included 25 participants, 21 of which responded to a workshop pre-survey that provided us with information about their background and home institutions. All participants except one were housed in Mathematics departments, and the group represented a diverse group of institutions and positions (see Figure 1). Less than a third of survey respondents reported that they prefer to lecture most of the time, and more than 70% reported that they like to have students work in groups on problems in class, and more than 60% report they frequently ask students to explain their thinking to the whole class when they teach. This was significant to our research because it suggests our sample is part of the sizable subset of undergraduate STEM faculty working to teach in student-centered ways, and the choice to attend the workshop also points to an interest, outside the RUME community, in research-based instructional approaches.

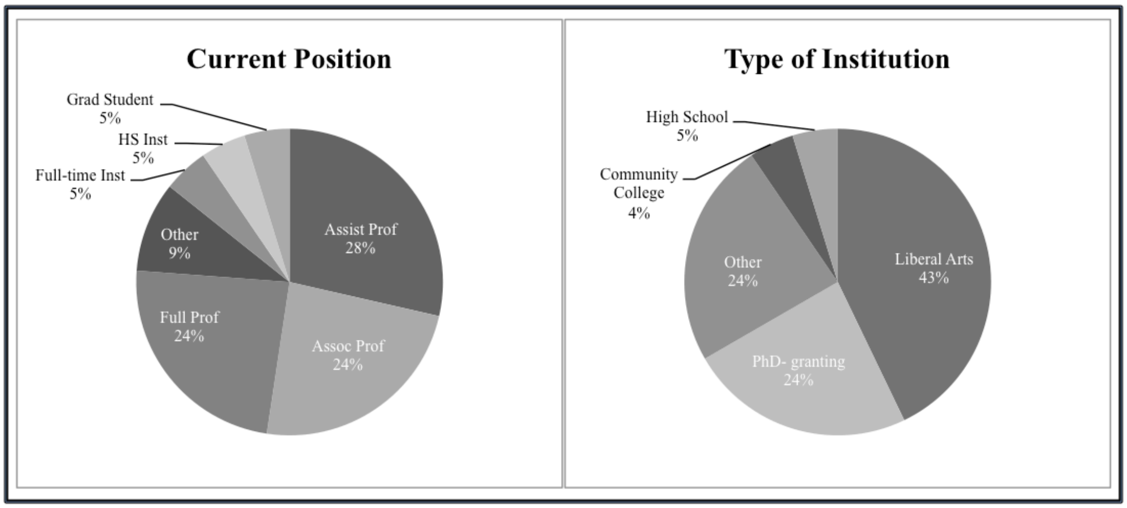

Figure 1. Position and Institution Types of Survey Respondents

In each of the two 2-hour workshop sessions, all whole-group and breakout segments were video- and/or audio-recorded for subsequent analysis. We began analyzing this recorded data by generating content logs to document the sequence of events in each segment of the workshop. Each content log was organized in a table with four columns: timestamp, description of events, focus of talk, and other comments. The 'focus of talk' column aimed to help us track whether the focus of talk was on the mathematics (M), the teacher (T), or students (S), and whether it was the facilitator or participants who were doing that talking. From these content logs, we generated summaries of each session to describe the focus and use of the time along with initial characterizations of the participants' pedagogical reasoning. We noted stark differences in the conversations of the linear algebra breakout group as compared to those in the abstract algebra group, so we decided to conduct our analysis as a comparative case study of these two groups (Yin, 2003). Content logs were then used to generate codes for the focus of participants' talk during the session and to identify conversational moments (selectively transcribed for closer analysis) when participants were engaged in pedagogical reasoning.

Findings

Our preliminary findings are two-fold: First, differences in facilitation appear to have played a role in how participants engaged in the mathematics on the first day of the workshop. Second, the nature of participants' engagement with the mathematics was related to their subsequent pedagogical reasoning. In this preliminary report, we focus primarily on the different ways in which participants in two of the breakout groups engaged in the mathematics on the first day of the workshop, and the variation in facilitation that may help explain those differences in engagement. In our presentation, we will provide further elaboration on differences in subsequent pedagogical reasoning of the two groups.

