Lacking confidence and resources despite having value: A potential explanation for learning goals and instructional tasks used in undergraduate mathematics courses for prospective secondary teachers

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In this paper, I report on an interview-based study of 9 mathematicians to investigate the process of choosing tasks for undergraduate mathematics courses for prospective secondary teachers. Participants were asked to prioritize complementary learning goals and tasks for an undergraduate mathematics course for prospective secondary teachers and to rate their confidence in their ability to teach with those tasks and goals. While the mathematicians largely valued task types and goals that mathematics education researchers have proposed to be beneficial for such courses, the mathematicians also largely expressed lack of confidence in their ability to teach with these task types and goals. Expectancy-value theory, in combination with these findings, is proposed as one account of why, despite consensus about broad aims of mathematical preparation for secondary teaching, these aims may be inconsistent with learning opportunities afforded by actual tasks and goals used.

Key words: secondary teacher education, mathematicians' instructional dispositions

Each year, many prospective secondary teachers are enrolled in undergraduate programs intended to prepare them to apply mathematical knowledge to their future teaching practice. The field has called for improving these programs, including teachers' mathematical preparation. In many programs, mathematicians teach the mathematics courses for these programs. However, there are few studies of how mathematicians teach (Speer, Smith, & Horvath, 2010), including how mathematicians make instructional decisions.

Scholars in teacher education have argued that mathematical knowledge for teaching develops through reasoning mathematically in ways that interact with pedagogical considerations, and that such reasoning should play a prominent role in teachers' preparation and continual development (e.g., Ball, 2000; Gallimore & Stigler, 2003; Mason & Davis, 2013; Shulman, 1986). Prospective teachers, as those who have not yet accrued experience teaching their own class, are unlikely to be able to contextualize mathematical knowledge into how it would apply to teaching. Thus from prospective teachers' viewpoint, even ostensibly useful knowledge may seem irrelevant to future practice and they therefore may not be invested in learning—a viewpoint that has been shown in multiple studies of prospective secondary teachers (e.g., Goulding, Rodd, & Hatch, 2003) Tasks that are "practice-based" (Ball & Bass, 2003)—those that engage the doer in mathematical reasoning situated in a pedagogical context provided—can potentially bridge this disconnect. By engaging in such tasks, pre-service teachers could apply mathematics in ways that are authentic to the demands of teaching (Stylianides & Stylianides, 2014; Ball, 2000). Moreover, practicing teachers' achievement on assessments using these tasks correlates positively with their student outcomes and teaching quality (Baumert et al., 2010; Hill, Rowan, & Ball, 2005; Hill et al., 2008; Rockoff, Jacob, Kane, & Staiger, 2011). Practice-based tasks, then, could play a potentially powerful role in the mathematical preparation of teachers by giving prospective teachers a window into teaching that engages them in mathematics with which they may otherwise not engage.

As the first part of the full paper will elaborate, such tasks do appear to be common to specialized courses[1] for elementary but not secondary level teaching. Given prospective secondary teachers' perception of their mathematics coursework as irrelevant, the reported study investigated: *Which task types used in specialized courses for secondary teaching are prioritized by mathematicians who may teach them, why, and for what purposes?*

The goal of the study is to identify reasons why practice-based tasks may have had a slower adoption in specialized courses for secondary level teaching as compared to elementary. This small scale study was designed to elicit potential reasons by conducting think-aloud interviews with mathematicians (n = 9), in which the mathematicians were asked to prioritize tasks and goals for use in a specialized course for secondary level teaching. The results of the study are four hypotheses that to be examined in a future, larger scale study:

1. Mathematicians generally value practice-based tasks but lack confidence in using practice-based tasks for specialized courses for secondary level teaching.
2. Mathematicians are generally more confident about teaching tasks from a secondary from an advanced perspective than practice-based tasks, even if they may value it less than practice-based tasks.
3. The confidence of a mathematician for using practice-based tasks is mediated by perceived access to resources where practice-based tasks are paired with pedagogical guidance about questions or prompts to use with prospective teachers.
4. Mathematicians frame programmatic goals in terms of assessment and lesson-level or task-level goals in terms of instruction.

These hypotheses suggest a potential reason why practice-based tasks are not commonly integrated into specialized courses at the secondary level, and this reason runs contrary to the idea that mathematicians, due to their training in the discipline of mathematicians, may simply value discipline-based more than practice-based problems. Instead, practice-based tasks may not be common because mathematicians may not feel that they can adequately teach or design such tasks, even though they would like to be able to. Additionally, the hypotheses are significant in that investigating them may explain why, despite the appearance of consensus about the programmatic aims of mathematics teacher education as evidenced by policy documents co-written by leaders of mathematics and mathematics education (CBMS, 2001; CBMS, 2012), the aims may not be coherent with the learning opportunities afforded by tasks and goals used in practice. If broad aims, tasks, and lesson-level goals are not consistent, it will be hard to improve mathematical preparation for secondary teaching in any substantive way. I take up this issue in the conclusion.

