Exploring pre-service teachers' mental models of doing math

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This preliminary study explores the mental models pre-service teachers hold of doing math. Mental models are cognitive structures people use while reasoning about the world. The mental models related to mathematics would influence a teacher's pedagogical decisions and thus influence the mental model of mathematics that their students would construct. In this study, pre-service elementary teachers drew images of mathematicians doing math and of themselves doing math. Using comparative judgements, they selected an image that best represented a mathematician doing math. Most images of mathematicians doing math were of a man in front of a blackboard filled with mathematical symbols. The mathematicians appeared happy. In contrast, many images of participants showed them to be unhappy or in confused states. The preliminary results suggest that their shared mental model of doing math is naïve and shaped by limited experiences with mathematics in the classroom.

Key words: Mental Models, Drawing Research, Pre-service Teacher Mathematics Beliefs

In a recent article of the *MAA FOCUS* magazine, Francis Su, newly installed president of the MAA was asked the following question, "What is your earliest memory of doing mathematics?" Dr. Su spoke of solving arithmetic problems on worksheets, prior to being of kindergarten age, given to him by his father. He further clarified that, at that time, this was what he believed mathematics to be (Peterson, 2015). What does it mean to do math? Being able to better understand this notion and clearly explain it is imperative, especially for those involved in education, as beliefs about it drive curricular and pedagogical decisions. Additionally, lack of a clear vision hampers efforts to help people learn mathematics and to recruit future mathematicians into the field, endangering mathematics as a whole. This current study aims to explore mental models held by pre-service elementary teachers to better understand their perceptions of what it means to *do* math.

Mental Model Theory

Mental model theory is a theory of how people reason about the world. A mental model is a cognitive structure that is constructed by an individual as a representation of a possibly real, imaginary, or hypothetical external reality (Gentner, 2002; Jacob & Shaw, 1999; Johnson-Laird, Girotto, & Legrenzi, 1998; Jones, Ross, Lynam, Perez, & Leitch, 2011). Due to cognitive limitations of an individual, a model cannot contain every detail of the reality and thus are not complete or technically accurate representations (Gentner, 2002; Jones et al., 2011; Norman, 1983/2014). However, a model will have structural features in common with the represented domain and be as iconic as possible (Johnson-Laird, 2004). Thus, structural relations present in the reality will have analogous representations in the individual's mental model (Johnson-Laird, 1998).

An individual constructs a mental model through experience, by perceiving or imagining the reality, or by understanding discourse and gaining formal knowledge (Jacob & Shaw, 1999; Johnson-Laird et al., 1998; Jones et al., 2011). Because of how models are constructed, a mental model is contextually bound, constrained by an individual's experiences with the represented domain (Norman, 1983/2014). In addition to experience, goals and motives for construction of the model influence the structural aspects of the reality that end up being represented in the model (Jones et al., 2011). An individual uses mental models as conceptual frameworks through which to interpret, understand, and reason about the world (Gentner

2002; Jacob & Shaw, 1999). New information filters through the model (Jones et al., 2011), and the individual reasons about situations, leading to predictions and decisions through mental manipulations of the models (Johnson-Laird, 2005).

Reasoning about unfamiliar situations occurs as an individual constructs a new mental model by appealing to existing models. An individual imports structural relationships from a mental model of a similar domain via analogical thinking. As described by Collins and Gentner (1987), "People construct generative models by using analogy to map the rules of transition and interaction from known domains into unfamiliar domains. Analogy is a major way in which people derive models of new domains" (p. 263). Thus, new mental models are created from existing mental models of situations that appear analogous to the new situation.

Closely related to mental models is the notion of a mental image. Mental models are considered to be more general than a mental image. As some features of a mental model may not be visualizable, a mental image refers to the visualizable aspects. Hence, underlying any mental image is a mental model, with the image being the projection of the mental model's visualizable aspects (Johnson-Laird, 1998; Johnson-Laird, Girotto, & Legrenzi, 1998).

Although mental models are typically explored as an individual's model, research has been conducted into models shared by cultural groups. One such model is the *shared* mental model, which is the "overlapping mental representation of knowledge by members of a team" (Van den Bossche, Gijselaers, Segers, Woltjer, & Kirschner, 2011, p. 285). It is generally accepted that for teams to function in an effective manner, the members of the team must share a mental model (Langan-Fox, Code, & Langfield-Smith, 2000).

In addition to representing physical aspects of a particular domain, mental models also incorporate an individual's beliefs related to the domain. Therefore, mental models are reflective of belief systems (Libarkin, Beilfuss, & Kurdziel, 2003; Norman, 1983/2014). The connection to belief systems can be used as a means to explore an individual's mental model. Due to being internal constructs, mental models are difficult to explore. While one method of exploration is the direct questioning of an individual's beliefs, people generally have difficulty clearly articulating their beliefs (Gentner, 2002). As a result, novel methods are necessary to construct an external representation of an internal mental model (Jones et al., 2011).

