



*The Special Interest Group of the Mathematical Association of America on Research in Undergraduate Mathematics Education presents its nineteenth*

*Conference on Research in  
Undergraduate Mathematics  
Education*

*Program*

*Marriott City Center  
Pittsburgh, Pennsylvania  
February 25-27, 2016*

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# Conference Planning Committee

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Temple University

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# SIGMAA on RUME



On behalf of the Mathematical Association of America, West Virginia University's Department of Mathematics, and the RUME Conference Planning Committee, we welcome you to the 18<sup>th</sup> Annual Conference on Research in Undergraduate Mathematics Education! We have assembled an outstanding program of talks and events for this year's conference. We hope that you find the conference intellectually stimulating, productive in making connections with colleagues, and socially rewarding.

All talks in this year's program have online links to their accompanying short papers, which we refer to as the **RUME Conference Reports**. Authors who present at the conference also have the option of submitting a 15-page paper for peer-review consideration for publication in the **RUME Conference Proceedings**, to be published online later this spring. Guidelines for the RUME Conference Proceedings long papers are available on the conference webpage. Submissions to the RUME Conference Proceedings are eligible for the Best Paper Award.

On behalf of the RUME Executive Committee, we would like to thank the RUME Conference Planning Committee for their efforts towards making this a successful conference. Our sincere thanks goes to Christine Andrews-Larson, Jason Belnap, Stacy Brown, Paul Dawkins, Jessica Ellis, Estrella Johnson, Karen Allen Keene, Elise Lockwood, Ami Mamolo, Jason Martin, Pablo Mejia-Ramos, Kevin Moore, Annie Selden, Keith Weber, Megan Wawro, Aaron Weinberg, and Michelle Zandieh. We want to give special thanks to Jason Belnap for organizing the working group process. We also wish to thank the WVU group for their work in making this conference successful.

This year we received and reviewed 169 preliminary, poster, contributed, and theoretical report proposals. This work required the time and energy of over 90 reviewers, who prepared evaluations and feedback for authors. We could not carry out the conference without their contribution. Thank you to the 2016 reviewers!

If you have comments and suggestions about the submission or review process, please feel free to share them with members of the planning committee (wearing blue nametags) during the conference or send them to Tim at [timatunh@gmail.com](mailto:timatunh@gmail.com).

Local organizers will be wearing gold "host committee" name badges, and feel free to ask any of them questions you may have about the conference or Pittsburgh. Should you have any questions, concerns, or compliments about the conference, please do not hesitate to talk to any of us. You will receive an online Conference Evaluation form after the conference and be sure to share your suggestions with us, as we're always looking for ways to improve. Enjoy and have a wonderful conference!

Sincerely,

Nicole Engelke  
Local Organizer

Tim Fukawa-Connelly  
Program Committee Chairperson

# **RUME 2015 Conference Proceedings and Best Paper Award Information**

All authors who present at the 2015 conference will have the option of submitting a 15-page paper for publication in the RUME Conference Proceedings. All long papers will be reviewed by 2-3 reviewers and considered for the best paper award.

All long papers must follow the long paper formatting guidelines (available on the conference website under the “Guidelines” link) and will be reviewed according to the proposal review criteria.

## ***LONG PAPER REVIEW CRITERIA***

- Does the proposal explore a significant issue/questions relevant to RUME?
  - How does it relate to prior research on related topics/issues?
  - Is the theoretical perspective clearly outlined?
  - Is there an appropriate choice of research methodology?
  - Is it clear what the conclusions/main claims are?
  - Are those supported by data?
  - Does the research contribute to teaching practice/theory development? Submission of a long paper is not required of participants but is strongly encouraged. Acceptance to the RUME Conference Proceedings is dependent upon reviewers’ recommendations. All long papers are **due Sunday, March 22, 2015**. To submit a long paper, check the conference website under the “Guidelines” link:
- Go to: <https://www.easychair.org/conferences/?conf=rume18>
  - Log into EasyChair. Note: You will need to use the login name and password you used before to submit your proposal.
  - Click “New Submission” in the header
  - Enter the title and other information for the paper
  - Select “Proceedings Long Paper” for the paper category
  - Upload either a .doc or .docx version of your paper (do not upload a .pdf file) Questions regarding long papers should be emailed to Tim Fukawa-Connelly, the Chair of the Program Committee, at [timatunh@gmail.com](mailto:timatunh@gmail.com).

Thursday	Friday Breakfast from 7-8:30	Saturday Breakfast from 7-8:30
8:00 AM – 12:00 PM RUME Working Group Meetings	8:35 AM – 9:05 AM Session 8 – Contributed Reports	8:35 AM – 9:05 AM Session 19 – Contributed Reports
	9:15 AM – 9:45 AM Session 9 – Preliminary Reports	9:15 AM – 9:45 AM Session 20 – Preliminary Reports
	9:45 AM – 10:15 AM Coffee Break	9:45 AM – 10:15 AM Coffee Break
	10:15AM – 10:45 AM Session 10 – Preliminary Reports	10:15AM – 10:45 AM Session 21 – Contributed Reports
	10:55 AM – 11:25 AM Session 11 – Contributed Reports	10:55 AM – 11:25 AM Session 22 – Preliminary Reports
1:00 PM – 1:15 PM Opening Session Grand Ballroom Salons 2O4	11:35 AM – 12:05 PM Session 12 – Mixed Reports –4	11:35 AM – 12:05 PM Session 23 – Mixed Reports
1:25 PM – 1:55 PM Session 1 – Contributed Reports --4	12:05 – 1:05 PM Lunch Grand Ballroom Salons 2O4	12:05 PM – 1:10 PM Lunch Grand Ballroom Salons 2O4
2:05 PM – 2:35 PM Session 2 – Contributed Reports --4	1:05 PM – 1:35 PM Session 13 – Contributed Reports	1:10 PM – 1:40 PM Session 24 – Contributed Reports
2:35 PM – 3:05 PM Coffee Break	1:45 PM – 2:15 PM Session 14 – Contributed Reports	1:50 PM – 2:20 PM Session 25 – Contributed Reports
3:05 PM – 3:35 PM Session 3 – Contributed Reports --4	2:25 PM – 2:55 PM Session 15 – Contributed Reports	2:30 PM – 3:00 PM Session 26 – Contributed Reports
3:45 PM – 4:15 PM Session 4 – Contributed Reports	2:55 PM – 3:25 PM Coffee Break	3:00 PM – 3:30 PM Coffee Break
4:20 PM – 4:50 PM Session 5 – Preliminary Reports	3:25 PM – 3:55 PM Session 16 – Preliminary Reports	3:30 PM – 4:00 PM Session 27 – Preliminary Reports
5:00 PM – 5:30 PM Session 6 – Contributed Reports	4:05 PM – 4:35 PM Session 17 – Contributed Reports	4:10 PM – 4:40 PM Session 28 – Contributed Reports
5:40 PM – 6:10 PM Session 7 – Contributed Reports	4:45 PM – 5:15 PM Session 18 – Preliminary Reports	4:50 PM – 5:20 PM Session 29 – Contributed Reports
	5:30 PM – 6:30 PM Plenary Session <i>Dr. David Stinson</i> Grand Ballroom Salons 2O4	5:25 PM – 6:20 PM Poster Session 2 & Cash Bar Grand Foyer
6:10 PM – 7:00 PM Poster Session 1 & Reception Grand Foyer		
7:00 PM – 9:30 PM Dinner & Plenary Session <i>Dr. Peg Smith</i> Grand Ballroom Salons 2O4	6:30 PM Dinner On Your Own	6:30 PM – 9:15 PM Dinner, Awards Banquet & Plenary Session <i>Dr. Sean Larsen</i> Grand Ballroom Salons 2O4

<b>THURSDAY, FEBRUARY 25, 2016</b>	
<b>1:00 - 1:15 pm</b> <i>Grand Ballroom Salons 2-4</i>	<b>OPENING SESSION</b>
<b>1:25 - 1:55 pm</b> <i>Marquis A</i>	<b>SESSION 1 – CONTRIBUTED REPORTS</b>  <b>Prospective teachers’ evaluations of students’ proofs by mathematical induction</b>  Hyejin Park  This study examines how prospective secondary teachers validate several proofs by mathematical induction (MI) from hypothetical students and how their work with proof validations relates to how they grade their students’ proofs. When asked to give criteria for evaluating a student’s argument, participants wished to see a correct base step, inductive step, and algebra. However, participants prioritized the base step and inductive step over assessing the correctness of the algebra when validating and grading students’ arguments. All of the participants gave more points to an argument that presented only the inductive step than to an argument that presented only the base step. Two of the participants accepted the students’ argument addressing only the inductive step as a valid proof. Further studies are needed to determine how prospective teachers evaluate their students’ arguments by MI if many algebraic errors are present, especially in the inductive step.  1 <a href="#">Paper</a>

<p><i>Marquis B</i></p>	<p><b>Analyzing students' interpretations of the definite integral as concept projections</b></p> <p>Joseph F. Wagner</p> <p>This study of beginning and upper-level undergraduate physics students extends earlier research on students' interpretations of the definite integral. Using Wagner's (2006) transfer-in-pieces framework and the notion of a concept projection, fine-grained analyses of students' understandings of the definite integral reveal a greater variety and sophistication in some students' use of integration than previous researchers have reported. The dual purpose of this work is to demonstrate and develop the utility of concept projections as a means of investigating knowledge transfer, and to critique and build on the existing literature on students' conceptions of integration.</p> <p>9</p> <p><a href="#">Paper</a></p>
<p><i>Marquis C</i></p>	<p><b>Organizational features that influence departments' uptake of student-centered instruction: Case studies from inquiry-based learning in college mathematics</b></p> <p>Sandra Laursen</p> <p>Active learning approaches to teaching mathematics and science are known to increase student learning and persistence in STEM disciplines, but do not yet reach most undergraduates. To broadly engage college instructors in using these research-supported methods will require not only professional development and support for individuals, but the engagement of departments and institutions as organizations. This study examines four departments that implemented inquiry-based learning (IBL) in college mathematics, focusing on the question, "What explicit strategies and implicit departmental contexts help or hinder the uptake of IBL?" Based on interview data and documents, the four departmental case studies reveal strategies used to support IBL instructors and engage colleagues not actively involved. Comparative analysis highlights how contextual features supported (or not) the spread and sustainability of these teaching reforms. We use Bolman and Deal's (1991) framework to analyze the structural, political, human resource and symbolic elements of these organizational strategies and contexts.</p> <p><a href="#">Paper</a></p> <p>62</p>

<p><i>Grand Ballroom 5</i></p>	<p><b>An interconnected framework for characterizing symbol sense</b></p> <p>Margaret Kinzel</p> <p>Algebraic notation can be a powerful mathematical tool, but not all seem to develop “symbol sense,” the ability to use that tool effectively across situations. Analysis of interview data identified three interconnected viewpoints: looking at, with, and through the notation. The framework and implications for instruction will be presented.</p> <p><a href="#">Paper</a></p> <p>27</p>
<p><b>2:05 - 2:35 pm</b></p>	<p><b>SESSION 2 – CONTRIBUTED REPORTS</b></p>
<p><i>Marquis A</i></p>	<p><b>Students’ explicit, unwarranted assumptions in “proofs” of false conjectures</b></p> <p>Kelly Bubp</p> <p>Although evaluating, refining, proving, and refuting conjectures are important aspects of doing mathematics, many students have limited experiences with these activities. In this study, undergraduate students completed prove-or-disprove tasks during task-based interviews. This paper explores the explicit, unwarranted assumptions made by six students on tasks involving false statements. In each case, the student explicitly assumed an exact condition necessary for the statement in the task to be true although it was not a given hypothesis. The need for an ungiven assumption did not prompt any of these students to think the statement may be false. Through prompting from the interviewer, two students overcame their assumption and correctly solved the task and two students partially overcame it by constructing a solution of cases. However, two other students were unable to overcome their assumptions. Students making explicit, unwarranted assumptions seems to be related to their limited experience with conjectures.</p> <p><a href="#">Paper</a></p> <p>11</p>

<p><i>Marquis B</i></p>	<p><b>Physics: Bridging the symbolic and embodied worlds of mathematical thinking</b></p> <p>Clarissa Thompson, Sepideh Stewart and Bruce Mason</p> <p>Physics spans understanding in three domains – the Embodied (Real) World, the Formal (Laws) World, and the Symbolic (Math) World. Expert physicists fluidly move among these domains. Deep, conceptual understanding and problem solving thrive in fluency in all three worlds and the facility to make connections among them. However, novice students struggle to embody the symbols or symbolically express the embodiments. The current research focused on how a physics instructor used drawings and models to help his students develop more expert-like thinking and move among the worlds.</p> <p><a href="#">Paper</a></p> <p>41</p>
<p><i>Marquis C</i></p>	<p><b>Inquiry-based learning in mathematics: Negotiating the definition of a pedagogy</b></p> <p>Zachary Haberler and Sandra Laursen</p> <p>Inquiry-based learning is one of the pedagogies that has emerged in mathematics as an alternative to traditional lecturing in the last two decades. There is a growing body of research and scholarship on inquiry-based learning in STEM courses, as well as a growing community of practitioners of IBL in mathematics. However, despite the growth of IBL research and practice in mathematics, wide uptake of IBL remains hamstrung in part by the lack of a sophisticated discussion of its definition. This paper offers a first step toward addressing this problem by describing how a group of IBL practitioners define IBL, how they adopt IBL to fit their specific teaching needs, and how differences in definitions and perceptions of IBL have constrained and enabled its diffusion to new instructors.</p> <p><a href="#">Paper</a></p> <p>55</p>

<i>Grand Ballroom 5</i>	<p><b>Student resources pertaining to function and rate of change in differential equations</b></p> <p>George Kuster</p> <p>While the importance of student understanding of function and rate of change are themes across the research literature in differential equations, few studies have explicitly focused on how student understanding of these two topics grow and interface with each other while students learn differential equations. Extending the perspective of Knowledge in Pieces (diSessa, 1993) to student learning in differential equations, this research explores the resources relating to function and rate of change that students use to solve differential equations tasks. The findings reported herein are part of a larger study in which multiple students enrolled in differential equations were interviewed periodically throughout the semester. The results culminate with two sets of resources a student used relating to function and rate of change and implications for how these concepts may come together to afford an understanding of differential equations.</p> <p><a href="#">Paper</a></p> <p>124</p>
<b>2:35 - 3:05 pm</b>	<b>COFFEE BREAK</b>
<b>3:05 - 3:35 pm</b>	<b>SESSION 3 – CONTRIBUTED REPORTS</b>

<p><i>Marquis A</i></p>	<p><b>A new perspective to analyze argumentation and knowledge construction in undergraduate classrooms</b></p> <p>Karen Keene, Derek Williams and Celethia McNeil</p> <p>Using argumentation to help understand how learning in a classroom occurs is a compelling and complex task. We show how education researchers can use an argumentation knowledge construction framework (Weinberger &amp; Fischer, 2006) from research in online instruction to make sense of the learning in an inquiry oriented differential equations classroom. The long term goal is see if there are relationships among classroom participation and student outcomes. The research reported here is the first step: analyzing the discourse in terms of epistemic, social, and argumentative dimensions. The results show that the epistemic dimension can be better understood by identifying how students verbalize understanding about a problem, the conceptual space around the problem, the connections between the two and the connections to prior knowledge. In the social dimension, we can identify if students are building on their learning partners' ideas, or using their own ideas, and or both.</p> <p><a href="#">Paper</a></p> <p>42</p>
<p><i>Marquis B</i></p>	<p><b>Prototype images of the definite integral</b></p> <p>Steven Jones</p> <p>Research on student understanding of definite integrals has revealed an apparent preference among students for graphical representations of the definite integral. Since graphical representations can potentially be both beneficial and problematic, it is important to understand the kinds of graphical images students use in thinking about definite integrals. This report uses the construct of "prototypes" to investigate how a large sample of students depicted definite integrals through the graphical representation. A clear "prototype" group of images appeared in the data, as well as related "almost prototype" image groups.</p> <p>15</p> <p><a href="#">Paper</a></p>

