Divergent definitions of inquiry-based learning in undergraduate mathematics

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Inquiry-based learning is becoming more important and widely practiced in undergraduate mathematics education. As a result, research about inquiry-based learning is similarly becoming more common, including questions of the efficacy of such methods. Yet, thus far, there has been little effort on the part of practitioners or researchers to come to a description of the range(s) of practice that can or should be understood as inquiry-based learning. As a result, studies, comparisons and critiques can be dismissed as not using the appropriate definition, without adjudicating the quality of the evidence or implications for research and teaching. Through a large-scale literature review and surveying of experts in the community, this study begins the conversation about possible areas of agreement that would allow for a constituent definition of inquiry-based learning and allow for differentiation with non-inquiry pedagogical practices.

Keywords: Inquiry, Inquiry Based Learning, Inquiry Oriented Learning, Definition

Over the past few years, a growing amount of literature has been published on undergraduate inquiry based mathematics education. This type of education puts the teacher in the place of a guide who has the role of asking thought provoking questions. From this, students learn by working through questions and frustrations to gain a deep understanding of a particular concept and reflect on what they just learned and what implications may be. The change in undergraduate teaching practices, in classes such as calculus and linear algebra that have traditional curricula, means that there is a concurrent growth in professional development, publications about teaching and curriculum, and research on inquiry-based instruction. Additionally, this increase in research, professional development and implementation has spurned an upcoming special interest group of the mathematical association of america in inquiry based learning.

However, despite the growth of published materials on inquiry, we argue that the term is not consistently defined, and some publications do not define it at all. The lack of a definition has been the source of some debate in the past. One published paper claimed that inquiry based learning does not work (Kirshchner, Sweller & Clark, 2006); however, criticisms of this paper centered on the fact that the authors misunderstood what inquiry based learning is and were over simplifying it as unguided discovery (Hmelo-Silver, Duncan, & Chinn, 2007). In this paper we survey the literature for uses of the term inquiry and survey current experts in undergraduate mathematics inquiry to learn how they define inquiry. In this survey some participants point out that people continue to over-simplify inquiry, a problem for the field if inquiry is going to be promoted as more effective than directive methods. This paper is the beginning of a discussion about defining inquiry based learning in an undergraduate mathematics classroom in order to allow for meaningful discussion and evaluation.

Background

Inquiry based mathematics education has become more popular in undergraduate settings. However, the term "inquiry" lacks a clear and concise definition. Instead, throughout recent literature, we identified six major themes when defining the term inquiry. These six themes are all distinct, and, they show up individually and in clusters in papers about IBL in undergraduate mathematics.

The first theme is student ownership of knowledge (student ownership), Johnson (2014) described the idea as "Learners regard the knowledge they acquire as their own personal knowledge they are responsible for". To add, in a separate paper, inquiry is defined as students being encouraged to create knowledge by themselves (Ko & Mesa, 2014).

The second theme is new knowledge building on existing knowledge (knowledge building). For example it is stated that "Inquiry mathematics allows students to… find new ways to use prior knowledge to understand equations" (Keene & McNeil, 2014).

The third theme is students participating in mathematics (doing math). Johnson added that inquiry is the "Expansion of what is experimentally real" while learning mathematical skills is "Synonymous with becoming a participant in the community" (2014).

The fourth theme is the importance of the student/instructor relationship (student/instructor relationship). This relationship, as many papers state, is crucially important because it "Enables instructors to have a deeper understanding of students and learning" (Ko & Mesa, 2014) and "teachers need to understand student thinking" in inquiry (Larson, Wawro, Zandieh, Rasmussen, Plaxco, Czeranko, 2014), a statement that arises from Rasmussen and Kwon's "Inquiry-Oriented Learning" (IOL), where the instructor inquiring into student learning is a key component (2007).

The fifth theme is the importance of student to student interaction (peer involvement). Rasmussen and Kwon explain how part of inquiry includes a student's' ability to "routinely explain and justify their thinking, listen to and attempt to make sense of others' ideas" (2007).

The sixth theme is better alignment with how people learn which leads to increased student success (student success). Overall, students who take inquiry based mathematics classes do better in other classes because they have gained the necessary tools to be able to decipher future problem sets (Mantini, Trigalet, Davis, 2014; Yoshinobu & Jones, 2012). A recent study showed some evidence that students in an IBL calculus class that covered fewer topics did at least as well as their directively taught peers when they took calculus II (Laursen et al, 2014).