The facilitators of both the linear algebra and the abstract algebra group intended for the entire hour of the first day's breakout group to be focused on working through the mathematics in the respective task sequences. However, our content logs revealed the Abstract Algebra group spent a much larger portion of their hour-long breakout session on the first day of the workshop working through the math (86% of the time spent working through the math) than did the Linear Algebra group (30% of the time spent working through the math). Additionally, the nature of the mathematical talk of the two groups differed in that the abstract algebra group appeared to engage in the mathematics much more deeply than did the linear algebra group. We coded talk into six categories: logistics (e.g. "Does everyone have a handout?"), introductions, implementation questions (e.g. "How many times a week does your class meet?"), pedagogical moves (e.g. "I have them present their work as soon as we finish a task"), discussing mathematics (e.g. "They [students] came to different conclusions based on whether or not they considered all linear combinations"), and doing mathematics (e.g. "We want to show the additive inverse we'd expect from the big group stays in the small group."). Table 2 summarizes each breakout group's conversational focus on day 1 according to these categories.

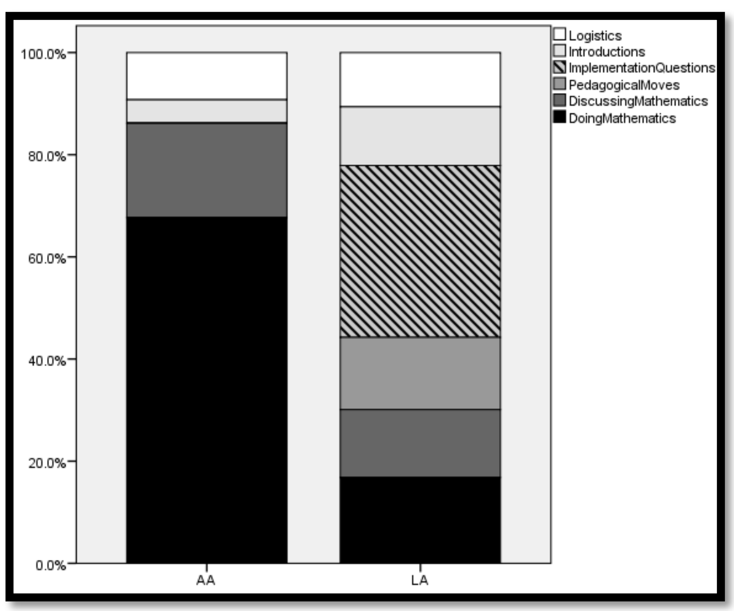

Figure 2: Conversational focus during Day 1 Breakout Groups

It was initially unclear why such differences in participants' engagement in the mathematics were observed. Analysis of the facilitation across the two breakout groups points to several factors that help explain these differences.

At the outset of the Abstract Algebra breakout session, the facilitator handed out the task statement and asked participants to turn and work together in small groups. As participants begin working, she circulated amongst them, listened, and occasionally chimed in with questions about their thought processes, methods, or mathematical assumptions. One participant later described the work of this breakout group, saying that the facilitator "modeled some instructor behaviors… pressing us on why we were thinking how we were thinking, pressing toward the subtlety without leading. We realized that because we understand abstract algebra, there are a bunch of similar ways to work with the definitions that are not obviously equivalent to a new user."