Rationale for Interview Design and Relation to Literature

I took the perspective that instructors use tasks to accomplish particular goals. Because specialized course goals are likely to be based on ideas about mathematics and teaching, and goal attainment in general is influenced by a number of cognitive and affective factors, the study design drew from literature in mathematics teacher education and cognitive science.

The role of practice-based tasks in mathematics teacher education

Practice-based tasks. I use the phrase "practice-based" in reference to Ball and Bass's (2003) description of mathematical knowledge for teaching as a "practice-based" theory. Practice-based mathematics tasks, of which examples include those used in the Learning

[1] In this paper, I use the term *specialized courses* to refer to courses designed primarily for prospective teachers, which are intended to address mathematics broadly useful for teaching a grade band within K-12 mathematics.

Mathematics for Teaching (LMT, 2008) and COACTIV (Baumert et al., 2010) instruments, are those for which successful performance on the tasks require mathematical reasoning based on inferences about the pedagogical context (Hill, Schilling, & Ball, 2004; Lai, Jacobson, & Thames, 2013). Pedagogical context refers to elements of teaching and learning provided by the task, including the purpose of an embedded teacher or information about students. Task (d) in Table 1 (d) is an example of a practice-based task.

Tasks potentially used in specialized courses for secondary teaching include tasks addressing secondary mathematics from an advanced standpoint, secondary mathematics with connections to tertiary mathematics, practice-based contexts, and common content knowledge. These are the four task types used in the interviews. The first three represent goals for specialized courses for secondary teaching as described in the guiding document *The Mathematical Education of Teachers II* (CBMS, 2012). The last type, common content knowledge (Ball, Thames, & Phelps, 2008), represents proficiency (NRC, 2001) at secondary level content. Examples of each task type used in the study are provided in Table 1.

Table 1. Task types and examples

(a) *Secondary mathematics from an advanced standpoint*	(b) *Secondary mathematics with connections to tertiary mathematics*[2]
Suppose $x \neq 0$. Prove that $x^0 = 1$. You may use the additive law of exponents ($a^{b+c} = a^b a^c$ for all $a \in \mathbb{R}$, $b, c \geq 0$, and $b, c \in \mathbb{Z}$) and the definition that $a^1 = a$ for all $a \in \mathbb{R}$	During a lesson on exponentiation, Ms. Waller's students came across the expression $\left((-4)^{\frac{1}{2}}\right)^2$. Two students obtained different answers when they tried to evaluate this expression. **Anna**: *I got* -4. *I started with* $(-4)^{\frac{1}{2}} = \sqrt{-4}$. *And* $\sqrt{-4} = 2i$. *So* $(2i)^2 = 4i^2 = -4$, *and so* $\left((-4)^{\frac{1}{2}}\right)^2 = -4$. **Brenda**: *My answer was* 4. *I did* $\left((-4)^{\frac{1}{2}}\right)^2 = (-4)^{\frac{1}{2}\cdot 2} = (-4)^{2\cdot\frac{1}{2}} = \left((-4)^2\right)^{\frac{1}{2}} = 16^{\frac{1}{2}} = \sqrt{16} = 4$. Explain the apparent contradiction between Anna's and Brenda's answers in terms of a multi-valued exponential function.
(c) *Practice-based*[3]	(d) *Common content knowledge*
Ms. Madison wants to pick one example from the previous day's homework on simplifying radicals to review at the beginning of today's class. Which of the following radicals is best for setting up a discussion about different solution paths for simplifying radical expressions? 1. $\sqrt{54}$ 2. $\sqrt{72}$ 3. $\sqrt{120}$ 4. $\sqrt{124}$ 5. Each of them would work equally well. Explain your reasoning.	Find three different pairs of functions g and h such that $$g \circ h = (x+3)^2.$$

Common content knowledge was included because it is necessary for teaching, and also to represent the viewpoint that teachers only need to be able to do the mathematics their

[2] Adapted from an instrument developed by the Educational Testing Service © 2013, with permission

[3] Source: Thames (2006), p. 6.

students need to learn to do. Although this viewpoint is not often expressed in the teacher education literature, it nonetheless implicitly or explicitly presents itself in our culture. I see this viewpoint as underlying the design of studies on teacher effectiveness using teachers' SAT or ACT as a proxy for knowledge (e.g., Rockoff, Jacob, Kane, & Staiger, 2011).