<p><i>Marquis C</i></p>	<p><b>The graphical representation of an optimizing function</b></p> <p>Renee Larue and Nicole Infante</p> <p>Optimization problems in first semester calculus present many challenges for students. In particular, students are required to draw on previously learned content and integrate it with new calculus concepts and techniques. While this can be done correctly without considering the graphical representation of such an optimizing function, we argue that consistently considering the graphical representation provides the students with tools for better understanding and developing their optimization problem-solving process. We examine seven students' concept images of the optimizing function, specifically focusing on the graphical representation, and consider how this influences their problem-solving activities.</p> <p><a href="#">Paper</a></p> <p>93</p>
<p><i>Grand Ballroom 5</i></p>	<p><b>Support for proof as a cluster concept: An empirical investigation into mathematicians' practice</b></p> <p>Keith Weber</p> <p>In a previous RUME paper, I argued that proof in mathematical practice can profitably be viewed as a cluster concept in mathematical practice. I also outlined several predictions that we would expect to hold if proof were a cluster concept. In this paper, I empirically investigate the viability of some of these predictions. The results of the studies confirmed these predictions. In particular, prototypical proofs satisfy all criteria of the cluster concept and their validity is agreed upon by most mathematicians. Arguments that satisfy only some of the criteria of the cluster concept generate disagreement amongst mathematicians with many believing their validity depends upon context. Finally, mathematicians do not agree on what the essence of proof is.</p> <p><a href="#">Paper</a></p> <p>116</p>
<p><b>3:45 - 4:15 pm</b></p>	<p><b>SESSION 4 - CONTRIBUTED REPORTS</b></p>

<p><i>Marquis A</i></p>	<p><b>A Study of Common Student Practices for Determining the Domain and Range of Graphs</b></p> <p>Peter Cho, Benjamin Norris and Deborah Moore-Russo</p> <p>This study focuses on how students in different postsecondary mathematics courses perform on domain and range tasks regarding graphs of functions. Students often focus on notable aspects of a graph and fail to see the graph in its entirety. Many students struggle with piecewise functions, especially those involving horizontal segments. Findings indicate that Calculus I students performed better on domain tasks than students in lower math course students; however, they did not outperform students in lower math courses on range tasks. In general, student performance did not provide evidence of a deep understanding of domain and range.</p> <p><a href="#">Paper</a></p> <p>5</p>
<p><i>Marquis B</i></p>	<p><b>On symbols, reciprocals and inverse functions</b></p> <p>Rina Zazkis and Igor Kontorovich</p> <p>In mathematics the same symbol – superscript (-1) – is used to indicate an inverse of a function and a reciprocal of a rational number. Is there a reason for using the same symbol in both cases? We analyze the responses to this question of prospective secondary school teachers presented in a form of a dialogue between a teacher and a student. The data show that the majority of participants treat the symbol <math>x^{-1}</math> as a homonym, that is, the symbol is assigned different and unrelated meanings depending on a context. We exemplify how knowledge of advanced mathematics can guide instructional interaction</p> <p><a href="#">Paper</a></p> <p>39</p>

<p><i>Marquis C</i></p>	<p><b>Interpreting proof feedback: Do our students know what we're saying?</b></p> <p>Robert C. Moore, Martha Byrne, Timothy Fukawa-Connelly and Sarah Hanusch</p> <p>Instructors often write feedback on students' proofs even if there is no expectation for the students to revise and resubmit the work. However, it is not known what students do with that feedback or if they understand the professor's intentions. To this end, we asked eight advanced mathematics undergraduates to respond to professor comments on four written proofs by interpreting and implementing the comments. We analyzed the student's responses through the lenses of communities of practice and legitimate peripheral participation. This paper presents the analysis of the responses from one proof.</p> <p><a href="#">Paper</a></p> <p>87</p>
<p><i>Grand Ballroom 5</i></p>	<p><b>Student responses to instruction in rational trigonometry</b></p> <p>James Fanning</p> <p>I discuss an investigation on students' responses to lessons in Wildberger's (2005a) rational trigonometry. First I detail background information on students' struggles with trigonometry and its roots in the history of trigonometry. After detailing what rational trigonometry is and what other mathematicians think of it I describe a pre-interview, intervention, post interview experiment. In this study two students go through clinical interview pertaining to solving triangles before and after instruction in rational trigonometry. The findings of this study show potential benefits of students studying rational trigonometry but also highlight potential detriments to the material.</p> <p><a href="#">Paper</a></p> <p>6</p>
<p><b>4:20 - 4:50 pm</b></p>	<p><b>SESSION 5 - PRELIMINARY REPORTS</b></p>

<p><i>Marquis A</i></p>	<p><b>Use of strategic knowledge in a mathematical bridge course: Differences between an undergraduate and graduate</b></p> <p>Darryl Chamberlain Jr. and Draga Vidakovic</p> <p>The ability to construct proofs has become one of, if not the, paramount cognitive goal of every mathematical science major. However, students continue to struggle with proof construction and, particularly, with proof by contradiction construction. This paper is situated in a larger research project on the development of an individual's understanding of proof by contradiction in a transition-to-proof course. The purpose of this paper is to compare proof construction between two students, one graduate and one undergraduate, in the same transition-to-proof course. The analysis utilizes Keith Weber's framework for Strategic Knowledge and shows that while both students readily used symbolic manipulation to prove statements, the graduate student utilized internal and flexible procedures to begin proofs as opposed to the external and rigid procedures utilized by the undergraduate.</p> <p><a href="#">Paper</a></p> <p>7</p>
<p><i>Marquis B</i></p>	<p><b>Classifying combinations: Do students distinguish between different types of combination problems?</b></p> <p>Elise Lockwood, Nicholas Wasserman and William McGuffey</p> <p>In this paper we report on a survey designed to test whether or not students differentiated between two different types of problems involving combinations – problems in which combinations are used to count unordered sets of distinct objects (a natural, common way to use combinations), and problems in which combinations are used to count ordered sequences of two (or more) indistinguishable objects (a less obvious application of combinations). We hypothesized that novice students may recognize combinations as appropriate for the first type but not for the second type, and our results support this hypothesis. We briefly discuss the mathematics, share the results, and offer implications and directions for future research.</p> <p><a href="#">Paper</a></p> <p>30</p>

<p><i>Marquis C</i></p>	<p><b>A case study of developing self-efficacy in writing proof frameworks</b></p> <p>Ahmed Benkhalti, Annie Selden and John Selden</p> <p>This case study documents the progression of one non-traditional individual's proof-writing through a semester. We analyzed the videotapes of this individual's one-on-one sessions working through our course notes for an inquiry-based transition-to-proof course. Our theoretical perspective informed our work with this individual and included the view that proof construction is a sequence of (mental, as well as physical) actions. It also included the use of proof frameworks as a means of initiating a written proof. This individual's early reluctance to use proof frameworks, after an initial introduction to them, was documented, as well as her later acceptance of, and proficiency with, them. By the end of the first semester, she had developed considerable facility with both the formal-rhetorical and problem-centered parts of proofs and a sense of self-efficacy.</p> <p><a href="#">Paper</a></p> <p>44</p>
<p><i>Grand Ballroom 5</i></p>	<p><b>Results from the Group Concept Inventory: Exploring the role of binary operation in introductory group theory task performance</b></p> <p>Kathleen Melhuish and Jodi Fasteen</p> <p>Binary operations are an essential, but often overlooked topic in advanced mathematics. We present results related to student understanding of operation from the Group Concept Inventory, a conceptually focused, group theory multiple-choice test. We pair results from over 400 student responses with 30 follow-up interviews to illustrate the role binary operation understanding played in tasks related to a multitude of group theory concepts. We conclude by hypothesizing potential directions for the creation of a holistic binary operation understanding framework.</p> <p><a href="#">Paper</a></p> <p>56</p>

<p><i>City Center A</i></p>	<p><b>Online calculus homework: The student experience</b></p> <p>Andrew Krause</p> <p>The MAA advertises that the online homework system WeBWorK is used successfully at over 700 colleges and universities, and the institution selected for my study has implemented WeBWorK universally across all calculus courses. I used a mixed method approach to examine how students experience online calculus homework in order to provide insights as to how online homework might be improved. In particular, I examined the behaviors, perceptions, and resources associated with online homework. A survey was administered to all students in the mainstream calculus course that provides quantitative information about general trends and informs further questioning. For example, more than half of students reported that they never study calculus with classmates nor in office hours. In tandem with the large survey, I also closely studied the online homework experience of 4 students through screen recordings and interviews.</p> <p><a href="#">Paper</a></p> <p>68</p>
<p><i>City Center B</i></p>	<p><b>Students' symmetric ability in relation to their use and preference for symmetry heuristics in problem solving</b></p> <p>Meredith Muller and Eric Pandiscio</p> <p>Advanced mathematical problem solving is marked by efficient and fluid use of multiple solution strategies. Symmetric arguments are apt heuristics and eminently useful in mathematics and science fields. Research suggests that mathematics proficiency is correlated with spatial reasoning. We define symmetric ability as fluency with mentally visualizing, manipulating, and making comparisons among 2D objects under rotation and reflection. We hypothesize that symmetric ability is a distinct sub-ability of spatial reasoning which is more accessible to students due to inherent cultural biases for symmetric balance. Do students with varying levels of symmetric ability use or prefer symmetric arguments in problem solving? How does symmetric ability relate to insight in problem solving? Results from a pilot study indicate that, among undergraduates, there is high variation in symmetric ability. Further, students with higher symmetric ability tend towards more positive attitudes about mathematics. Methods, future research, and implications are discussed.</p> <p><a href="#">Paper</a></p> <p>49</p>
<p><b>5:00 – 5:30 pm</b></p>	<p><b>SESSION 6 – CONTRIBUTED REPORTS</b></p>

<p><i>Marquis A</i></p>	<p><b>Ways of understanding and ways of thinking in using the derivative concept in applied (non-kinematic) contexts</b></p> <p>Steven Jones</p> <p>Much research on students' understanding of derivatives in applied contexts has been done in kinematics-based contexts (i.e. position, velocity, acceleration). However, given the wide range of applied derivatives in other fields of study that are not based on kinematics, this study focuses on how students interpret and reason about applied derivatives in non-kinematics contexts. Three main ways of understanding or ways of thinking are described in this paper, including (1) invoking time, (2) overgeneralization of implicit differentiation, and (3) confusion between derivative expression and original formula.</p> <p><a href="#">Paper</a></p> <p>31</p>
<p><i>Marquis B</i></p>	<p><b>Graphs of inequalities in two variables</b></p> <p>Kyunghee Moon</p> <p>In this study, I analyze how preservice secondary teachers represented and explained graphs of three inequalities—a linear, a circular, and a parabolic—in two variables. I then suggest new ways to explain graphs of inequalities, i.e. some alternatives to the solution test, based on the preservice teachers' thought processes and by incorporating the idea of variation. These alternatives explain graphs of inequalities as collections of rays or curves, which is similar to graphs of functions as collections of points in one variable functions and as collections of curves in two variable functions. I conclude the study by applying the alternatives to the solving of optimization problems and discussing the implications of these alternatives for future practice and research.</p> <p><a href="#">Paper</a></p> <p>34</p>

<p><i>Marquis C</i></p>	<p><b>Mathematicians' grading of proofs with gaps</b></p> <p>David Miller, Nicole Engelke-Infante and Keith Weber</p> <p>In this study, we presented nine mathematics professors with three proofs containing gaps and asked the professors to assign the proofs a grade in the context of a transition-to-proof course. We found that the participants frequently deducted points from proofs that were correct and assigned grades based on their perceptions of how well students understood the proofs. The professors also indicated that they expected lecture proofs in the transition-to-proof course to have the same rigor as those demanded of students, but lecture proofs could be less rigorous than the rigor demanded of students in advanced mathematics courses. This presentation will focus on participants' rationales for these beliefs.</p> <p><a href="#">Paper</a></p> <p>92</p>
<p><b>6:10 – 6:40 pm</b></p>	<p><b>SESSION 7 – CONTRIBUTED REPORTS</b></p>
<p><i>Marquis A</i></p>	<p><b>Covariational and parametric reasoning</b></p> <p>Teo Paoletti and Kevin Moore</p> <p>Researchers have argued that students can develop foundational meanings for a variety of mathematics topics via quantitative and covariational reasoning. We extend this research by examining two students' reasoning that we conjectured created an intellectual need for parametric functions. We first describe our theoretical background including different conceptions of covariation researchers have found useful when analyzing students' activities constructing and representing relationships between covarying quantities. We then present two students' activities during a teaching experiment in which they constructed and reasoned about covarying quantities and highlight aspects of the students' reasoning that we conjecture created an intellectual need for parametric functions. We conclude with implications the students' activities and reasoning have for future research and curriculum design.</p> <p><a href="#">Paper</a></p> <p>20</p>

<p><i>Marquis B</i></p>	<p><b>Re-claiming during proof production</b></p> <p>David Plaxco</p> <p>In this research, I set out to elucidate the construct of Re-Claiming - a way in which students' conceptual understanding relates to their proof activity. This construct emerged during a broader research project in which I analyzed data from individual interviews with three students from a junior-level Modern Algebra course in order to model the students' understanding of inverse and identity, model their proof activity, and explore connections between the two models. Each stage of analysis consisted of iterative coding, drawing on grounded theory methodology (Charmaz, 2006; Glaser &amp; Strauss, 1967). In order to model conceptual understanding, I draw on the form/function framework (Saxe, et al., 1998). I analyze proof activity using Aberdeen's (2006a, 2006b) extension of Toulmin's (1969) model of argumentation. Reflection across these two analyses contributed to the development of the construct of Re-Claiming, which I describe and explore in this article.</p> <p><a href="#">Paper</a></p> <p>125</p>
<p><i>Marquis C</i></p>	<p><b>Limitations of a "chunky" meaning for slope</b></p> <p>Cameron Byerley, Hyunkyoungh Yoon and Patrick Thompson</p> <p>This paper will investigate the question "What mathematical meanings do high school mathematics teachers hold for slope?" It will also investigate to what extent these meanings build on an image of quotient as a measure of relative size. The data comes from the administration of the diagnostic instrument named "Meanings for Mathematics Teaching Secondary Math" (MMTsm).</p> <p><a href="#">Paper</a></p> <p>95</p>
<p><b>6:10 – 7:00 pm</b> <i>Grand Foyer</i></p>	<p><b>POSTER SESSION &amp; RECEPTION</b></p>

**Clearing the way for mindset changed through formative assessment**

Rebecca Dibbs and Jennie Patterson

One of the reasons for the exodus in STEM majors is the introductory calculus curriculum. Although there is evidence that curricula like CLEAR calculus promoted significant gains in students' growth mindset, it is unclear how this curriculum promotes mindset changes. The purpose of this case study was to investigate which features of CLEAR Calculus promoted positive changes in students' mindsets. After administering the Patterns of Adaptive Learning Scale to assess students' initial mindset in one section of calculus, four students were selected for interviews. Although participants were selected for maximal variation in their mindset at the beginning of the course, there were a lot of similar themes in their interviews. Students cited that CLEAR Calculus curriculum challenges them in ways that facilitates deeper comprehensive learning than that of a traditional calculus course.