Efforts continue in order to improve methods for constructing external representations of internal mental models. For example, some recent studies have explored mental models via participant-made drawings. Included among the models explored via drawings are elementary and middle school students' mental models of circuits (Jabot & Henry, 2007), pre-service teachers mental models of themselves as teachers of science (Thomas, Pederson, & Finson, 2001), pre-service agriculture teachers' mental models of effective teaching (Robinson, Kelsey, & Terry, 2013), and pre-service teachers' mental models of the environment (Moseley, Desjean-Perrotta, & Utley, 2010). While not explicitly using mental model theory, other studies have used a drawing methodology to explore pre-service elementary teachers' visual images of themselves as mathematics teachers (Utley & Showalter, 2007) and middle and secondary students images of mathematicians at work (Aguilar, Rosas, Zavaleta, & Robo-Vazquez, 2014; Picker & Berry, 2000; Rock & Shaw, 2000).

In their work, Picker and Berry (2000) suggested a model for how a cultural image of mathematicians and their work is formed. A young learner, someone unfamiliar with the stereotypical cultural view of mathematics, begins school. Through exposure to cultural stereotypes and memes via media, adults, and peers, through interaction with teachers lacking rich images of mathematics who could otherwise challenge stereotypes, through a pedagogy that reinforces stereotypes, and through the lack of clear intervention by the mathematics community, students begin forming their mental model of mathematics. Stereotypes fill the void left vacant by desirable alternatives, and the student's mental model is validated through

experience. The student can now take his or her place in the culture and perpetuate the shared mental model. Teachers play a key early role in inculcating students into the stereotypes of mathematics and thus could rescue students from entering the vicious cycle. However, the teachers would need to hold a healthy model of mathematics themselves to have much effect, as a teacher's beliefs influence the mathematical experiences they have with their students and so influence the model that the students form (Mewborn & Cross, 2007). If students do not have healthy images of mathematics, they may choose to pursue other vocations, potentially robbing society of valuable mathematical innovation. Thus, exploring pre-service teachers' mental models related to mathematics is of importance.

Doing Math

From a survey of twenty-five post-secondary mathematics professors, Latterell and Wilson (2012) formulated a working definition stating that in order to be considered doing math, mathematicians must be *creating* new mathematics. Schoenfeld (1994) stated, "research – what most mathematicians would call *doing* mathematics – consists of making contributions to the mathematical community's knowledge store" (p. 66). As a result of their definition, Latterell and Wilson specifically excluded teachers of mathematics from being considered as mathematicians and only included mathematics professors as being mathematicians doing math if they were engaged in research mathematics. The general populace does not necessarily hold to this same definition.

Through a survey of children in grades K-8, Rock and Shaw (2000) determined that the students believed mathematicians did the same kind of math the students did in the classroom, only with larger numbers. Students also tended to believe mathematicians solved the hard problems no one else wanted to do. Many images drawn by the participants showed a mathematician in a classroom setting. Picker and Berry (2000) found similar results when they explored the images that 12-13 year olds had of mathematicians at work. About one-fifth of the drawings were of a teacher. The images of mathematicians adhered to some stereotypes found in the research of images of scientists; most of the images were of men, and some of the drawings resembled Einstein. In a follow-up prompt, the plurality of students mentioned that mathematicians were hired to teach math, suggesting that students actually do not have a clear idea of what mathematicians did. As a result, Picker and Berry suggested that mathematicians and their work were basically invisible to the students. Fralick, Kearn, Thompson, and Lyons (2009) studied the drawings of middle schoolers to explore perceptions of scientists and engineers at work. Approximately one-fourth of the drawings of engineers contained no discernible action. Other drawings showed engineers more in the role of a "worker bee" rather than suggesting that engineering required mental efforts, leading to the conclusion that students' images of engineers and their work were naïve or incomplete.

From a study of images of mathematicians at work created by high-achieving high school students attending a mathematics and science school, Aguilar, Rosas, Zavaleta, and Romo-Vázquez (2014) discovered that while the images were mostly male figures and contained many images of teachers, the students had a richer conception of what mathematicians did. They suggested this richer view developed from more exposure to advanced mathematics. Also, since many of the images contained items found in school settings, the students' limited interactions with math, mainly in the schools, heavily influenced their image of doing math.

Due to the important role that teachers and the school setting play in the formation of a student's mental model of mathematics, this study will explore the following questions.

1. What shared mental model of *doing mathematics* is held by pre-service elementary teachers in a mathematics content course?

2. To what extent do pre-service elementary teachers' mental models of themselves doing mathematics align with this shared model?

To address these questions, this study will use mental model theory to explore drawings of "doing math" generated by the participants. In this study, drawings created by the participants are taken to be external representations created by them of their own mental images, which are in turn the projections of the visualizable aspects of the corresponding internal mental model. An individual's mental model is influenced by the culture to which he or she belongs and thus forms a shared mental model. The formation of the shared mental model occurs in a fashion as described by Picker and Berry (2000).