Note that Task (b) in Table 1 might be thought of as practice-based, but it is classified first as a connections to tertiary task because of its reference to complex analysis which is not a secondary topic. Moreover, the level of inference about pedagogical context needed for the task is arguably less than that of the example practice-based task.

Practice-based tasks in prospective teachers' development of mathematical knowledge for teaching. Practice-based tasks may play an especially critical role in teacher preparation. Multiple researchers have commented on the potential of *practicing* secondary teachers to learn and apply mathematics from mathematics courses and tasks that do not provide pedagogical context, and to see these mathematical experiences as relevant to their teaching (e.g., Watson, 2008; Thompson, Carlson, & Silverman, 2008; Kleickmann et al., 2013). Yet *prospective* secondary teachers' documented perception of the irrelevance of their undergraduate mathematical experiences (Goulding, Rodd, & Hatch, 2003; Ticknor, 2012; Wasserman, Villaneuva, Mejia-Ramos, & Weber, 2015) suggests that even if the tasks they worked on drew on relevant mathematics, a different approach or at least supplement to teaching and learning is needed in order for the tasks to influence thinking during and outside of class (Doyle, 1988). Practice-based tasks, by situating mathematics in teaching, could play such a role (Stylianides & Stylianides, 2014; Ball, 2000). I am not arguing that all tasks in specialized courses should be practice-based but rather than some tasks should be, and that the tasks should be tightly connected to the mathematical theory developed, whether the theory is from an advanced standpoint, or with connections to tertiary mathematics, or an alternative that somehow connects to secondary mathematics teaching.

The availability of practice-based tasks integrated into the mathematical goals of a specialized course is greater at the elementary and middle levels than secondary level. As elaborated in the full paper, evidence for this assertion includes an analysis of the content and tasks of textbooks commonly used in and policy documents guiding the curriculum for specialized courses (e.g., Bassarear, 2011; Beckmann, 2003; Bremigan, Bremigan, & Lorch, 2011; CBMS 2012; Parker & Baldridge, 2004; Sultan & Artzt, 2011; Usiskin, Peressini, Marchisotto, & Stanley, 2003). Tasks provided were identified as practice-based or not, and the key mathematical knowledge needed for the tasks were compared to the knowledge from theorems or explicitly stated mathematical goals of the section of the chapter they were contained in. Textbooks and policy documents for specialized courses for elementary and middle grades teaching incorporate practice-based tasks. On the other hand, for specialized courses for secondary teaching, the most commonly used textbooks do so less centrally. When these textbooks do incorporate practice-based tasks, the tasks do not tightly connect to a mathematical theory being developed, and so can be treated as asides rather than a central part of the course.

Cognitive and affective factors influencing goal attainment

Broadly speaking, many studies have shown that a person's success in attaining a goal is strongly shaped by how much the person values the goal intrinsically, the person's confidence that they could attain the goal, and the quality of the person's ability to conceive of implementation intentions (statements of the form "If X happens, then I will do goal-attaining behavior Y"). (See Eccles and Wigfield (2002)'s review of research on the effects of motivational beliefs and values on goal attainment, and Gollwitzer and Sheeran (2006)'s review on the effect of implementation intention on goal attainment).

To represent “confidence”, I used the notion of expectancy, that is, a person’s belief about how well they will do at a task (Atkinson, 1964), as used in Eccles and colleagues’ extensively validated expectancy-value theory that relative value and perceived probability of success influence achievement-related choices (e.g., Eccles, 1983; Eccles, Wigfield, Harold, & Blumenfeld, 1993). The phrasing of this study’s interview questions on expectancy and value were adapted from those described in Eccles, Wigfield, Harold, and Blumenfeld (1993). To represent capacity for implementation intention, the interview design included asking mathematicians to articulate anticipating prospective teacher thinking on a task and how the mathematicians would respond in order to move the teachers toward a particular learning goal. While a separate study is planned for examining these implementation intentions, statements regarding expectancy and value expressed during this interview portion were used in the analysis for the present study.

Data, Interview Design, and Method

Mathematicians who self-reported as “having taught a course designed primarily for prospective secondary teachers or would be interested in teaching such a course were the opportunity made available” were recruited for the study via a national network of US mathematicians interested in mathematics education. Interviews of 9 mathematicians were conducted, each approximately 90 minutes in length. The mathematicians were located in 6 different states, had between 0-12 years of teaching specialized courses for secondary teaching, and 0-10 years of teaching specialized courses for elementary teaching. All mathematicians had previously taught or were teaching prospective or practicing teachers.