In the Linear Algebra session, the facilitator (who is one of the authors) also distributed the mathematical tasks for participants to work on, but devoted several minutes at the beginning to introductions in an effort to help create a safe, collaborative environment for sharing. However, this opportunity for participants to contextualize their institution (size and type of college, which students take linear algebra, etc.) also seemed to encourage speakers to bring up topics not directly related to the mathematics at hand (airing concerns about content coverage, logistics, student prerequisites, etc.). Additionally, there were moments in which the facilitator -- endeavoring to strike a balance between math instructor and professional development leader, and to respect the assumed content knowledge of her group members – inadvertently set norms that discouraged participants from fully engaging with the mathematics. Specifically, the facilitator began small group work by asking participants to begin by working on the second task in a sequence of related tasks, rather than starting with the first task, positioning the initial mathematical work as "easy." We posit that this created a setting with implicit professional risk: the participants were more or less strangers to one another, and several expressed feeling out of practice with the subject, so this potentially increased the pressure on those already feeling vulnerable. Later, in an attempt to refocus the group on the math, the facilitator suggested working through a task from the vantage point of a "typical student." Interestingly, instead of easing the mathematical pressure, this also created a barrier, with at least two participants later remarking that they didn't know how their students might approach the task. Finally, one of the participants had previously used the materials and was introduced as a potential resource for other instructors. Many of the digressions from the mathematics in the Linear Algebra group came in the form of specific questions directed to this participant.

Initial analysis suggests that this set-up up participants' mathematical engagement on the first day of the workshop was consequential for participants' subsequent pedagogical reasoning; in our presentation we will provide more detail on the differences in this subsequent reasoning.

Questions for Audience

- Might different content lend itself to participants engaging differently (e.g. linear algebra vs. abstract algebra)? If so, how can we account for this?
- Can we define or operationalize the kinds of tasks or activities that are productive for advancing participants' pedagogical reasoning?
- In what ways is it (or might it be) important for the facilitator to be inquiry-oriented in their stance toward participants' pedagogical reasoning (during workshop facilitation)?

References

Fairweather, J. (2008). Linking evidence and promising practices in science, technology, engineering, and mathematics (STEM) undergraduate education.*Board of Science Education, National Research Council, The National Academies, Washington, DC*.

Henderson, C., Beach, A., & Finkelstein, N. (2011). Facilitating change in undergraduate STEM instructional practices: An analytic review of the literature. *Journal of Research in Science Teaching*, *48*(8), 952-984.

Horn, I.S. (2007). Fast kids, slow kids, lazy kids: Framing the mismatch problem in math teachers' conversations, *Journal of the Learning Sciences, 16(1),* 37-79.

Hurtado, S., Eagan, K., Pryor, J. H., Whang, H., & Tran, S. (2012). Undergraduate teaching faculty: The 2010-2011 HERI faculty survey. Retrieved from http://www.heri.ucla.edu.ezp1.lib.umn.edu/monographs/HERI-FAC2011-Monograph-Expanded.pdf

Kogan, M., & Laursen, S. L. (2013). Assessing long-term effects of inquiry-based learning: A case study from college mathematics. *Innovative higher education*, 1-17.

Kwon, O. N., Rasmussen, C., & Allen, K. (2005). Students' retention of mathematical knowledge and skills in differential equations. *School Science and Mathematics*, *105*(5), 227-239.

Larsen, S., Johnson, E., & Bartlo, J. (2013). Designing and scaling up an innovation in abstract algebra. *The Journal of Mathematical Behavior*.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge university press.

President's Council of Advisors on Science and Technology (PCAST) (2012). Engage to excel: Producing one million additional college graduates with Degrees in Science, Technology, Engineering, and Mathematics. Washington, DC: The White House.

Rasmussen, C., & Ellis, J. (2013). Who is switching out of calculus and why. In *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 73-80).

Rasmussen, C., & Kwon, O. N. (2007). An inquiry-oriented approach to undergraduate mathematics. *The Journal of Mathematical Behavior*, *26*(3), 189-194.

Wawro, M., Rasmussen, C., Zandieh, M, Sweeney, G., & Larson, C. (2012). An inquiry-oriented approach to span and linear independence: The case of the Magic Carpet Ride sequence. *PRIMUS,* 22(7), 1-23.

Wawro, M., Rasmussen, C., Zandieh, M., & Larson, C. (2013). Design research within undergraduate mathematics education: An example from introductory linear algebra. In N. Nieveen & T. Plomp (Eds.), *Educational design research – Part B: Illustrative cases* (pp 905-925). Enschede, the Netherlands: SLO.

Yin, R. K. (2003). Case study research: Design and methods (3rd ed.). Thousand Oaks, CA: Sage.