[Paper](#)

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**Student interest in calculus I**

Derek Williams

This reports on a secondary analysis of data collected by the Mathematical Association of America's Characteristics of Successful Programs in College Calculus (2015). Survey data were collected from more than 700 instructors, and roughly 14,000 students making these data ideal for multiple level analysis techniques (Raudenbush & Bryk, 2002). Here, these data are used to analyze students' interest in Calculus I. Results suggest that students with higher frequencies of presenting to their classmates, collaborating with peers, working individually, explaining their work, and taking Calculus I with an experienced instructor tend to be more interested in class

[Paper](#)

	<p><b>Using reading journals in calculus</b></p> <p>Tara Davis and Anneliese Spaeth</p> <p>In parallel studies during the Fall 2015 semester, we examined the effects of assigning reading journals in a first semester calculus course. At the beginning of the semester, students were given instructions about how to read the textbook. On alternating weeks, students were asked to complete journal assignments - these included taking reading notes, responding to a prompt question, and reflecting upon any confusing portions of the reading. A comparison between student quiz scores from weeks during which journals were assigned and quiz scores from weeks during which no journals were assigned will be given, and implications for teaching will be discussed.</p> <p><a href="#">Paper</a></p>
	<p><b>Enriching student’s online homework experience in pre-calculus courses: Hints and cognitive supports</b></p> <p>Nathan Wakefield and Wendy Smith</p> <p>As part of reforming our Pre-Calculus courses, we realized that reforms to instruction needed to be accompanied by reforms to the homework. We utilized the online homework system WeBWorK but recognized our students wanted more support on missed questions. WebWorK “hints” provided us an avenue to ask students leading questions to prompt thinking over procedures. Preliminary data show many students are using these hints and the hints are working as intended. We plan to expand hints beyond our Pre-Calculus courses. The open source nature of WeBWorK provides an opportunity for hints to be implemented on a wide scale.</p> <p><a href="#">Paper</a></p> <p>134</p>

**Calculus students' understanding of the vertex of the quadratic function in relation to the concept of derivative**

Annie Childers and Draga Vidakovic

The purpose of this study was to gain insight into thirty Calculus I students' understanding of the relationship between the concept of vertex of a quadratic function and the concept of the derivative. APOS (action-process-object-schema) theory (Asiala et al., 1996) was used in analysis on student written work, think-aloud, and follow up group interviews. Students' personal meanings of the vertex, including misconceptions, were explored, and how students relate the vertex to the understanding of the derivative. Results give evidence of students' lack of connection between different problem types which use the derivative to find the vertex. Implications and suggestions for teaching are made based on the results. Future research is suggested as a continuation to improve student understanding of the vertex of quadratic functions and the derivative.

[Paper](#)

**An insight from a developmental mathematics workshop**

Eddie Fuller, Jessica Deshler, Marjorie Darrah, Xiangming Wu and Marcela Mera Trujillo

In this report, we present data from 404 students in a developmental mathematics course at a large research university and try to better understand academic and non-academic factors that predict their success. This work is the first step in a larger project to understand when science, technology, engineering, and mathematics (STEM) intending students who begin in developmental mathematics courses are successful and continue to be successful in higher-level mathematics courses. To gain some preliminary insight, we analyze SAT and ACT mathematics scores for STEM and non-STEM majors who succeeded in our developmental mathematics course and also look at personality traits and anxiety levels in these students. Specifically, we sought to answer the following questions for STEM intending students: (i) what SAT and ACT mathematics scores correlate with success in developmental mathematics? and (ii) what other non-academic factors predict success in developmental mathematics?

[Paper](#)

**College-educated adults on the autism spectrum and mathematical thinking**

Jeffrey Truman

This study examines the mathematical learning of adults on the autism spectrum, currently or formerly undergraduate students. I aim to expand on previous research, which often focuses on younger students in the K-12 school system. I have conducted various interviews with current and former students. The interviews involved a combination of asking for the interviewee's views on learning mathematics, self-reports of experiences (both directly related to courses and not), and some particular mathematical tasks. I present some preliminary findings from these interviews and ideas for further research.

[Paper](#)

**Colloquial mathematics in mathematics lectures**

Kristen Lew, Victoria Krupnik, Joe Olsen, Timothy Fukawa-Connelly and Keith Weber

In this poster, we focus on mathematics professors' use of colloquial mathematics where they express mathematical ideas using informal English. We analyzed 80-minute lectures in advanced mathematics from 11 different mathematics professors. We identified each instance where mathematicians expressed a mathematical idea using informal language. In the poster, we use this as a basis to present categories of the metaphorical images that professors use to help students comprehend the mathematics that they are teaching.

[Paper](#)

**Talking about teaching: Social networks of instructors of undergraduate mathematics**

Naneh Apkarian

The RUME community has focused on students' understandings of and experiences with mathematics. This project sheds light on another part of the higher education system – the departmental culture surrounding undergraduate mathematics instruction. This paper reports on the interactions of members of a single mathematics department, centered on their conversations about undergraduate mathematics instruction. Social network analysis of this group sheds important light on the informal structure of the department.

[Paper](#)

**Preliminary genetic decomposition for implicit differentiation and its connections to multivariable calculus**

Sarah Kerrigan

Derivatives are an important concept in undergraduate mathematics and across the STEM fields. There have been many studies on student understanding of derivatives, from graphing derivatives to applying them in different scientific areas. However, there is little research on how students construct an understanding of multivariable calculus from their understanding of single variable calculus. This poster uses APOS theory to hypothesize the mental reflections and constructions students need to make in order to solve and interpret an implicit differentiation problem and examine the connections to multivariable calculus. Implicit differentiation is often the first time students are introduced to the notion of a function defined by two dependent variables, a concept vital in multivariable calculus. Investigating how students initially reconcile this new idea of two variable functions can provide knowledge of how students think about multivariable calculus.

[Paper](#)

**Physics Students' Construction and Use of Differential Elements in Multivariable Coordinate Systems**

Benjamin Schermerhorn and John Thompson

As part of an effort to examine students' understanding of non-Cartesian coordinate systems when using vector calculus in the physics topics of electricity and magnetism, we interviewed four pairs of students. In one task, developed to force them to be explicit about the components of specific coordinate systems, students construct differential length and volume elements for an unconventional spherical coordinate system. While all pairs eventually arrived at the correct elements, some unsuccessfully attempted to reason through spherical or Cartesian coordinates, but recognized the error when checking their work. This suggests students' difficulty with differential elements comes from an incomplete understanding of the systems.

[Paper](#)

**Supporting undergraduate teachers' instructional change**

George Kuster and William Hall

Teaching Inquiry-oriented Mathematics: Establishing Supports (TIMES) is an NSF-funded project designed to study how we can support undergraduate instructors as they implement changes in their instruction. One factor in the disconnect between the development and dissemination of student-centered curricula are the challenges that instructors face as they work to implement these curricular innovations. For instance, researchers investigating mathematicians' efforts to teach in student-centered ways have identified a number of challenges including: developing an understanding of student thinking, planning for and leading whole class discussions, and building on students' solution strategies and contributions. This research suggests a critical component needed to take curricular innovations to scale: supports for instructional change. In this poster we address our current research efforts to support undergraduate teachers' instructional change.

[Paper](#)

**RUME- and Non-RUME-track students' motivations of enrolling in a RUME graduate course**

Ashley Berger, Rebecca Grider, Juliana Bucher, Mollie Mills-Weis, Fatma Bozkurt, and Milos Savic

The purpose of this ongoing study is to investigate students' motivations in taking a graduate-level RUME course. Seven individual semi-structured interviews were conducted with graduate students enrolled in a RUME course at a large Midwestern university that has a RUME Ph.D. option in the mathematics department. Our analysis of those interviews utilized two theoretical frameworks: Self-Determination Theory (Ryan & Deci, 2000) and Hannula's (2006) needs and goals structure. Preliminary analysis of the interviews indicates that non-RUME-track students are extrinsically, need-motivated, while RUME-track students are intrinsically, goal-motivated when taking a RUME course. The researchers conjecture that knowing what influences non-RUME-track students may aid in closing the gap between the mathematical and RUME communities.

[Paper](#)

**Teaching and learning linear algebra in terms of community of practice**

Deniz Kardes Birinci, Karen Bogard Givvin and James W. Stigler

Communities of practice (CoP) are defined as groups of people who share a concern, a set of problems, or a passion about a topic, and who interact in an ongoing basis to deepen their knowledge and expertise. The purpose of this study is to examine the process of teaching and learning linear algebra within this theoretical framework. In this research, we used an ethnographic case study design to study three linear algebra instructors and their students at a large public university. The instructors have different educational and cultural backgrounds. Data included observations, a Linear Algebra Questionnaire, and semi-structured interviews. We observed significant differences in teaching methods between the instructors.

[Paper](#)

**Using the chain rule to develop secondary school teachers' Mathematical Knowledge for Teaching, focused on the rate of change**

Zareen Rahman, Debasmita Basu, Karmen Yu and Aminata Adewumi

The unit described in this study was designed to connect secondary and advanced mathematical topics. It focused on how the knowledge of chain rule impacts secondary teachers' understanding and teaching of rate of change so that they can address students' misconceptions. This project is informed by the idea of Mathematical Knowledge for Teaching, which encompasses both subject-matter knowledge and pedagogical content knowledge of teachers. The goal was to enhance secondary school teachers' teaching of the rate of change and the unit featured tasks connecting rate of change problems as seen in high school algebra to the concept of chain rule. The unit was designed to engage mathematics teachers in discourse about the content learned at the college level to content that is taught at the secondary school level.

[Paper](#)

**Exploring the factors that support learning with digitally-delivered activities and testing in community college algebra**

Shandy Hauk and Bryan Matlen

A variety of computerized interactive learning platforms exist. Most include instructional supports in the form of problem sets. Feedback to users ranges from "Correct!" to offers of hints and partially to fully worked examples. Behind-the-scenes design of such systems varies as well – from static dictionaries of problems to "intelligent" and responsive programming that adapts assignments to users' demonstrated skills, timing, and an array of other learning theory-informed data collection within the computerized environment. This poster presents background on digital learning contexts and invites lively conversation with attendees on the research design of a study aimed at assessing the factors that influence teaching and learning with such systems in community college elementary algebra classes.

[Paper](#)

**What would the research look like? Knowledge for teaching mathematics capstone courses for future secondary teachers**

Shandy Hauk, Eric Hsu and Natasha Speer

Mathematics Capstone Course Resources is a 14-month proof-of-concept development project. Collaborators across three sites aim to: (1) develop and pilot two multi-media activities for advanced pre-service secondary mathematics teacher learning, (2) create guidance for college mathematics faculty for effective use of the materials with target audiences, and (3) gather information from instructors and students to inform future work to develop additional modules and to guide subsequent research on the implementation of the materials. The goal of this poster presentation is to provide information about capstone module development and brainstorm research design suggestions with the long term aim of developing a grant proposal to research the knowledge college mathematics faculty use to effectively teach mathematics to future teachers.

[Paper](#)

**Students' experiences and perceptions of an inquiry-based model of supplemental instruction for calculus**

Karmen Yu

The Inquiry-Based Instructional Support (IBIS) workshop model is part of an innovative degree program designed to prepare elementary mathematics teachers. The reason behind IBIS workshops was to support students enrolled in "historically difficult" mathematics courses, such as Calculus I and Calculus II. The design of IBIS workshop was framed and guided by Peer-Led Team Learning (PLTL) (Gosser & Roth, 1998) and Complex Instruction (Cohen, 1994). During workshop, students work in small groups and engage in "groupworthy" mathematical tasks that promote their conceptual understanding of Calculus topics (Cohen, 1994). A pilot study was conducted to evaluate the workshop structure and these tasks. In order to assess students' workshop experiences, follow-up interviews were conducted. Students' responses indicated that their workshop experiences helped to promote the development of their problem solving skills and highlighted the critical roles of thinking and reasoning in learning Calculus with understanding.

[Paper](#)

	<p><b>An overview of research on the arithmetic mean in university introductory statistics courses</b></p> <p>Sam Cook</p> <p>There is a dearth of research on the arithmetic mean at the university level. This poster will cover overlap of several studies (some unpublished) on university students' understanding of the mean and university statistics instructors' beliefs about their students' understandings of the mean.</p> <p><a href="#">Paper</a></p>
	<p><b>Reasoning about changes: a frame of reference approach</b></p> <p>Surani Joshua</p> <p>In a RUME 18 Theoretical Report my co-authors and I presented our cognitive description of a conceptualized frame of reference, consisting of mental commitments to units, reference points, and directionality of comparison when thinking about measures. Here I present a pilot study on how a focus on conceptualizing a frame of reference impacts students' ability to reason quantitatively about changes. The two-part empirical study consisted of clinical interviews with several students followed by teaching interviews with three students chosen because of their varying abilities to conceptualize a frame of reference. My initial evidence shows that the ability to conceptualize a frame of reference greatly benefits students as they attempt to reason with changes.</p> <p><a href="#">Paper</a></p>
<p><b>7:00 - 9:30 pm</b> <i>Grand Ballroom Salons 2-4</i></p>	<p><b>DINNER AND PLENARY</b></p> <p><i>Peg Smith</i></p>
<p><b>FRIDAY, FEBRUARY 26, 2016</b></p>	
<p><b>8:35 - 9:05 am</b></p>	<p><b>SESSION 8 - CONTRIBUTED REPORTS</b></p>

<p><i>Marquis A</i></p>	<p><b>Developmental mathematics students' use of representations to describe the intercepts of linear functions.</b></p> <p>Anne Cawley</p> <p>This paper reports findings from a pilot study that investigated the way that six college students enrolled in a developmental workshop worked through a task of nine problems on linear functions. Specifically, I investigated two aspects, the order that students completed the problems and what sources of information the students used to find the requested features, and also the types of representations (symbolic, graphical, or numerical) students used to describe the intercepts of the function. Findings suggest that students have an overwhelming reliance on the graph of the linear function and that there is variation in the number of representations used to describe the intercepts (Single, Transitional, and Multi Users). Because the graphical representation is a preferred representation, it may be wise to build student knowledge from this representation, making connections to other representations. This study contributes to understanding the mathematical knowledge that developmental mathematics students bring to the classroom.</p> <p><a href="#">Paper</a></p> <p>24</p>
<p><i>Marquis B</i></p>	<p><b>A case study of a mathematic teacher educator's use of technology</b></p> <p>Kevin Laforest</p> <p>The use of technology in mathematics classrooms remains an important focus in mathematics education due to the proliferation of technology in society and a lag in the implementation of technology in classrooms. In this paper, I present data from clinical interviews with a mathematics teacher educator (MTE) and observations from that MTE's class in order to discuss his use of technology. Specifically, I describe three themes that emerged from the MTE's technology use and how they relate to his epistemological stance. These themes are: (a) his developing a classroom environment around the use of technology, (b) technology providing a precise and dynamic environment, and (c) his using technology to help engender students' mental imagery. Finally, I discuss how the ideas emerging from this paper can be helpful for the mathematics education community.</p> <p><a href="#">Paper</a></p> <p>72</p>