Each interview included these five parts: (a) Task Goal Sort (b) Goal Sort (c) Task Sort (e) Overarching Goal Sort (e) Wish List. In Task Goal Sort, mathematicians were asked to prioritize learning goals for prospective teachers in the context of using a particular task, and to describe what specifically they would anticipate prospective teachers thinking, how they would know, and how they would respond so as to move the class toward the intended goal. In Goal Sort, mathematicians were then asked to prioritize the goals for “how important are these goals for mathematical preparation for secondary teaching”, independent of the task. Table 2 describes these goals and task. In Task Sort, mathematicians were presented with a set of 6 tasks and asked to prioritize them for “how well each task represents what secondary teachers should learn in their mathematical preparation”. Table 1 provides a sample of 4 of the tasks used. The task types represented the set presented to each mathematician were: practice-based (Table 1c), tertiary connections situated in a pedagogical context (Table 1b), secondary from advanced standpoint (Table 1a), a variant whose mathematics matched the advanced standpoint task but situated in a pedagogical context, another secondary from advanced standpoint task addressing different mathematics and also situated in a pedagogical context, and common content knowledge (Table 1d). The Overarching Goal Sort used the same prompt as the Goal Sort with generic goals that paralleled those in the Goal Sort, also shown in Table 2. In Wish List, mathematicians were asked to describe the resources they felt they would need to “get better at teaching courses designed primarily for prospective teachers.

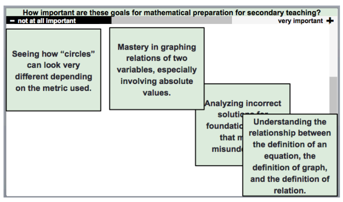


Fig 1. Card sort interface

In Goal Sort, Task Sort, and Overarching Goal Sort, mathematicians expressed their prioritizations by “sorting” the cards containing the goals and tasks horizontally, where more to the left/right meant lower/higher priority. Figure 1 shows a picture of this interface. They were then asked to sort the cards vertically by expectancy, where lower/higher meant “less/more confident that, if asked, that you could create or learn to create opportunities for teachers to do well at [these kinds of tasks/this goal].” Cards could overlap. Mathematicians trained on the interface by placing the cards “do math while drinking coffee” and “make mathematical puns” horizontally and vertically where more to the left/right meant “enjoy less/more” and lower/higher meant “less/more confident that, if asked, you could create or learn to create opportunities for fellow mathematicians to [do math while drinking coffee/make mathematical puns]”. Most mathematicians placed the coffee card at the very top, and the puns card on the very bottom. This activity was to ensure that study participants understood the notion of expectancy and that leftmost/rightmost and upmost/downmost represented extremes.

For each card for each participant, cards were assigned horizontal and vertical coordinates with values between 1 and 5 based on the approximate location of the center of the card as placed by the participant. Horizontal coordinates represented value and vertical represented expectancy. Interview transcripts were chunked into statements of beliefs, reasons, goals, and resources. The collection of statements were analyzed for themes (Glaser & Strauss, 1967). Patterns noted in card placements were triangulated with interview statements.

Table 2. Task and goals sorted by participants

Task Goal Sort/Goal Sort	*Overarching Goal Sort*
Understanding the relationship between the definition of an equation, the definition of graph, and the definition of relation.	Connecting ideas from higher mathematics to secondary mathematics
Seeing how “circles” can look very different depending on the metric used.	Experiencing secondary mathematics as a rigorous, challenging, coherent body of mathematics.
Analyzing incorrect solutions for foundational ideas that may be misunderstood.	Analyzing mathematical teaching situations
Mastery in graphing relations of two variables, especially involving absolute values.	Ensuring that teachers themselves would be able to do the problems that they are responsible for teaching K-12 students how to do.
Description of Task used in Task Goal Sort [4]	
Which of the following best shows the graph of $|x| + |y| = 6$? (a) [picture of a circle] (b) [diamond] (c) [shaded triangle in quadrant I] (d) [square] (e)[shape similar to a four pointed hypocycloid]	

Results

I summarize the logic of how the results generated the hypotheses described in the beginning of this proposal, with more elaboration in the full paper.

Hypotheses 1 and 2: *Mathematicians generally value practice-based tasks and goals but lack confidence in using practice-based task sand goals for specialized courses for secondary level teaching. Mathematicians are generally more confident about teaching tasks from tertiary connections and advanced viewpoint than practice-based tasks, even if they may value them less than practice-based tasks.* These hypotheses are supported by the general trend that practice-based goals and tasks were generally placed more right than down (below the 45^o line), representing higher value and lower expectancy than other types of tasks; and advanced standpoint and tertiary connections tasks were generally more left than up (above the 45^o line). Table 3 shows scatterplots of card sort placements.