Methodology

The study was conducted at a regional university in the southeastern United States. Participants in the study were undergraduate students in a teacher preparation program. The students were enrolled in one of three sections of a mathematics content course for pre-service teachers. The course was the third in a sequence of four mathematics content courses required by the program. Forty-six students were enrolled in the sections. The students were divided between two disciplines, early childhood education (31, 67.4%) and special education (15, 32.6%). Of these students, 4 (8.7%) are male and 42 (91.3%) are female. Additionally, 2 are Hispanic (4.3%), 10 are African-American (21.7%), and 34 (73.9%) are Caucasian.

During the sixth week of classes, students responded in an at-home activity to the following four prompts: 1.) Draw a picture of a mathematician. 2.) Draw a picture of a mathematician doing math. 3.) Draw a picture of you doing math. 4.) Draw a picture of one of your students doing math. Students had approximately one week to create the drawings. The drawings were subsequently collected and scanned to create electronic files.

The *Mathematician* and *Mathematician doing Math* drawings were uploaded to the No More Marking website (nomoremarking.com). During the ninth week, for an at-home activity, students were invited to perform comparative judgments on the two sets of drawings with the following questions, respectively: 1.) Which best represents who a mathematician is? 2.) Which best represents a mathematician doing math? Furthermore, participants were instructed to compare each drawing and choose the one they believed best answered the questions, to give honest responses, and to not judge the pictures on artistic merit. Each participant made 40 comparisons per data set.

Based upon the results of the comparative judgments, the top 11 drawings for each set were compiled into files. During week eleven, students reviewed each set of drawings and listed features common among the drawings. They then compared and contrasted their drawing they made of *You doing Math* to the *Mathematician doing Math* images, explaining why they believed their drawings were either similar or dissimilar.

Finally, during the twelfth week, for an at-home activity, students were shown the image selected through comparative judgment as the best representative of a *Mathematician doing Math* and answered the following prompts: 1.) Why do you believe this picture was selected as the best representation of a mathematician doing math? 2.) To what extent does this picture align with your beliefs of what it means for a mathematician to do math? 3.) To you, what does it mean to be a mathematician?

Participant drawings and responses will be explored for common themes using an open coding procedure. The drawing of a mathematician doing math selected through comparative judgement as the most representative will be used as an initial model to construct the shared mental model the class had of a mathematician doing math. The themes from the drawings of themselves doing math will be used to construct a shared mental model of the pre-service teachers doing math. The themes from the drawings and written responses will be used to

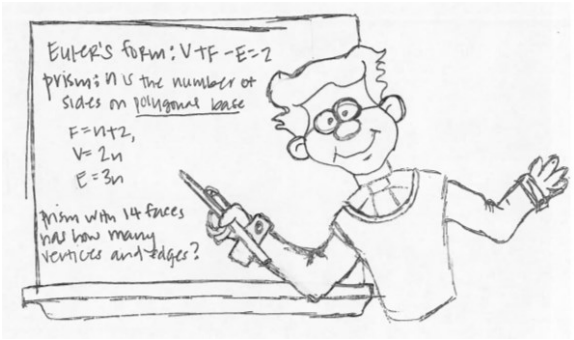

Figure 1. Image selected as best representing a mathematician doing math.

triangulate the results. A comparison will be made of the mental models of a mathematician doing math and the pre-service teacher doing math.

Discussion and Implications

As data analysis is currently ongoing, early results and implications will be offered. Figure 1 shows the image selected through comparative judgement of a mathematician doing math. The selected image apparently shows a teacher discussing Euler's formula and prisms. This suggests that experience and context played a key role in the students' mental model. For them, a math teacher teaching represented a mathematician doing math. Additionally, the content on the board can be explained by the content of the mathematics course the students were enrolled in, geometry. This image of a mathematician doing math fits in with the students' previous experiences with mathematics as being a classroom subject involving formulae and facts to be memorized.

When considering the next ten highly selected drawings, every single one showed a person standing in front of a chalkboard covered in math symbols. The symbols were all related to mathematical content that would have been experienced within a classroom. The person was either apparently teaching the content or pondering the problem. Moreover, while over 90% of the participants were female, very few participants actually drew mathematicians that could be considered female. Of the top 11 selected drawings, only 2 could potentially represent a female mathematician.

While the drawings of the mathematicians generally appeared to be happy or in a pensive state, many of the drawings that participants made of themselves doing math showed people who were dismayed, stressed, or upset. Even drawings that suggested the participant as a teacher lacked confidence. Also, some drawings showed participants in more of a student role or working with very basic math, suggesting students had trouble viewing themselves as potential mathematicians.

Overall, the images suggest that students' limited experience with math and mathematicians has led to a mental model that is very naïve. Unless these students experience an intervention, this model will continue to be reinforced as they experience struggles with math, commiserating with each other, while witnessing the apparent ease with which their teacher interacts with math.

Audience Questions

1. Should teachers of mathematics be considered mathematicians?
2. What implications would there be in comparing images of mathematicians doing math and the participants doing math, and would these implications be worthwhile?

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