<i>Marquis C</i>	<p><b>Example construction in the transition-to-proof classroom</b></p> <p>Sarah Hanusch</p> <p>Accurately constructing examples and counterexamples is an important component of learning how to write proofs. This study investigates how one instructor of a transition-to-proof course taught students to construct examples, and how her students reacted to the instruction.</p> <p><a href="#">Paper</a></p> <p>100</p>
<b>9:15 - 9:45 am</b>	<b>SESSION 9 - PRELIMINARY REPORTS</b>
<i>Marquis A</i>	<p><b>Perturbing practices: The effects of novel didactic objects on instruction</b></p> <p>Krysten Pampel</p> <p>The advancement of technology has significantly changed the practices of numerous professions, including teaching. When a school first adopts a new technology, established classroom practices are perturbed. These perturbations can have both positive and negative effects on teachers' abilities to teach mathematical concepts with the new technology. Therefore, before new technology can be introduced into mathematics classrooms, we need to better understand how technology effects instruction. Using interviews and classroom observations, I explored perturbations in classroom practice as an instructor implemented novel didactic objects. In particular, the instructor was using didactic objects designed to lay the foundation for developing a conceptual understanding of rational functions through the coordination of relative magnitudes of the numerator and denominator. The results are organized according to a framework that captures leader actions, communication, expectations of technology, roles, timing, student engagement, and mathematical conceptions.</p> <p><a href="#">Paper</a></p> <p>8</p>

<p><i>Marquis B</i></p>	<p><b>Investigating student-learning gains from video lessons in a flipped calculus course</b></p> <p>Cassie Williams and John Siegfried</p> <p>The flipped classroom has garnered attention in post-secondary mathematics in the past few years, but much of the research on this model has been on student perceptions rather than its effect on the attainment of learning goals. Instead of comparing to a “traditional” model, in this study we investigated student-learning gains in two flipped sections of Calculus I. In this session, we will focus on the question of determining learning gains from delivering content via video outside of the classroom. In particular, we will compare student-learning gains after watching more conceptual videos versus more procedural ones. We will share qualitative and quantitative data gathered from surveys and quizzes, as well as results from in-class assessments.</p> <p><a href="#">Paper</a></p> <p>17</p>
<p><i>Marquis C</i></p>	<p><b>Students’ formalization of pre-packaged informal arguments</b></p> <p>Melissa Mills and Dov Zazkis</p> <p>We gave pairs of students enrolled in a graduate analysis class tasks in which they were provided with a video taped informal argument for why a result held and asked to produce a rigorous proof of this result. This provided a lens into students’ formalization process and the various roles these informal arguments played in each pair’s proving process. Comparing across several pairs of participants revealed 3 distinct roles informal arguments can play in proving, 1) solely as a starting point, 2) as a reference that can be continually returned to during the proving process, and 3) as a convincing argument that does not inform the proving process.</p> <p><a href="#">Paper</a></p> <p>43</p>

<p><i>Grand Ballroom 5</i></p>	<p><b>Students' Perceptions of Learning College Algebra Online using Adaptive Learning Technology</b></p> <p>Lori Ogden</p> <p>Adaptive learning technology was used in the teaching of an online college algebra course. As students worked on the mastery goals set for them, the technology helped students identify content that they already understood and other content that they had yet to master. Goal orientation theory suggests that when learning is mastery-oriented, a student's motivation to learn may improve (Ames &amp; Archer, 1988). Qualitative methodology was used to describe how students perceived the instruction of their college algebra course and their learning in the course. Preliminary findings suggested that an adaptive teaching approach may help build students' confidence because they can control the pace of instruction and chose where to focus their effort without drawing negative attention to themselves.</p> <p><a href="#">Paper</a></p> <p>57</p>
<p><i>City Center A</i></p>	<p><b>Effect of emphasizing a dynamic perspective on the formal definition of limit</b></p> <p>Jeremy Sylvestre and William Hackborn</p> <p>We attempt to determine the efficacy of using an alternate, equivalent formulation of the formal definition of the limit in a first-year university calculus course in aiding the understanding of the definition and of alleviating the development of common misconceptions concerning the limit.</p> <p><a href="#">Paper</a></p> <p>60</p>

<i>City Center B</i>	<p><b>Eliciting mathematicians' pedagogical reasoning</b></p> <p>Christine Andrews-Larson, Valerie Peterson and Rachel Keller</p> <p>Given the prevalence of work in the RUME community to examine student thinking and develop instructional materials based on this research, we argue it is important to document the ways in which undergraduate mathematics instructors make sense of this research to inform their own teaching. We draw on Horn's notion of pedagogical reasoning in order to analyze video recorded conversations of over twenty mathematicians who elected to attend a workshop on inquiry-oriented instruction at a large national mathematics conference. In this context, we examine the questions: (1) How do undergraduate mathematics instructors engage in efforts to make sense of inquiry-oriented instruction? (2) How does variation in facilitation relate to instructors' reasoning about these issues? Preliminary findings suggest that differences in facilitation relate to how participants engaged in the mathematics, and that the nature of participants' engagement with the mathematics was related to their subsequent pedagogical reasoning.</p> <p><a href="#">Paper</a></p> <p>85</p>
9:45 - 10:15 am	<b>COFFEE BREAK</b>
10:15 - 10:45 am	<b>SESSION 10 - PRELIMINARY REPORTS</b>
<i>Marquis A</i>	<p><b>Does it converge? A look at second semester calculus students' struggles determining convergence of series</b></p> <p>David Earls and Eyob Demeke</p> <p>Despite the multitude of research that exists on student difficulty in first semester calculus courses, little is known about student difficulty determining convergence of sequences and series in second semester calculus courses. In our preliminary report, we attempt to address this gap specifically by analyzing student work from an exam question that asks students to determine the convergence of a series. We develop a framework that can be used to help analyze the mistakes students make when determining the convergence of series. In addition, we analyze how student errors relate to prerequisites they are expected to have entering the course, and how these errors are unique to knowledge about series.</p> <p><a href="#">Paper</a></p> <p>108</p>

<p><i>Marquis B</i></p>	<p><b>An example of a linguistic obstacle to proof construction: Dori and the hidden double negative</b></p> <p>Annie Selden and John Selden</p> <p>This paper considers the difficulty that university students' may have when unpacking an informally worded theorem statement into its formal equivalent in order to understand its logical structure, and hence, construct a proof. This situation is illustrated with the case of Dori who encountered just such a difficulty with a hidden double negative. She was taking a transition-to-proof course that began by having students first prove formally worded "if-then" theorem statements that enabled them to construct proof frameworks, and thereby, make initial progress on constructing proofs. But later, students were presented with some informally worded theorem statements to prove. We go on to consider the question of when, and how, to enculturate students into the often informal way that theorem statements are normally written, while still enabling them to progress in their proof construction abilities.</p> <p><a href="#">Paper</a></p> <p>63</p>
<p><i>Marquis C</i></p>	<p><b>Student characteristics and online retention: Preliminary investigation of factors relevant to mathematics course outcomes</b></p> <p>Claire Wladis, Alyse Hachey and Katherine Conway</p> <p>There is evidence that students drop out at higher rates from online than face-to-face courses, yet it is not well understood which students are particularly at risk online. In particular, online mathematics (and other STEM) courses have not been well-studied in the context of larger-scale analyses of online dropout. This study surveyed online and face-to-face students from a large U.S. university system. Results suggest that for online courses generally, grades are significant predictors of differential online versus face-to-face performance and that student parents may be particularly vulnerable to poor online course outcomes. Native-born students were also vulnerable online. The next stage of this research will be to analyze the factors that are relevant to online versus face-to-face retention in mathematics (and other STEM) courses specifically.</p> <p><a href="#">Paper</a></p> <p>69</p>

<p><i>Grand Ballroom 5</i></p>	<p><b>Student interpretation and justification of “backward” definite integrals</b></p> <p>Vicki Sealey and John Thompson</p> <p>The definite integral is an important concept in calculus, with applications throughout mathematics and science. Studies of student understanding of definite integrals reveal several student difficulties, some of which are related to determining the sign of an integral. Clinical interviews of 5 students gleaned their understanding of “backward” definite integrals, i.e., integrals for which the lower limit is greater than the upper limit and the differential is negative. Students initially invoked the Fundamental Theorem of Calculus to justify the negative sign. Some students eventually accessed the Riemann sum appropriately but could not determine how to obtain a negative quantity this way. We see the primary obstacle here as interpreting the differential as a width, and thus an unsigned quantity, rather than a difference between two values.</p> <p><a href="#">Paper</a></p> <p>80</p>
<p><i>City Center A</i></p>	<p><b>Divergent definitions of inquiry-based learning in undergraduate mathematics</b></p> <p>Samuel Cook, Sarah Murphy and Tim Fukawa-Connelly</p> <p>Inquiry-based learning is becoming more important and widely practiced in undergraduate mathematics education. As a result, research about inquiry-based learning is similarly becoming more common, including questions of the efficacy of such methods. Yet, thus far, there has been little effort on the part of practitioners or researchers to come to a description of the range(s) of practice that can or should be understood as inquiry-based learning. As a result, studies, comparisons and critiques can be dismissed as not using the appropriate definition, without adjudicating the quality of the evidence or implications for research and teaching. Through a large-scale literature review and surveying of experts in the community, this study begins the conversation about possible areas of agreement that would allow for a constituent definition of inquiry-based learning and allow for differentiation with non-inquiry pedagogical practices.</p> <p><a href="#">Paper</a></p> <p>88</p>

<i>City Center B</i>	<p><b>IVT as a starting point for multiple real analysis topics</b></p> <p>Steve Strand</p> <p>The proof of the Intermediate Value Theorem (IVT) provides a rich and approachable context for motivating many concepts central to real analysis, such as: sequence and function convergence, completeness of the real numbers, and continuity. As a part of the development of local instructional theory, an RME-based design experiment was conducted in which two post-calculus undergraduate students developed techniques to approximate the root of a polynomial. They then adapted those techniques into a (rough) proof of the IVT.</p> <p><a href="#">Paper</a></p> <p>118</p>
<p><b>10:55 – 11:25 am</b></p>	<p><b>SESSION 11 – CONTRIBUTED REPORTS</b></p>
<i>Marquis A</i>	<p><b>Symbolizing and Brokering in an Inquiry Oriented Linear Algebra Classroom</b></p> <p>Michelle Zandieh, Megan Wawro and Chris Rasmussen</p> <p>The purpose of this paper is to explore the role of symbolizing and brokering in fostering classroom inquiry. We characterize inquiry both as student inquiry into the mathematics and instructor’s inquiry into the students’ mathematics. Disciplinary practices of mathematics are the ways that mathematicians go about their profession and include practices such as conjecturing, defining, symbolizing, and algorithmatizing. In this paper we present examples of students and their instructor engaging in the practice of symbolizing in four ways. We integrate this analysis with detail regarding how the instructor serves as a broker between the classroom community and the broader mathematical community.</p> <p><a href="#">Paper</a></p> <p>98</p>

<p><i>Marquis B</i></p>	<p><b>Mathematicians' ideas when formulating proof in real analysis</b></p> <p>Melissa Troudt</p> <p>This report presents some findings from a study that investigated the ideas professional mathematicians find useful in developing mathematical proofs in real analysis. This research sought to describe the ideas the mathematicians developed that they deemed useful in moving their arguments toward a final proof, the context surrounding the development of these ideas in terms of Dewey's theory of inquiry, and the evolving structure of the personal argument utilizing Toulmin's argumentation scheme. Three research mathematicians completed tasks in real analysis thinking aloud in interview and at-home settings and their work was captured via video and Livescribe technology. The results of open iterative coding as well as the application of Dewey's and Toulmin's frameworks were three categories of ideas that emerged through the mathematicians' purposeful recognition of problems to be solved and their reflective and evaluative actions to solve them.</p> <p><a href="#">Paper</a></p> <p>114</p>
<p><i>Marquis C</i></p>	<p><b>Reinventing the multiplication principle</b></p> <p>Elise Lockwood and Branwen Schaub</p> <p>Counting problems offer opportunities for rich mathematical thinking, yet students struggle to solve such problems correctly. In an effort to better understand students' understanding of a fundamental aspect of combinatorial enumeration, we had two undergraduate students reinvent a statement of the multiplication principle during an eight-session teaching experiment. In this presentation, we report on the students' progression from a nascent to a sophisticated statement of the multiplication principle, and we highlight two key mathematical issues that emerged for the students through this process. We additionally present potential implications and directions for further research.</p> <p><a href="#">Paper</a></p> <p>16</p>
<p><b>11:35 - 12:05 pm</b></p>	<p><b>SESSION 12 - MIXED REPORTS</b></p>

<p><i>Marquis A</i></p>	<p><b>Results from a national survey of abstract algebra instructors: Math ed is solving problems they don't have</b></p> <p>Tim Fukawa-Connelly, Estrella Johnson and Rachel Keller</p> <p>There is significant interest from policy boards and funding agencies to change students' experiences in undergraduate mathematics classes. Abstract algebra specifically has been the subject of reform initiatives, including new curricula and pedagogies, since at least the 1960s; yet there is little evidence about whether these change initiatives have proven successful. Pursuant to answering this question, we conducted a survey of abstract algebra instructors to generally investigate typical practices, and more specifically, their knowledge, goals, and orientations towards teaching and learning. On average, moderate levels of satisfaction were reported with regard to the course itself or student outcomes; moreover, little interest in, or knowledge of, reform practices or curricula were identified. We found that 77% of respondents spend the majority of class time lecturing – not surprising when considering 82% reported the belief that lecture is the most effective way to teach.</p> <p><a href="#">Paper</a></p> <p>122</p>
<p><i>Marquis B</i></p>	<p><b>Investigating a mathematics graduate student's construction of a hypothetical learning trajectory</b></p> <p>Ashley Duncan</p> <p>This study reports results of how a teacher's mathematical meanings and instructional planning decisions transformed while participating in and then generating a hypothetical learning trajectory on angles, angle measure and the radius as a unit of measurement. Using a teaching experiment methodology, an initial clinical interview was designed to reveal the teacher's meanings for angles and angle measure and to gain information about the teacher's instructional planning decisions. The teacher participated in a researcher generated HLT designed to promote the construction of productive meanings for angles and angle measure and then constructed her own HLT for her students. The initial interview revealed that the teacher had several unproductive meanings for angles and angle measure that caused the teacher perturbations while participating in the tasks of the researcher generated HLT. This participation allowed her to construct different meanings for angles and angle measure which changed her instructional planning decisions.</p> <p><a href="#">Paper</a></p> <p>117</p>

<p><i>Marquis C</i></p>	<p><b>On the axiomatic formalization of mathematical understanding</b></p> <p>Daniel Cheshire</p> <p>This study adopts a property-based perspective to investigate the forms of abstraction, instantiation, and representation used by undergraduate topology students when acting to understand and use the concept of a continuous function as it is defined axiomatically. Based on a series of task-based interviews, profile cases are being developed to compare and contrast the distinct ways of thinking and processes of understanding observed by students undergoing this transition. A framework has been established to interpret the participants' interactions with the underlying mathematical properties of continuous functions while they reconstructed their concept images to reflect a topological (axiomatic) structure. This will provide insight into how such properties can be successfully incorporated into students' concept images and accessed; and which obstacles prevent this. Preliminary results reveal several coherent categories of participants' progression of understanding. This report will outline these profiles and seek critical feedback on the direction of the described research.</p> <p><a href="#">Paper</a></p> <p>101</p>
<p><i>Grand Ballroom 5</i></p>	<p><b>Struggling to comprehend the zero-product property</b></p> <p>John Paul Cook</p> <p>The zero-product property (ZPP), typically stated as 'if <math>ab=0</math> then <math>a=0</math> or <math>b=0</math>,' is an important property in school algebra (as a technique for solving equations) and abstract algebra (as the defining characteristic of integral domains). While the struggles of secondary mathematics students to employ the ZPP are well-documented, it unclear how undergraduate students preparing to take abstract algebra understand the ZPP as they enter abstract algebra. To this end, this paper documents students' understanding of the ZPP while also investigating how students might be able to develop and harness their own intuitive understandings of the property. Preliminary findings, in addition to characterizing how students reason with and understand the ZPP, indicate that the ZPP is no less trivial for advanced undergraduate students than it is for secondary mathematics student<sup>2</sup></p> <p><a href="#">Paper</a></p> <p>102</p>