Hypothesis 3. The confidence of a mathematician for using practice-based tasks is mediated by perceived access to resources where practice-based tasks are paired with pedagogical guidance about questions or prompts to use with prospective teachers. This hypothesis emerged from themes in statements about resources made by mathematicians, and

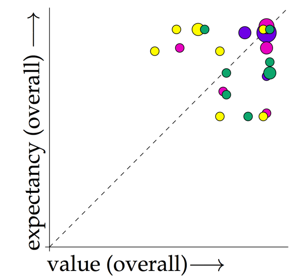
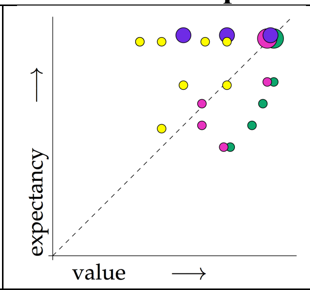
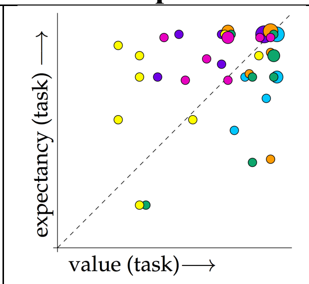
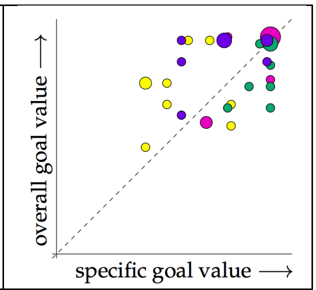
[4] Source: Begle (1972), p. 42

the statements about expectancy made by the mathematicians who expressed relatively higher expectancy about practice-based tasks.

Hypothesis 4. Mathematicians frame programmatic goals in terms of assessment and lesson-level or task-level goals in terms of instruction. This hypothesis was generated by comparing values ascribed to parallel goals in Goal Sort and Overarching Goal Sort. When looking over the interview chunks for reasons and beliefs concerning the placement of overarching goals, participants who placed types in Overarching Goal Sort differently than in the Goal Sort, tended to, in the Overarching Goal Sort, bring up the theme of assessment. That is, they appeared to frame programmatic goals in terms of certifying knowledge and more specific goals in terms of instruction; not one participant who placed them differently brought up the difference explicitly suggesting that the differing frames of assessment and instruction may not have been adopted deliberately.

Limitations of the study. As the findings of this study are based on a small scale study with limited examples from each type, the findings at most suggest hypotheses that bear examination in larger scale studies. Alternative explanations may account for the findings. For instance that participants happened to prefer or not prefer the particular examples of specific goals and tasks, but had other examples or variants of the goals and tasks been used, then the results may have been different. However, there are ways in which the findings are consistent with other literature. For example, if Hypothesis 2 is true, expectancy-value theory would predict that many specialized courses would be characterized by tertiary connections and an advanced standpoint, corroborating Murray and Star (2013).

Table 3. Scatterplots of card sort placements

			
Overarching Goal Sort Value vs. Expectancy	Goal Sort Value vs. Expectancy	Task Sort Value vs. Expectancy	Values in Goal Sort vs. Overarching Goal Sort
Key: Green = Practice Based, Yellow = Tertiary Connections, Pink = Advanced Standpoint, Purple = Common Content Knowledge. Larger circles represent more participants placing the card in that approximate location.			

Implications

The main aim of specialized courses is to prepare teachers to learn and apply mathematical knowledge to their future teaching. The CBMS (2012) policy document takes this position, signifying broad agreement in this aim. Practice-based tasks could play an important role in carrying this aim to fruition, but are not being used. The findings of this study suggest the uncommonness of practice-based tasks in specialized courses for secondary teaching is not explained by the idea that mathematicians do not value practice-based tasks. In fact, almost all participants remarked unprompted on the value of “tasks like the ones on the colored cards” that had practice-based elements, and almost all participants mentioned a wish for a repository of such tasks. The lack of practice-based tasks may be better explained by mathematicians’ lack of confidence in using, accessing, and designing practice-based tasks. However, it is an open question as to what such a resource would look like and how it would be indexed.

Another implication of this work has to do with how discussion of goals actually drives programmatic and instructional decisions. If there is a difference in how parallel goals are prioritized when thinking about them on the program level and on the course level, then actions taken are likely to be incoherent. Increased awareness may be needed for the frames used in discussion and be clear when we are discussing overall certification or moment-to-moment instructional decisions.

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