These six themes appear to inform the concept of inquiry based mathematics education. With this, however, comes some debate on the definition of inquiry in regards to undergraduate mathematics education. In this study, our aim is to survey the entire undergraduate inquiry community to better understand how the community defines inquiry.

Methods

Literature search

To orient ourselves, we attempted to collect all papers about inquiry-based instruction, broadly defined, in undergraduate mathematics education. In order to do so, the second author searched a database mutli-search that included over 20 databases, including JStor, ERIC, Academic Search Premier and more using criteria such as, 'inquiry-oriented,' 'inquiry-based,''guided-discovery,' and 'realistic mathematics education' always in conjunction with undergraduate mathematics. Additionally, the second author searched the conference proceedings of the three most recent SIGMAA-RUME conferences. Moreover, we asked experts in the field for recommendations of other articles. For each identified article, the second author carried out two tasks; first identifying the author(s) and any instructors of inquiry-based courses and adding them to a list of undergraduate faculty who teach or do research on inquiry-based courses. The second action was to extract the definition of inquiry-based, inquiry-oriented, or guided-discovery (hereafter shortened to inquiry-based) that the author(s) gave in the paper. If the authors did not specify a definition, that was also noted. Based on the literature review, we noted that many papers took as unproblematic the definition of inquiry-based teaching. As a

result, the first and second author also attempted to characterize the instruction that was described. There appeared to be significant differences in the range of practices that authors, even of research papers, described as inquiry-based instruction, meaning, there appeared to be little agreement about the defining features and the types of experiences that students might have, thus making large-scale evaluation of the efficacy problematic.

Participants

The population of this study is any author of a peer reviewed research article or conference proceeding dealing with inquiry in the undergraduate mathematics classroom, a person who has experience in professional development of inquiry in undergraduate mathematics classroom or a person who has authored a textbook or support materials with inquiry in the undergraduate classroom in mind that the first and second authors identified via their literature review and subsequent snowballing technique. The authors identified 67 persons who fit in one of these 3 categories in an attempt to be as exhaustive as possible. All 67 members of the population were invited to complete the survey and 18 persons participated. The participants included 10 mathematics educators, 6 mathematicians and 2 STEM educators, 4 of the mathematics educators were doctoral students while none of the mathematicians or STEM educators were.

Survey and Coding

The researchers developed a nine question survey with 2 demographic questions addressing employment position and asking each participant to identify themselves as a mathematician, math educator, scientist, science/STEM educator or other. There were 7 free response questions centering on various aspects and understandings of inquiry. Survey responses were then independently coded by each researcher using a method most closely associated with grounded theory (Strauss & Corbin, 1990). Each researcher read through the responses keeping in mind the 6 categories identified in the literature review, student ownership, knowledge building, doing math, student-instructor relationship, peer involvement and student success. The research team developed a coding manual to identify when a response invoked a particular code. For example, we coded a response as invoking the notion of student ownership when it included such phrases as:

- Students are doing the intellectual work of discovering
- Students should as much as possible be responsible for the acquisition of knowledge
- Investigation… generated by the learner
- Student/learner engagement via their own problem solving… and active involvement
- The problems are designed to encourage students to… contract their own justifications for their conclusions
- Developing their own ideas

In addition, researchers independently created their own codes for responses that did not fit in an identified category.

Results

We report three preliminary results. The first of which describes commonalities among the different definitions. In particular, the definitions that the respondents provided overwhelmingly focused on three particular ideas; that the instructor-student relationship is different than in a traditional class (sometimes described as a guide or facilitator), that student curiosity is important and should be nurtured, and that the classes include peer-to-peer interactions. Here we present a representative examples of such a definition;

- A mathematician offered the definition: I always go with the AIBL.org folks on this one: "What this means is that we define IBL broadly, and support the use of a wide range of teaching methods in mathematics courses consistent with courses where students are (a) deeply engaged in rich mathematical tasks, and (b) have ample opportunities to collaborate with peers (where collaboration is defined broadly)."

In this example the themes of instructor-student relationship, peer-to-peer discussion, and student curiosity are represented. Moreover, we note that the adjective 'rich' does not have a clear definition, meaning that different observers could come to different conclusions about whether or not students are engaged in such a task.

Most definitions, 17 of the 18 participants, gave relatively few criteria, typically two or three criteria. There were instances where a respondent gave multiple sub-criteria describing, for example, the notion of student mathematical responsibility such as the mathematician who offered the following definition:

- I believe that there are two essential elements to IBL. Students should as much as possible be responsible for:
    - 1. guiding the acquisition of knowledge and
    - 2. validating the ideas presented. (Students should not, that is, be looking to the instructor as the sole authority.)