<p><b>12:05 – 1:05 pm</b> Grand Ballroom Salons 2-4</p>	<p><b>LUNCH</b></p>
<p><b>1:05 – 1:35 pm</b></p>	<p><b>SESSION 13 – CONTRIBUTED REPORTS</b></p>
<p><i>Marquis A</i></p>	<p><b>Examining student attitudes and mathematical knowledge inside the flipped classroom experience</b></p> <p>Matthew Voigt</p> <p>Flipped classrooms or hybrid online courses are becoming increasingly prevalent at the undergraduate level as institutions seek cost-saving measures while also desiring to implement technological innovations to attract 21st century learners. This study examined undergraduate pre-calculus students' (N=427) experiences, attitudes and mathematical knowledge in a flipped classroom format compared to students in a traditional lecture format. Our initial results indicate students in the flipped format were more positive about their overall classroom experiences, were more confident in their mathematical abilities, were more willing to collaborate to solve mathematical problems, and achieved slight higher gains in mathematical knowledge. Contrary to prior research, this study indicated that a majority of students in the flipped classroom would take the class again in the same format, but of concern is the gender disparity, indicating that female students are much more likely to resist taking a class in a flipped format.</p> <p><a href="#">Paper</a></p> <p>26</p>
<p><i>Marquis B</i></p>	<p><b>Students' meanings of a (potentially) powerful tool for generalizing in combinatorics</b></p> <p>Elise Lockwood and Zackery Reed</p> <p>In this paper we provide two contrasting cases of student work on a series of combinatorial tasks that were designed to facilitate generalizing activity. These contrasting cases offer two different meanings (Thompson, 2013) that students had about what might externally appear to be the same tool – a general outcome structure that both students spontaneously developed. By examining the students' meanings, we see what made the tool more powerful for one student than the other and what aspects of his combinatorial reasoning and his ability to generalize prior work were efficacious. We conclude with implications and directions for further research.</p> <p><a href="#">Paper</a></p> <p>66</p>

<i>Marquis C</i>	<p><b>Framework for Mathematical Understanding for Secondary Teaching: A Mathematical Activity perspective</b></p> <p>M. Kathleen Heid and Patricia Wilson</p> <p>A framework for mathematical understanding for secondary teaching was developed from analysis of the mathematics in classroom events. The Mathematical Activity perspective describes the mathematical actions that characterize the nature of the mathematical understanding that secondary teachers could productively use.</p> <p><a href="#">Paper</a></p> <p>78</p>
<b>1:45 – 2:15 pm</b>	<b>SESSION 14 – CONTRIBUTED REPORTS</b>
<i>Marquis A</i>	<p><b>Integrating oral presentations in mathematics content courses for pre-service teachers</b></p> <p>Sayonita Ghosh Hajra and Abeer Hasan</p> <p>In this paper we report on a study of assessment-based oral presentation tasks in a mathematics content course for pre-service teachers at a public university in the western United States. We used statistical inference to test for the significance of the observed improvement in pre-service teachers’ attitudes towards using oral presentation tasks in their mathematical learning and towards teacher preparation. Our results suggest that use of oral presentation improves pre-service teachers’ attitudes and beliefs towards mathematics learning. Moreover, responses to the post-presentation questionnaire provide insights on the benefits of using oral presentation tasks in mathematics courses for pre-service teachers.</p> <p><a href="#">Paper</a></p> <p>4</p>

<p><i>Marquis B</i></p>	<p><b>Gender, switching, and student perceptions of Calculus I</b></p> <p>Jessica Ellis and Rebecca Cooper</p> <p>We analyze survey data to explore how students' reported perceptions of their Calculus I experiences relate to their gender and persistence in calculus. We draw from student free-responses from universities involved in a comprehensive US national study of Calculus I. We perform a thematic analysis on the data, identifying quantitative patterns within themes and analyzed student responses to better understand these patterns. Our analyses indicate that female students report negative affect towards themselves more often than males, and that female students discuss their high school preparation differently than males. We discuss how these potential factors may influence student persistence in calculus.</p> <p><a href="#">Paper</a></p> <p>76</p>
<p><i>Marquis C</i></p>	<p><b>Mary, Mary, is not quite so contrary: Unless she's wearing Hilbert's shoes</b></p> <p>Stacy Brown</p> <p>Researchers (Leron, 1985; Harel &amp; Sowder, 1998) have argued that students' lack a preference for indirect proofs and have argued that the lack of preference is due to a preference for constructive arguments. Recent empirical research (author, 2015), however, which employed a comparative selection task involving a direct proof and an indirect proof of the contraposition form, found no evidence of a lack of preference for indirect proof. Recognizing that indirect proofs of the contradiction form may differ from those that employ the contraposition, this study documents students' proof preferences and selection rationales when engaging in a comparative selection task involving a direct proof and an indirect proof of the contradiction form.</p> <p><a href="#">Paper</a></p> <p>81</p>
<p><b>2:25 - 2:55 pm</b></p>	<p><b>SESSION 15 - CONTRIBUTED REPORTS</b></p>

<p><i>Marquis A</i></p>	<p><b>Graphing habits: “I just don’t like that”</b></p> <p>Kevin Moore, Teo Paoletti, Irma Stevens and Natalie Hobson</p> <p>Students’ ways of thinking for graphs remain an important focus in mathematics education due to both the prevalence of graphical representations in the study of mathematics and the persistent difficulties students encounter with graphs. In this report, we draw from clinical interviews to report ways of thinking (or habits) undergraduate students maintain for assimilating graphs. In particular, we characterize aspects intrinsic to students’ ways of thinking for graphs that inhibited their ability to represent covariational relationships they conceived to constitute some phenomenon or situation. As an example, we illustrate that students’ ways of thinking for graphs were not productive for their representing a relationship such that neither quantity’s value increased or decreased monotonically.</p> <p><a href="#">Paper</a></p> <p>2</p>
<p><i>Marquis B</i></p>	<p><b>The Effect of Mathematics Hybrid Course on Students’ Mathematical Beliefs</b></p> <p>Kuo-Liang Chang, Roxanne Brinkerhoff and Ellen Backus</p> <p>Computer-based courses (e.g., online or hybrid) have significantly changed the design of pedagogy and curriculum in the past decade, which include online teaching and learning on mathematics education. As beliefs play an essential role on achievement, the impact of computer-based courses on mathematical beliefs is still underdeveloped. In particular, we are interested in whether mathematics hybrid class (blend of online and face-to-face) has different impact on students’ mathematical beliefs compared to regular face-to-face class. A two-by-two design of instruction method (hybrid vs. regular) and mathematics performance (high vs. low) was employed. The results showed that both hybrid and regular class students believed understanding and memorization were equally important in mathematics learning. Hybrid class students showed more flexibility in selecting solution methods compared to regular class students on their beliefs about problem solving.</p> <p><a href="#">Paper</a></p> <p>13</p>

<p><i>Marquis C</i></p>	<p><b>Using the effect sizes of subtasks to compare instructional methods: A network model</b></p> <p>Garry Johns, Christopher Nakamura and Curtis Grosse</p> <p>Networks have become increasingly important in studying air pollution, energy use, genetics and psychology. These directed graphs also have features that may be useful in modeling student learning by answering questions such as the following: How can we determine if one teaching approach has better outcomes than a second method? In this paper we present a framework for dividing an approach into subtasks, assigning a numerical value (such as an effect size) to each subtask and then combining these values to determine an overall effectiveness rating for the original approach. This process allows researchers to investigate potential causes for student achievement rather than simple correlations, and can compare the effectiveness of a method for various types of students or instructors.</p> <p><a href="#">Paper</a></p> <p>18</p>
<p><b>2:55 – 3:25 pm</b></p>	<p><b>COFFEE BREAK</b></p>
<p><b>3:25 – 3:55 pm</b></p>	<p><b>SESSION 16 – PRELIMINARY REPORTS</b></p>
<p><i>Marquis A</i></p>	<p><b>Effect of teacher prompts on student proof construction</b></p> <p>Margaret Morrow and Mary Shepherd</p> <p>Many students have difficulty learning to construct mathematical proofs. In an upper level mathematics course using inquiry based methods, while this is some research on the types of verbal discourse in these courses, there is little, if any, research on teachers’ written comments on students’ work. This paper presents some very preliminary results from ongoing analysis from one teacher’s written prompts on students’ rough drafts of proofs for an Abstract Algebra course. The teacher prompts will initially be analyzed through a framework proposed by Blanton &amp; Stylianou (2014) for verbal discourse and the framework will be modified in the course of the analysis. Can we understand if a type of prompt is “better” in some sense in getting students to reflect on their work and refine their proofs? It is anticipated that teacher prompts in the form of transactive questions are more effective in helping students construct proofs.</p> <p><a href="#">Paper</a></p> <p>64</p>

<p><i>Marquis B</i></p>	<p><b>Secondary teachers confronting mathematical uncertainty: Reactions to a teacher assessment item on exponents</b></p> <p>Heejoo Suh, Heather Howell and Yvonne Lai</p> <p>Teaching is inherently uncertain, and teaching secondary mathematics is no exception. We take the view that uncertainty can present opportunity for teachers to refine their practice, and that undergraduate mathematical preparation for secondary teaching can benefit from engaging pre-service teachers in tasks presenting uncertainty. We examined 13 secondary teachers' reactions to mathematical uncertainty when engaged with concepts about extending the domain for the operation of exponentiation. The data are drawn from an interview-based study of items developed to assess mathematical knowledge for teaching at the secondary level. In our findings, we characterize ways in which teachers either denied or mathematically investigated the uncertainty. Potential implications for instructors include using mathematical uncertainty to provide an opportunity for undergraduates to learn both content and practices of the Common Core State Standards. The proposal concludes with questions addressing how undergraduate mathematics instructors could use uncertainty as a resource when teaching preservice teachers.</p> <p><a href="#">Paper</a></p> <p>70</p>
<p><i>Marquis C</i></p>	<p><b>Students' concept image of tangent line compared to their understanding of the definition of the derivative</b></p> <p>Brittany Vincent and Vicki Sealey</p> <p>Our research explores first-semester calculus students' understanding of tangent lines and the derivative concept through a series of three interviews conducted over the course of one semester. Using a combination of Zandieh's (2000) derivative framework and Tall and Vinner's (1981) notions of concept image and concept definition, our analysis examines the role that students' concept image of tangent lines plays in their conceptual understanding of the derivative concept. Preliminary results seem to indicate that students are more successful when their concept image of tangent includes the limiting position of secant lines, as opposed to a tangent line as the line that touches the curve at one point.</p> <p><a href="#">Paper</a></p> <p>89</p>

<p><i>Grand Ballroom 5</i></p>	<p><b>Unraveling, synthesizing and reweaving: Approaches to constructing general statements.</b></p> <p>Duane Graysay</p> <p>Learning progressions for the development of the ability to look for and make use of mathematical structure would benefit from understanding how students in mathematics-focused majors might construct such structures in the form of general statements. The author recruited ten university students to interviews focused on tasks that asked for the reconstruction of a general statement to accommodate a broader domain. Through comparative analysis of responses, four major categories of approaches to such tasks were identified. This preliminary report describes in brief those four categories.</p> <p><a href="#">Paper</a></p> <p>102</p>
<p><i>City Center A</i></p>	<p><b>Student conceptions of definite integration and accumulation functions</b></p> <p>Brian Fisher, Jason Samuels and Aaron Wangberg</p> <p>Prior research has shown several common student conceptualizations of integration among undergraduates. This report focuses on data from a written assessment of students' views on definite integration and accumulation functions to categorize student conceptualizations and report on their prevalence among the undergraduate population. Analysis of these results found four categorizations for student descriptions of definite integrals: antiderivative, area, an infinite sum of one dimensional pieces, and a limit of approximations. When asked about an accumulation function, student responses were grouped into three categorizations: those based on the process of calculating a single definite integral, those based on the result of calculating a definite integral, and those based on the relationship between changes in the input and output variables of the accumulation function. These results were collected as part of a larger study on student learning in multivariable calculus, and the implications of these results on multivariable calculus will be considered.</p> <p><a href="#">Paper</a></p> <p>113</p>

<p><i>City Center B</i></p>	<p><b>Supporting students in seeing sequence convergence in Taylor series convergence</b></p> <p>Jason Martin, Matthew Thomas and Michael Oehrtman</p> <p>Virtual manipulatives designed to increase student understanding of the concepts of approximation by Taylor polynomials and convergence of Taylor series were used in calculus courses at multiple institutions. 225 students responded to tasks requiring graphing Taylor polynomials, graphing Taylor series, and describing relationships between different notions of convergence. We detail significant differences observed between students who used virtual manipulatives and those that did not. We propose that the use of these virtual manipulatives promotes an understanding of Taylor series supporting an understanding consistent with the formal definition of pointwise convergence.</p> <p><a href="#">Paper</a></p> <p>115</p>
<p><b>4:05 – 4:35 pm</b></p>	<p><b>SESSION 17– CONTRIBUTED REPORTS</b></p>
<p><i>Marquis A</i></p>	<p><b>On the use of dynamic animations to support students in reasoning quantitatively</b></p> <p>Grant Sander</p> <p>This study addresses the well-documented issue that students struggle to write meaningful expressions and formulas to represent and relate the values of quantities in applied problem contexts. In developing an online intervention, we drew from research that revealed the importance of and processes involved in conceptualizing quantitative relationships to support students in conceptualizing and representing quantitative relationships in applied problem contexts. The results suggest that the use of dynamic animations with prompts that focus students’ attention on conceptualizing and relating quantities can be effective in supporting students in constructing meaningful expressions to represent the value of one quantity in terms of another, and formulas to define how two co-varying quantities change together.</p> <p><a href="#">Paper</a></p> <p>12</p>

<p><i>Marquis B</i></p>	<p><b>Inquiry-oriented instruction: A conceptualization of the instructional the components and practices</b></p> <p>George Kuster and Estrella Johnson</p> <p>In this paper we provide a characterization of inquiry-oriented instruction. We begin with a description of the roles of the tasks, the students, and the teacher in advancing the mathematical agenda. We then shift our focus to four main instructional components that are central to carrying out these roles: Generating student ways of reasoning, Building on student contributions, Developing a shared understanding, and Connecting to standard mathematical language and notation. Each of these four components is further delineated into a total of eight practices. These practices are defined and exemplified by drawing on the K-16 research literature. As a result, this conceptualization of inquiry-oriented instruction makes connections across research communities and provides a characterization that is not limited to undergraduate, secondary, or elementary mathematics education. The ultimate goal for this work is to serve as a theoretical foundation for a measure of inquiry-oriented instruction.</p> <p><a href="#">Paper</a></p> <p>45</p>
<p><i>Marquis C</i></p>	<p><b>Design research on inquiry-based multivariable calculus: Focusing on students' argumentation and instructional design</b></p> <p>Oh Nam Kwon, Younggon Bae and Kuk Hwan Oh</p> <p>In this study, researchers design and implement an inquiry-based multivariable calculus course to enhance students' argumentation in mathematical discussions. This research aims to understand the students' argumentation in proof constructions activities, and to derive the characteristics of three sites of intervention: instructional design, classroom interaction, and the instructor's role. Over the course of 14 weeks, 18 freshmen mathematics education majors participated in this study. Multiple sources of data were collected, students' reasoning in the classroom discussions were analyzed within the Toulmin's argumentation structure, and the instructional interventions were gradually revised according to the iterative cyclic process of the design research. The students' argumentation structures presented in the classroom gradually developed into more complicated forms as the study progressed, and the researchers conclude that the interventions were effective at improving students' arguments.</p> <p><a href="#">Paper</a></p> <p>67</p>
<p><b>4:45 – 5:15 pm</b></p>	<p><b>SESSION 18 – PRELIMINARY REPORTS</b></p>

<p><i>Marquis A</i></p>	<p><b>The case of an undergraduate mathematics cohort of African American males striving for mathematical excellence</b></p> <p>Christopher Jett</p> <p>Historically Black Colleges and Universities (HBCUs) provide a different milieu as it pertains to supporting students academically in all disciplines, and this study champions an HBCU effort within the context of undergraduate mathematics. Specifically, it highlights the case of a cohort of 16 African American male mathematics majors at an all-male HBCU. The overarching research question sought to delve deeper into these participants' educational experiences. Using qualitative research methods grounded in critical race theory, preliminary data show that these African American male mathematics majors were affirmed racially and mathematically in their undergraduate mathematics space.</p> <p><a href="#">Paper</a></p> <p>74</p>
<p><i>Marquis B</i></p>	<p><b>Developing an open-ended linear algebra assessment: initial findings from clinical interviews</b></p> <p>Muhammad Haider, Khalid Bouhjar, Kelly Findley, Ruby Quea and Christine Andrews-Larson</p> <p>The primary goal of this study was to design and validate a conceptual assessment in undergraduate linear algebra course. We work toward this goal by conducting semi-structured clinical interviews with 8 undergraduate students who were currently enrolled or had previously taken linear algebra. We try to identify the variety of ways students reasoned about the items with the intent of identifying ways in which the assessment measured or failed to measure students' understanding of the intended topics. Students were interviewed while they completed the assessment and interview data was analyzed by using an analytical tool of concept image and concept definition of Tall and Vinner (1981). We identified two themes in students' reasoning: the first theme involves students reasoning about span in terms of linear combinations of vectors, and the second one involves students struggling to resolve the number of vectors given with the number of entries in each vector.</p> <p><a href="#">Paper</a></p> <p>75</p>

<p><i>Marquis C</i></p>	<p><b>Exploring tensions: Leanne’s story of supporting pre-service mathematics teachers with learning disabilities</b></p> <p>Robyn Ruttenberg-Rozen and Ami Mamolo</p> <p>This paper presents a case study of a mathematics teacher educator, Leanne, and her story of trying to support the development of two pre-service elementary school teachers with recognized learning disabilities. We analyze data through a lens of mathematical knowledge for teaching, focusing in particular on concerns and tensions about (i) maintaining academic rigor while meeting the emotional, cognitive and pedagogical needs of her students, (ii) seemingly opposing pedagogies between special education and mathematics education practices, and (iii) equitable opportunities for teachers with disabilities and the consequences for their potential pupils. We offer an analysis of Leanne’s personal struggle, highlighting implications for teacher education and offering recommendations for future research.</p> <p><a href="#">Paper</a></p> <p>83</p>
<p><i>Grand Ballroom 5</i></p>	<p><b>The Complement of RUME: What's Missing From Our Research?</b></p> <p>Natasha Speer and Dave Kung</p> <p>The Research in Undergraduate Mathematics Education (RUME) community has generated a substantial literature base on student thinking about ideas in the undergraduate curriculum. However, not all topics in the curriculum have been the object of research. Reasons for this include the relatively young age of RUME work and the fact that research topics are not necessarily driven by the content of the undergraduate curriculum. What topics remain largely untouched? We give a preliminary analysis, with a particular focus on concepts in the standard calculus sequence. Uses for this kind of analysis of the literature base in the education of novice researchers and potential future directions for further analyses are discussed.</p> <p><a href="#">Paper</a></p> <p>86</p>

<p><i>City Center A</i></p>	<p><b>Exploring pre-service teachers' mental models of doing math</b></p> <p>Ben Wescoatt</p> <p>This preliminary study explores the mental models pre-service teachers hold of doing math. Mental models are cognitive structures people use while reasoning about the world. The mental models related to mathematics would influence a teacher's pedagogical decisions and thus influence the mental model of mathematics that their students would construct. In this study, pre-service elementary teachers drew images of mathematicians doing math and of themselves doing math. Using comparative judgements, they selected an image that best represented a mathematician doing math. Most images of mathematicians doing math were of a man in front of a blackboard filled with mathematical symbols. The mathematicians appeared happy. In contrast, many images of participants showed them to be unhappy or in confused states. The preliminary results suggest that their shared mental model of doing math is naïve and shaped by limited experiences with mathematics in the classroom.</p> <p><a href="#">Paper</a></p> <p>90</p>
<p><i>City Center B</i></p>	<p><b>'It's not an English class': Is correct grammar an important part of mathematical proof writing at the undergraduate level?</b></p> <p>Kristen Lew and Juan Pablo Mejia-Ramos</p> <p>We studied the genre of mathematical proof writing at the undergraduate level by asking mathematicians and undergraduate students to read seven partial proofs based on student-generated work and to identify and discuss uses of mathematical language that were out of the ordinary with respect to what they considered standard mathematical proof writing. Preliminary results indicate the use of correct grammar is necessary in proof writing, but not always addressed in transition-to-proof courses.</p> <p><a href="#">Paper</a></p> <p>103</p>
<p><b>5:30 – 6:30 pm</b> <i>Grand Ballroom Salons 2-4</i></p>	<p><b>PLENARY SESSION</b></p> <p><i>David Stinson</i></p>
<p><b>6:30 pm</b></p>	<p><b>DINNER ON YOUR OWN</b></p>
<p><b>SATURDAY, FEBRUARY 21, 2015</b></p>	
<p><b>8:35 – 9:05 am</b></p>	<p><b>SESSION 19 – CONTRIBUTED REPORTS</b></p>

<p><i>Marquis A</i></p>	<p><b>Adaptations of learning glass solutions in undergraduate STEM education</b></p> <p>Shawn Firouzian, Chris Rasmussen and Matthew Anderson</p> <p>One of the main issues STEM faculty face is promoting student success in large-enrollment classes while simultaneously meeting students' and administrators' demands for the flexibility and economy of online and hybrid classes. The Learning Glass is an innovative new instructional technology that holds considerable promise for engaging students and improving learning outcomes. In this report we share the results of an efficacy study between an online calculus-based physics course using Learning Glass technology and a large auditorium-style lecture hall taught via document projector. Both courses were taught with the same instructor using identical content, including exams and homework. Our quasi-experimental design involved identical pre- and post-course assessments evaluating students' attitudes and behavior towards science and their conceptual learning gains. Results are promising, with equivalent learning gains for all students, including minority and economically disadvantaged students.</p> <p><a href="#">Paper</a></p> <p>65</p>
<p><i>Marquis B</i></p>	<p><b>Pre-service teachers' meanings of area</b></p> <p>Sayonita Ghosh Hajra, Betsy McNeal and David Bowers</p> <p>An exploratory study was conducted of pre-service teachers' understanding of area at a public university in the western United States. Forty-three pre-service teachers took part in the study. Their definitions of area and their responses to area-units tasks were recorded throughout the semester. We found a wide gap between pre-service teachers' meaning of area and their use of area-units. Initially, pre-service teachers had weak definitions of area. Over the semester, these definitions were refined, but misconceptions about area and area-units were illuminated in activities involving non-standard units and areas of irregular regions. We conclude that, despite detailed models of children's understanding of area, much work is needed to understand the learning trajectories of pre-service teachers, particularly when misconceptions exist.</p> <p><a href="#">Paper</a></p> <p>33</p>

<p><i>Marquis C</i></p>	<p><b>Personification as a lens into relationships with mathematics</b></p> <p>Dov Zazkis and Ami Mamolo</p> <p>Personification is the attribution of human qualities to non-human entities (Inagaki &amp; Hatano, 1987). Eliciting personification as a research method takes advantage of a naturally occurring means through which (some) people discuss the nuanced emotional relationships they have with those entities. In this paper, we introduce the eliciting personification method for exploring individuals' images of mathematics, as well as discuss an initial set of approaches for analyzing the resulting data. Data from both pre-service teachers and research mathematicians are discussed in order to illustrate the method.</p> <p><a href="#">Paper</a></p> <p>35</p>
<p><b>9:15 – 9:45 am</b></p>	<p><b>SESSION 20 – PRELIMINARY REPORTS</b></p>
<p><i>Marquis A</i></p>	<p><b>Calculus students' understanding of logical implication and its relationship to their understanding of calculus theorems</b></p> <p>Joshua Case and Natasha Speer</p> <p>In undergraduate mathematics, deductive reasoning is an important skill for learning theoretical ideas and is primarily characterized by the concept of logical implication. This plays roles whenever theorems are applied, i.e., one must first check if a theorem's hypotheses are satisfied and then make correct inferences. In calculus, students must learn how to apply theorems. However, most undergraduates have not received instruction in propositional logic. How do these students comprehend the abstract notion of logical implication and how do they reason conditionally with calculus theorems? Results from our study indicated that students struggled with notions of logical implication in abstract contexts, but performed better when working in calculus contexts. Strategies students used (successfully and unsuccessfully) were characterized. Findings indicate that some students use "example generating" strategies to successfully determine the validity of calculus implications. Background on current literature, results of our study, further avenues of inquiry, and instructional implications are discussed.</p> <p><a href="#">Paper</a></p> <p>28</p>

<p><i>Marquis B</i></p>	<p><b>Service-learning in a precalculus class: Tutoring improves the course performance of the tutor.</b></p> <p>Ekaterina Yurasovskaya</p> <p>We have introduced an experiment: as part of a Precalculus class, university students have been tutoring algebra prerequisites to students from the community via an academic service-learning program. The goal of the experiment was to improve university students' mastery of basic algebra and to quantitatively describe benefits of service-learning to students' performance in mathematics. At the end of the experiment, we observed 59% decrease of basic algebraic errors between experimental and control sections. The setup and analysis of the study have been informed by the theoretical research on service-learning and peer learning, both grounded in the constructivist theory of John Dewey.</p> <p><a href="#">Paper</a></p> <p>46</p>
<p><i>Marquis C</i></p>	<p><b>How well prepared are preservice elementary teachers to teach early algebra?</b></p> <p>Funda Gonulates, Leslie Nabors Olah, Heejoo Suh, Xueying Ji and Heather Howell</p> <p>This study aims to investigate undergraduate preservice teachers' knowledge in the domain of early algebra. We conducted 90-minute clinical interviews with 15 preservice teachers in their fourth year of a five-year teacher preparation program. These interview sessions collected preservice teachers' responses to a series of assessment items designed to measure their content knowledge for teaching early algebra, with follow up questions probing their content-based reasoning. We also collected self-report of their preparation in this content area. We found that the participants had difficulty on the items targeting the meaning and use of operational properties and also in evaluating the appropriate use of the equal sign when presented with different uses in student work. They reported that they had few opportunities to learn about early algebra as mathematical content and as a topic to teach. These findings can inform the development of teacher-education curricula and support materials.</p> <p><a href="#">Paper</a></p> <p>79</p>

<p><i>Grand Ballroom 5</i></p>	<p><b>Undergraduate students proof-reading strategies: A case study at one research institution</b></p> <p>Eyob Demeke and Mateusz Pacha-Sucharzewski</p> <p>Weber (2015) identified five effective proof-reading strategies that undergraduate students in proof-based courses can use to facilitate their proof comprehension. Following Weber’s (2015) study, we designed a survey study to examine how undergraduate students’ proof-reading strategies relate to what proficient learners of mathematics (mathematics professors and graduate students in mathematics) say undergraduates should employ when reading proofs. Our preliminary findings are: (i) Majority of the professors in our study claimed that undergraduates should use the strategies identified in Weber’s (2015) study, (ii) Professors’ response significantly differed from undergraduates’ in only two of the five proof-reading strategies described in Weber’s (2015) study (attempting to prove theorem before reading its proof and illustrating confusing assertions with examples), and finally (iii) Undergraduate students, for the most part, tend to agree with their professors’ preferred proof-reading strategies.</p> <p><a href="#">Paper</a></p> <p>106</p>
<p><i>City Center A</i></p>	<p><b>Beyond procedures: Quantitative reasoning in upper-division Math Methods in Physics</b></p> <p>Michael Loverude</p> <p>Many upper-division physics courses have as goals that students should ‘think like a physicist.’ While this is not well-defined, most would agree that thinking like a physicist includes quantitative reasoning skills: considering limiting cases, dimensional analysis, and using approximations. However, there is often relatively little curricular support for these practices and many instructors do not assess them explicitly. As part of a project to investigate student learning in math methods, we have developed a number of written questions testing the extent to which students in an upper-division course in Mathematical Methods in Physics can employ these skills. Although there are limitations to assessing these skills with written questions, they can provide insight to the extent to which students can apply a given skill when prompted.</p> <p><a href="#">Paper</a></p> <p>112</p>

<p><i>City Center B</i></p>	<p><b>A case for whole class discussions: Two case studies of the interaction between instructor role and instructor experience with a research-informed curriculum</b></p> <p>Aaron Wangberg, Elizabeth Gire, Brian Fisher and Jason Samuels</p> <p>This paper presents case studies of two instructors implementing a research informed multivariable calculus curriculum. The analysis, structured around social constructivist concepts, focuses on the interactions between the roles of the instructor in facilitating student discussions and the instructors' experiences with the activities. This study is a part of an effort to evaluate and improve the project's effectiveness in supporting instructors in implementing the activities to promote rich discussions with and among students. We find these instructors to be focused on their roles as facilitators for student-centered small-group discussion and that they choose not to have of whole class discussions. We argue that initiating whole class discussions would address concerns and negative experiences reported by the instructors.</p> <p><a href="#">Paper</a></p> <p>109</p>
<p><b>9:45 – 10:15 am</b></p>	<p><b>COFFEE BREAK</b></p>
<p><b>10:15 – 10:45 am</b></p>	<p><b>SESSION 21 – CONTRIBUTED REPORTS</b></p>
<p><i>Marquis A</i></p>	<p><b>A framework for examining the 2-D and 3-D spatial skills needed for calculus</b></p> <p>Nicole Engelke, Marjorie Darrah and Kristen Murphy</p> <p>Having well developed spatial skills is critical to success in many STEM fields such as engineering, chemistry, and physics; these skills are equally critical for success in mathematics. We present a framework for examining how spatial skills are manifested in math problems. We examine established spatial skills definitions and correlate them with the spatial skills needed to successfully solve a standard calculus problem – find the volume of a solid of revolution. This problem is deconstructed into steps and analyzed according to what 2-D and 3-D spatial skills are necessary to visualize the problem. The results of a pilot study in which we examine the spatial skills that first semester calculus students possess are presented along with the potential implications the students' skill level could bring to bear on the problem. We conclude with suggestions for remediation of these spatial skills in the calculus classroom and directions for future research.</p> <p><a href="#">Paper</a></p> <p>29</p>