In this case, the respondent gave two related descriptions of the student's actions in class that both relate to the 'responsibility' code. Similar are descriptions of the student role that include 'conjecturing' and 'questioning,' although focused on different aspects of student activities. This trend of giving relatively few criteria for a definition was inclusive of all categories of respondents, including mathematics education researchers.

The single most common definition used in research, given by four respondents, all mathematics education researchers, was that "students are inquiring into mathematics and the instructor is inquiring into student thinking." This definition is interesting in that the second clause gives some description about the instructor's role in the class; that the instructor is to be doing on a daily basis; investigating and understanding the student thinking about the mathematics. In terms of what the students are to be doing, the phrase 'students inquiring into the mathematics' is open to a wide-range of interpretations such that people could plausibly argue that almost any mathematical activity done by the students is inquiry. As a result, it appears that with this definition, the actions that the professor takes are more important than anything the students do.

Less commonly, participants described what types of activities the students should engage in. Only 3 respondents did so. When they did, they gave responses similar to the below:

- A math educator offered the following, noting it was used in research and served as a personal definition: I consider inquiry to involve student/learner engagement via their own problem solving, problem posing, questioning, and active involvement...this is as opposed to students/learners being passive participants in their learning of mathematics.

In these instances, the participants used terms such as problem-posing, questioning, conjecturing, and introducing key mathematical ideas. This gives much more explicit description by which an observer might decide whether a particular class is engaging in IBL. Similar in tone were definitions that suggested that students should 'regularly introduce key ideas.' and one qualified the statement by writing that 'as much as possible' students should be the

A second preliminary result is that participants are largely in agreement that there is not much agreement in the details of IBL. Three participants said they had never seen a published definition of it, three others said that they have experience with it being defined as simply group work or active learning, but that is not enough. One participant stated that "definitions in the literature are all over the map." Another participant thought the definition was sometimes used without consideration of the instructor's role in the inquiry. In all, only 4 participants stated they had not come across a definition of inquiry that did not fit with their definition, with one of the four saying "(Some definitions) are not quite as detailed as mine, but the spirit is usually the same."

A third preliminary result is that inquiry involves "students doing meaningful work" or "being active participants in mathematics"; however the description of what is meaningful work differs from participant to participant if they describe it at all. One participant suggests that the instructor must "put students in direct contact with mathematical questions, objects, and phenomena", another offers more specific criteria stating "This involves working through mathematical activities and classroom discussions where knowledge of a mathematical concept is developed based on the students' prior knowledge. (Students) are expected to participate in the learning of a mathematical concept. Since the goal is to understand a mathematical concept, asking questions and making mistakes is viewed as part of the learning process.". In addition, similar phrases such as "students are inquiring into mathematics" were common in the data, but what it means to "inquire into math" is not clear. The authors can guess what is meant by it and it may be assumed by the inquiry community, but it is not enough to be definitional. Further investigation is required to understand what range of tasks would be considered in doing meaningful work and what it means to "inquire into mathematics".

Discussion/Future Directions

Given the most common aspects of these definitions there are a wide-range of pedagogical practices that can be described in these terms. If students commonly engaged in group work, no matter the tasks, even doing exercises, as long as the students talk to each other, the students express questions, and the professor is more conversational it would fit within the most commonly given definitional criteria. Moreover, if the professor inquires about student thinking it would possibly fit the most commonly given definition. As a result, it appears that there is no set of criteria that describe a classroom that would allow observers to reliably differentiate between an IBL class and one that is somehow not; that is, where can researchers agree to differentiate between a lecture-class and an IBL class?

A next step is member checking the codes we identified in the data. We will reach back out to the entire population and ask them for ranked input on the themes identified from their free responses to gauge how important these experts believe each theme is. After member checking the data, we hope to offer a community definition of what inquiry based learning is for an undergraduate mathematics classroom.

Preliminary Report Questions

1. In the data the phrases "Student Centered" and "Student Responsibility" are used in very similar ways. Are these different, or are people using two words (centered and responsibility) for the same meaning?
2. A commonly stated characteristic of inquiry is students "doing math" or "participating in math". What does it mean to "do math"?
3. Are there characteristics of inquiry we have not coded or found in the data and you believe are important in informing a definition of inquiry?

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