<p><i>Marquis B</i></p>	<p><b>Ways in which engaging in someone else's reasoning is productive</b></p> <p>Naneh Apkarian, Chris Rasmussen, Hayley Milbourne, Tommy Dreyfus, Xuefen Gao and Matthew Voigt</p> <p>Typical goals for inquiry-oriented mathematics classrooms are for students to explain their reasoning and to make sense of others' reasoning. In this paper we offer a framework for interpreting ways in which engaging in the reasoning of someone else is productive for the person who is listening. The framework is the result of analysis of 10 individual problem-solving interviews with 10 mathematics education graduate students enrolled in a mathematics content course on chaos and fractals. The theoretical grounding for this work is the emergent perspective (Cobb &amp; Yackel, 1996), which views mathematical progress as a process of active individual construction and a process of mathematical enculturation. The framework captures the relationship between engaging with another's reasoning, decentering, elaborating justifications, and refining/enriching conceptions.</p> <p>25</p> <p><a href="#">Paper</a></p>
<p><i>Marquis C</i></p>	<p><b>Transforming graduate students' meanings for average rate of change</b></p> <p>Stacy Musgrave and Marilyn Carlson</p> <p>This report offers a brief conceptual analysis of average rate of change (AROC) and shares evidence that even mathematically sophisticated mathematics graduate students struggle to speak fluently about AROC. We offer data from clinical interviews with graduate teaching assistants who participated in at least one semester of a professional development intervention designed to support mathematics graduate students in developing deep and connected meanings of key ideas of precalculus level mathematics as part of a broader intervention to support mathematics graduate students in teaching ideas of precalculus mathematics meaningfully to students. The results revealed that the post-intervention graduate students describe AROC more conceptually than their pre-intervention counterparts, but many still struggle to verbalize a meaning for AROC beyond average speed, a geometric interpretation based on the slope of a secant line, or a computation.</p> <p><a href="#">Paper</a></p> <p>97</p>
<p><b>10:55 – 11:25 am</b></p>	<p><b>SESSION 22 – PRELIMINARY REPORTS</b></p>

<p><i>Marquis A</i></p>	<p><b>Supporting institutional change: A two-pronged approach related to graduate teaching assistant professional development</b></p> <p>Jessica Ellis, Jessica Deshler and Natasha Speer</p> <p>Graduate students teaching assistants (GTAs) are responsible for teaching a large percentage of undergraduate mathematics courses and many of them will go on to careers as educators. However, they often receive minimal training for their teaching responsibilities, and as a result often are not successful as teachers. In response, there is increased national interest in improving the way mathematics departments prepare their GTAs. In this report, we share the initial phases of joint work aimed at supporting institutions in developing or improving a GTA professional development (PD) program. We report on findings from analyses of a baseline survey designed to provide insights into the characteristics of current GTA PD programs in terms of their content, format and duration. Results indicate that there are many institutions seeking improvements to their GTA PD program, and that their needs are in line with the change strategies that the joint projects are employing.</p> <p><a href="#">Paper</a></p> <p>37</p>
<p><i>Marquis B</i></p>	<p><b>Why students cannot solve problems: An exploration of college students' problem solving processes by studying their organization and execution behaviors</b></p> <p>Kedar Nepal</p> <p>This qualitative study investigates undergraduate students' mathematical problem solving processes by analyzing their global plans for solving the problems. The students in three undergraduate courses were asked to write their global plans before they started to solve problems in their in-class quizzes and exams. The execution behaviors of their global plans and their success or failure in problem solving were explored by analyzing their solutions. Only student work that used clear and valid plans was analyzed, using qualitative techniques to determine the success (or failure) of students' problem solving, and also to identify the factors that were hindering students' efforts to solve problems successfully. Many categories of student errors were identified, and how those errors affected students' problem solving efforts will be discussed. This study is based on Garofalo and Lester's (1985), and also Schoenfeld's (2010) frameworks, which consist of some categories of activities or behaviors that are involved while performing a mathematical task.</p> <p><a href="#">Paper</a></p> <p>40</p>

<p><i>Marquis C</i></p>	<p><b>Supporting preservice teachers' use of connections and technology in algebra teaching and learning</b></p> <p>Eryn Stehr and Hyunyi Jung</p> <p>The Conference Board of the Mathematical Sciences recently advocated for making connections and incorporating technology in secondary mathematics teacher education programs, but programs across the United States incorporate such experiences to varying degrees. This study explores preservice secondary mathematics teachers' opportunities to expand their knowledge of algebra through connections and the use of technology and to learn how to use both to support teaching and learning of algebra. We explore the research question: What opportunities do secondary mathematics teacher preparation programs provide for PSTs to learn about connections and encounter technologies in learning algebra and learning to teach algebra? We examine data collected from five teacher education programs chosen from across the U.S. Our data suggest not all secondary mathematics teacher preparation programs integrate experiences with making connections of different types and using technology to enhance learning across mathematics and mathematics education courses. We present overall findings with exemplars.</p> <p><a href="#">Paper</a></p> <p>59</p>
<p><i>Grand Ballroom 5</i></p>	<p><b>Equity in Developmental Mathematics Students' Achievement at a Large Midwestern University</b></p> <p>Kenneth Bradfield</p> <p>With so many students entering college underprepared for the mainstream sequence of mathematics courses, mathematics departments continue to offer developmental or remedial courses with innovative methods of delivery. In order to support all students in their college education, researchers continue to investigate the effectiveness of undergraduate remediation programs with mixed results. This paper provides quantitative data from an NSF-funded project from a large Midwestern university over three years of a developmental mathematics course. Pre- and post-measures show that both urban and African-American students benefited the most from supplemental instruction in contrast to the online-only format. Based on these results, I offer recommendations for undergraduate mathematics departments to support equitable opportunities for all students ensuring a successful developmental mathematics program.</p> <p><a href="#">Paper</a></p> <p>104</p>

<p><i>City Center B</i></p>	<p><b>Mathematicians' rational for presenting proofs: A case study of introductory abstract algebra and real analysis courses</b></p> <p>Eyob Demeke and David Earls</p> <p>Proofs are essential to communicate mathematics in upper-level undergraduate courses. In an interview study with nine mathematicians, Weber (2012) describes five reasons for why mathematicians present proofs to their undergraduate students. Following Weber's (2012) study, we designed a mixed study to specifically examine what mathematicians say undergraduates should gain from the proofs they read or see during lecture in introductory abstract algebra and real analysis. Our preliminary findings suggest that: (i) A significant number of mathematicians said undergraduates should gain the skills needed to recognize various proof type and proving techniques, (ii) consistent with Weber's (2012) findings, only one mathematician said undergraduates should gain conviction from proofs, and finally (3) some mathematicians presented proof for reasons not described in Weber's (2012) study such as to help their students develop appreciation for rigor.</p> <p><a href="#">Paper</a></p> <p>110</p>
<p><i>City Center A</i></p>	<p><b>Student performance on proof comprehension tests in transition-to-proof courses</b></p> <p>Juan Pablo Mejia Ramos and Keith Weber</p> <p>As part of a project aimed at designing and validating three proof comprehension tests for theorems presented in a transition-to-proof course, we asked between 130 and 200 undergraduate students in several sections of one of these courses to take long versions (20 to 21 multiple-choice questions) of these tests. While analysis of these data is ongoing, we discuss preliminary findings about psychometric properties of these tests and student performance on these proof comprehension measures.</p> <p><a href="#">Paper</a></p> <p>123</p>
<p><b>11:35 - 12:05 pm</b></p>	<p><b>SESSION 23 - MIXED REPORTS</b></p>

<p><i>Marquis A</i></p>	<p><b>Opportunity to learn solving context-based tasks provided by business calculus textbooks: An exploratory study</b></p> <p>Thembinkosi Mkhathshwa and Helen Doerr</p> <p>The purpose of this study was to investigate the opportunities to learn how to solve realistic context-based problems that undergraduate business calculus textbooks in the United States offer to business and/or economics students. To do this, we selected and analyzed six different textbooks that are widely used in the teaching of business calculus nationwide. There are three major findings from this study: (1) a majority of the tasks in all the textbooks uses a camouflage context, (2) all the tasks in all the textbooks have matching information, and (3) only three textbooks had reflection tasks. The findings of this suggest that business calculus textbooks do not offer students rich and sufficient opportunities to learn how to solve realistic problems in a business and/or economic context.</p> <p><a href="#">Paper</a></p> <p>3</p>
<p><i>Marquis B</i></p>	<p><b>Students' conceptualizations and representations of how two quantities' change together</b></p> <p>Kristin Frank</p> <p>In this article I discuss the nature of two university precalculus students' meanings for functions and graphs. I focus on the ways in which these meanings influence how these students reasoned about and represented how two quantities change together. My analysis revealed that a student who views a graph as a static shape and does not see a graph as a representation of how two quantities change together will not be successful in constructing meaningful graphs, even in instances when she is able to reason about two quantities changing together. Students made progress in seeing graphs as emergent representations of how two quantities change together when they conceptualized the point <math>(x,y)</math> as a multiplicative object that represented the relationship between an <math>x</math> and <math>y</math> value.</p> <p><a href="#">Paper</a></p> <p>77</p>

<p><i>Marquis C</i></p>	<p><b>Classroom culture, technology, &amp; modeling: A case study of students' engagement with statistical ideas</b></p> <p>Dana Kirin, Jennifer Noll and Erin Glover</p> <p>Advances in technologies have changed the way statisticians do their work, as well as how people receive and process information. The case study presented here follows two groups of two students who participated in a reform-oriented curriculum that utilized technology to engage students with modeling and simulation activities to develop their statistical literacy, thinking, and reasoning. Our analysis applies a social theory of learning and a framework for student engagement as a means for studying students' development of statistical reasoning. In addition, we investigate the impact of a curriculum focused on modeling and simulation on the development of students' statistical reasoning skills.</p> <p><a href="#">Paper</a></p> <p>105</p>
<p><b>12:05 – 1:05 pm</b> <i>Grand Ballroom Salons 2-4</i></p>	<p><b>LUNCH</b></p>
<p><b>1:10 – 1:40 pm</b></p>	<p><b>SESSION 24 – CONTRIBUTED REPORTS</b></p>
<p><i>Marquis A</i></p>	<p><b>Student problem solving in the context of volumes of revolution</b></p> <p>Anand Bernard and Steven Jones</p> <p>The literature on problem solving indicates that focusing on strategies for specific types of problems may be more beneficial than seeking to determine grand, general problem solving strategies that work across large domains. Given this guideline, we seek to understand and map out different strategies students' used in the specific context of volumes of revolution problems from calculus. Our study demonstrates the complex nature of solving volumes of revolution problems based on the multitude of diverse paths the students in our study took to achieve the desired "epistemic form" of an integral expression for a given volume problem. While the large-grained, overarching strategy for these students did not differ much, the complexity came in how the student carried out each step in their overall strategy.</p> <p><a href="#">Paper</a></p> <p>14</p>

<p><i>Marquis B</i></p>	<p><b>Students' conceptions of factorials prior to and within combinatorial contexts</b></p> <p>Elise Lockwood and Sarah Erickson</p> <p>Counting problems offer rich opportunities for students to engage in mathematical thinking, but they can be difficult for students to solve. In this paper, we present a study that examines student thinking about one concept within counting, factorials, which are a key aspect of many combinatorial ideas. In an effort to better understand students' conceptions of factorials, we conducted interviews with 20 undergraduate students. We present a key distinction between computational versus combinatorial conceptions, and we explore three aspects of data that shed light on students' conceptions (their initial characterizations, their definitions of <math>0!</math>, and their responses to Likert-response questions). We present implications this may have for mathematics educators both within and separate from combinatorics, and we discuss possible directions for future research.</p> <p><a href="#">Paper</a></p> <p>32</p>
<p><i>Marquis C</i></p>	<p><b>When should research on proof-oriented mathematical behavior attend to the role of particular mathematical content?</b></p> <p>Paul Christian Dawkins and Shiv Karunakaran</p> <p>Because proving characterizes much mathematical practice, it continues to be a prominent focus of mathematics education research. Aspects of proving, such as definition use, example use, and logic, act as subdomains for this area of research. To yield such content-general claims, studies often downplay or try to control for the influence of particular mathematical content (analysis, algebra, number theory etc.) and students' mathematical meanings for this content. In this paper, we consider the possible negative consequences for mathematics education research of adopting such a domain-general characterization of proving behavior. We do so by comparing content-general and content-specific analyses of two proving episodes taken from the prior research of the two authors respectively. We intend to sensitize the research community to the role particular mathematical content can and should play in research on mathematical proving.</p> <p><a href="#">Paper</a></p> <p>48</p>
<p><b>1:50 – 2:20 pm</b></p>	<p><b>SESSION 25 – CONTRIBUTED REPORTS</b></p>

<p><i>Marquis A</i></p>	<p><b>Lacking confidence and resources despite having value: A potential explanation for learning goals and instructional tasks used in undergraduate mathematics courses for prospective secondary teachers</b></p> <p>Yvonne Lai</p> <p>In this paper, I report on an interview-based study of 9 mathematicians to investigate the process of choosing tasks for undergraduate mathematics courses for prospective secondary teachers. Participants were asked to prioritize complementary learning goals and tasks for an undergraduate mathematics course for prospective secondary teachers and to rate their confidence in their ability to teach with those tasks and goals. While the mathematicians largely valued task types and goals that mathematics education researchers have proposed to be beneficial for such courses, the mathematicians also largely expressed lack of confidence in their ability to teach with these task types and goals. Expectancy-value theory, in combination with these findings, is proposed as one account of why, despite consensus about broad aims of mathematical preparation for secondary teaching, these aims may be inconsistent with learning opportunities afforded by actual tasks and goals used.</p> <p><a href="#">Paper</a></p> <p>94</p>
<p><i>Marquis B</i></p>	<p><b>Helping instructors to adopt research-supported techniques: Lessons from IBL workshops</b></p> <p>Charles N. Hayward and Sandra L. Laursen</p> <p>Inquiry-based learning (IBL) is a research-supported form of active learning in mathematics. While studies continually show benefits of active learning, it is difficult to get faculty to adopt these methods. We present results from a set of intensive, one-week workshops designed to teach university mathematics instructors to use IBL. We use survey and interview data to explore why these workshops successfully got many participants (at least 58%) to adopt IBL. Results are framed through a three-stage theory of instructor change developed by Paulsen and Feldman (1995). We focus specifically on the first stage, 'unfreezing.' In this stage, instructors gain the motivation to change, so these findings may provide the most useful lessons for helping more instructors to adopt research-supported instructional strategies. One of the key factors for the high adoption of IBL was portraying it broadly and inclusively in a variety of contexts, rather than as a highly prescriptive method.</p> <p><a href="#">Paper</a></p> <p>36</p>

<p><i>Marquis C</i></p>	<p><b>Students' obstacles and resistance to Riemann sum interpretations of the definite integral</b></p> <p>Joseph F. Wagner</p> <p>Students use a variety of resources to make sense of integration, and interpreting the definite integral as a sum of infinitesimal products (rooted in the concept of a Riemann sum) is particularly useful in many physical contexts. This study of beginning and upper-level undergraduate physics students examines some obstacles students encounter when trying to make sense of integration, as well as some discomforts and skepticism some students maintain even after constructing useful conceptions of the integral. In particular, many students attempt to explain what integration does by trying to interpret the algebraic manipulations and computations involved in finding antiderivatives. This tendency, perhaps arising from their past experience of making sense of algebraic expressions and equations, suggests a reluctance to use their understanding of "what a Riemann sum does" to interpret "what an integral does."</p> <p><a href="#">Paper</a></p> <p>10</p>
<p><b>2:30 – 3:00 pm</b></p>	<p><b>SESSION 26 – CONTRIBUTED REPORTS</b></p>
<p><i>Marquis A</i></p>	<p><b>A national investigation of Precalculus through Calculus 2</b></p> <p>Chris Rasmussen, Naneh Apkarian, David Bressoud, Jessica Ellis, Estrella Johnson and Sean Larsen</p> <p>We present findings from a recently completed census survey of all mathematics departments that offer a graduate degree (Master's and/or PhD) in mathematics. The census survey is part of a larger project investigating department-level factors that influence student success over the entire progression of the introductory mathematics courses that are required of most STEM majors, beginning with Precalculus and continuing through the full year of single variable calculus. The findings paint a portrait of students' curricular experiences with Precalculus and single variable calculus, as well as the viewpoints held by departments of mathematics about that experience. We see that departments are not unaware of the value of particular features characteristic of more successful calculus programs, but that they are not always successful at implementation. However, our data also suggest hope for the future. Our work not only reveals what is currently happening, but also what is changing, how, and why.</p> <p><a href="#">Paper</a></p> <p>23</p>

<p><i>Marquis B</i></p>	<p><b>When nothing leads to everything: Novices and experts working at the level of a logical theory</b></p> <p>Stacy Brown</p> <p>Building on Antonini and Mariotti's (2008) theorization of mathematical theorem and research on students' meta-theoretical difficulties with indirect proof, this study examines mathematics majors and mathematicians: (1) responses and approaches to the validation tasks related to the assertion <math>S^* \rightarrow S</math>, when given a primary statement, <math>S</math>, of the form <math>\forall n, P(n) \Rightarrow Q(n)</math> and a secondary statement, <math>S^*</math> of the form used in proofs by contradiction; namely, <math>\exists n, P(n) \wedge \sim Q(n)</math>; and, (2) selection of a statement to prove given <math>S^*</math> and <math>S</math>. Findings indicate that novice proof writers responses differ from advanced students and mathematicians both in their approaches and selections, with novices tending to become entangled in natural language antonyms and to engage in the chunking of, rather than parsing of, quantified compound statements.</p> <p><a href="#">Paper</a></p> <p>82</p>
<p><i>Marquis C</i></p>	<p><b>Effects of dynamic visualization software use on struggling students' understanding of calculus: The case of David</b></p> <p>Julie Sutton and James Epperson</p> <p>Using dynamic visualization software (DVS) may engage undergraduate students in calculus while providing instructors insight into student learning and understanding. Results presented derive from a qualitative study of nine students, each completing a series of four individual interviews. We discuss themes arising from interviews with David, a student exploring mathematical relationships with DVS who earns a C in calculus. David prefers to visualize when solving mathematical tasks and previous research suggests that such students, while not the 'stars' of their mathematics classroom, may have a deeper understanding of mathematical concepts that their non-visualizing peers. Using modified grounded theory techniques, we examine evidence of uncontrollable mental imagery, the need to refocus David on salient aspects of the animations, instances when David's apparent conceptual knowledge is neither fully connected to nor supported by procedural knowledge, and David's failure to transfer knowledge when DVS was not offered during assessment.</p> <p><a href="#">Paper</a></p> <p>120</p>
<p><b>3:00 - 3:30 pm</b></p>	<p><b>COFFEE BREAK</b></p>
<p><b>3:30 - 4:00 pm</b></p>	<p><b>SESSION 27 - PRELIMINARY REPORTS</b></p>

<p><i>Marquis A</i></p>	<p><b>Measuring student conceptual understanding: The case of Euler’s method</b></p> <p>William Hall, Karen Keene and Nicholas Fortune</p> <p>This preliminary paper reports on early work for a differential equations concept inventory, which is being developed for an NSF-funded project to support mathematics instructors as they implement inquiry-oriented curricula. The goal is to assess student learning of differential equations. Preliminary results show that the iterative method of developing and field testing items, conducting student interviews, and modification may prove successful to complete a valid concept inventory. The field testing and piloting of questions concerning Euler’s method show that students do respond as the research suggests but that Euler’s method can be recreated by students and the correct response can be “figured out.”</p> <p><a href="#">Paper</a></p> <p>50</p>
<p><i>Marquis B</i></p>	<p><b>Developing mathematical knowledge for teaching in content courses for preservice elementary teachers</b></p> <p>Billy Jackson, Justin Dimmel, Meredith Muller</p> <p>In recent years, much attention in the teacher education literature has been given to ways in which inservice teachers develop facility with the construct known as mathematical knowledge for teaching (MKT). Much less is known about the ability of preservice teachers to construct MKT. To address this, the current preliminary report adds to the research base by investigating two primary questions: (1) Can teachers build MKT in their content courses?, and (2) Can teachers engage in meaningful mathematical discourse as a result of their content courses? The report examines the effects of a semester long course on number and operations designed to allow preservice elementary teachers opportunities to build different aspects of MKT. Very preliminary analysis shows that many students lack this knowledge upon entering the course, but most are able to begin to build a degree of facility in it by course completion.</p> <p><a href="#">Paper</a></p> <p>47</p>

<p><i>Marquis C</i></p>	<p><b>Obstacles in developing robust proportional reasoning structures: A story of teachers' thinking about the shape task</b></p> <p>Matt Weber, Amie Pierone and April Strom</p> <p>This paper presents some initial findings of an investigation focused on mathematics teachers' ways of thinking about proportional relationships, with an emphasis on multiplicative reasoning. Deficiencies in proportional reasoning among teachers can be serious impediments to the development of robust reasoning among their students. As such, this study focuses on how mathematics teachers reason through tasks that involve proportional reasoning by addressing the following two research questions: (1) In what ways do teachers reason through a specific task designed to elicit proportional reasoning? and (2) What difficulties do teachers encounter while reasoning through such tasks? This paper discusses the construction of a robust proportional reasoning structure in the context of a specific task and discusses one particular obstacle, which impedes the construction of such a structure.</p> <p><a href="#">Paper</a></p> <p>107</p>
<p><i>Grand Ballroom 5</i></p>	<p><b>Changes in assessment practices of calculus instructors while piloting research-based curricular activities</b></p> <p>Michael Oehrtman, Matthew Wilson, Michael Tallman and Jason Martin</p> <p>We report our analysis of changes in assessment practices of introductory calculus instructors piloting weekly labs designed to enhance the coherence, rigor, and accessibility of central concepts in their classroom activity. Our analysis compared all items on midterm and final exams created by six instructors prior to their participation in the program (355 items) with those they created during their participation (417 items). Prior exams of the six instructors were similar to the national profile, but during the pilot program increased from 11.3% of items requiring demonstration of understanding to 31.7%. Their questions involving representations other than symbolic expressions changed from 36.7% to 58.5% of the items. The frequency of exam questions requiring explanations grew from 4% to 15.1%, and they shifted from 0.8% to 4.1% of items requiring an open-ended response. We examine qualitative data to explore instructors' attributions for these changes.</p> <p><a href="#">Paper</a></p> <p>111</p>

<p><i>City Center A</i></p>	<p><b>A critical look at undergraduate mathematics classrooms: Detailing mathematics success as a racialized and gendered experience for Latin@ college engineers</b></p> <p>Luis Leyva</p> <p>Latin@s demonstrated an increase of nearly 75% in engineering degree completion over the last 15 years (National Science Foundation, 2015). However, Latin@s remain largely underrepresented across STEM disciplines with scholars calling for analyses of their undergraduate education experiences to improve retention (Cole &amp; Espinoza, 2008; Crisp, Nora, &amp; Taggart, 2009). With calculus as a gatekeeper into advanced STEM courses, undergraduate mathematics must be examined as a social experience for Latin@ engineering students. This report presents findings from a phenomenological study on mathematics success as a racialized and gendered experience among five Latin@ college engineers at a predominantly white institution. In light of recent calls for equity considerations in undergraduate mathematics education (Adiredja, Alexander, &amp; Andrews-Larson, 2015; Rasmussen &amp; Wawro, under review), this report focuses on Latin@ college engineers' mathematics classroom experiences with implications for establishing more positive and meaningful mathematics learning opportunities for Latin@s and other underrepresented populations in STEM.</p> <p><a href="#">Paper</a></p> <p>121</p>
<p><i>City Center B</i></p>	<p><b>Students' sense-making practices for video lectures</b></p> <p>Aaron Weinberg and Matthew Thomas</p> <p>There has been increased interest in the use of videos for teaching techniques such as "flipped" classrooms. However, there is limited evidence that connects the use of these videos with actual learning. Thus, there is a need to study the ways students experience and learn from videos. In this paper, we use sense-making frames as a tool to analyze student's video-watching. We describe preliminary results from interviews with 12 students who watched short videos on introductory statistics and probability concepts and discuss implications for student learning.</p> <p><a href="#">Paper</a></p> <p>126</p>
<p><b>4:10 – 4:40 pm</b></p>	<p><b>SESSION 28 – CONTRIBUTED REPORTS</b></p>

<p><i>Marquis A</i></p>	<p><b>Graduate students' pedagogical changes using iterative lesson study</b></p> <p>Sean Yee, Kimberly Rogers and Sima Sharghi</p> <p>Researchers at two universities implemented an iterative lesson study process with ten graduate student instructors (GSIs), five from each university's mathematics department. Over the span of two weeks, each group of GSIs met with a facilitator to collaboratively plan an undergraduate mathematics lesson, implement the lesson, revise their lesson plan, reteach the lesson to another class of students, and complete a final reflection. Using a multiple case study qualitative methodology, we thematically coded GSI consistencies and revisions to lesson planning during the iterative process according to the Principles to Actions national mathematical teaching practices. At both universities there were specific teaching practices that GSIs used throughout the iterative lesson study and specific teaching practices that GSIs revised. Identifying these teaching practices offers insight into the utility and value of iterative lesson study with graduate student instructors.</p> <p><a href="#">Paper</a></p> <p>51</p>
<p><i>Marquis B</i></p>	<p><b>Investigating the role of a secondary teacher's image of instructional constraints on his enacted subject matter knowledge</b></p> <p>Michael Tallman</p> <p>I present the results of a study designed to determine if there were incongruities between a secondary teacher's mathematical knowledge and the mathematical knowledge he leveraged in the context of teaching, and if so, to ascertain how the teacher's enacted subject matter knowledge was conditioned by his conscious responses to the circumstances he appraised as constraints on his practice. To address this focus, I conducted three semi-structured clinical interviews that elicited the teacher's rationale for instructional occasions in which the mathematical ways of understanding he conveyed in his teaching differed from the ways of understanding he demonstrated during a series of task-based clinical interviews. My analysis revealed that that the occasions in which the teacher conveyed/demonstrated inconsistent ways of understanding were not occasioned by his reacting to instructional constraints, but were instead a consequence of his unawareness of the mental activity involved in constructing particular ways of understanding mathematical ideas.</p> <p><a href="#">Paper</a></p> <p>52</p>

<p><i>Marquis C</i></p>	<p><b>Learning to think, talk, and act like an instructor: A framework for novice tertiary instructor teaching preparation programs</b></p> <p>Jessica Ellis</p> <p>In this report I present a framework to characterize novice tertiary instructor teaching preparation programs. This framework was developed through case study analyses of four graduate student teaching assistant professional development (GTA PD) programs at institutions identified as having more successful calculus programs compared to other institutions. The components of the framework are the structure of the program, the departmental and institutional culture and context that the program is situated within, and the types of knowledge and practices emphasized in the program. In this report I characterize one of the programs involved in the development of the framework as an example of how it is used. In addition to characterizing existing programs, this framework can be used to evaluate programs and aid in the development of new novice tertiary instructor teaching preparation programs.</p> <p><a href="#">Paper</a></p> <p>84</p>
<p><b>4:50 – 5:20 pm</b></p>	<p><b>SESSION 29 – CONTRIBUTED REPORTS</b></p>
<p><i>Marquis A</i></p>	<p><b>How should you participate? Let me count the ways</b></p> <p>Rachel Keller, Karen Zwanch and Steven Deshong</p> <p>Retention of students in STEM majors is an issue of national stability because government projections indicate our nation to need one million additional STEM majors by 2022 (PCAST, 2012); thusly, the current trends in attrition are alarming. Students leave STEM for various reasons, but poor experiences in Calculus I seem to be a significant contributing factor for many switchers, especially female students. Using data situated within a larger study (Characteristics of Successful Programs in College Calculus), the present report looks specifically at student participation and its influence on Calculus I success. Results indicate that while participation is significantly correlated with success, this effect is not uniformly distributed across types of participation or gender groups. Interestingly, overall success rates were equal, but gender differences were noted in frequency of participatory behaviors and distribution of grades; specifically, males (who earned more A grades) preferred in-class participation and females preferred out-of-class participatory activities.</p> <p><a href="#">Paper</a></p> <p>22</p>

<p><i>Marquis B</i></p>	<p><b>Probabilistic Thinking: An initial look at students' meanings for probability</b></p> <p>Neil Hatfield</p> <p>Probability is the central component that allows Statistics to provide a useful tool for many fields. Thus, the meanings that students develop for probability have the potential for lasting impacts. Using Thompson's (20015) theory of meanings, this report shares the results of examining 114 undergraduate students' conveyed meanings for probability after they received instruction.</p> <p><a href="#">Paper</a></p> <p>61</p>
<p><i>Marquis C</i></p>	<p><b>Fostering teacher change through increased noticing: Creating authentic opportunities for teachers to reflect on student thinking</b></p> <p>Alan O'Bryan and Marilyn Carlson</p> <p>This paper reports results from a case study focusing on a secondary teacher's sense-making as she was challenged to reinterpret her meanings for algebraic symbols and processes. Building from these opportunities, she redesigned lessons to gather information about how her students conceptualized quantities and how they thought of variables, terms, and expressions as representing those quantities' values. She then used this information to respond productively to her understanding of individual students' meanings and reasoning elicited during these lessons. We argue that this case study demonstrates the potential for coordinating quantitative reasoning with teacher noticing as a lens to support teacher learning and we recommend specific mathematical practices that can help teachers develop more focused noticing of students' mathematical meanings during instruction.</p> <p><a href="#">Paper</a></p> <p>71</p>
<p><b>5:25 – 6:20 pm</b> <i>Grand Foyer</i></p>	<p><b>POSTER SESSION</b></p>

**Impact of advanced mathematical knowledge on the teaching and learning of secondary mathematics**

Eileen Murray, Debasmita Basu, Matthew Wright

This poster presents preliminary data from an exploratory study that aims to advance our understanding of the nature of mathematics offered to prospective mathematics teachers by looking at mathematical connections. In this study, we investigate how in-service and pre-service middle school teachers make connections between tertiary and secondary mathematics as well as if and how the understanding of connections influences teachers' thoughts about teaching and learning mathematics.

[Paper](#)

**Analyzing classroom developments of language and notation for interpreting matrices as linear transformations.**

Ruby Quea, Christine Andrews-Larson

As part of a larger study of students reasoning in linear algebra, this research analyzes how students make sense of language and notation introduced by instructors when learning matrices as linear transformations. This paper examines the implementation of an inquiry-oriented instruction that consists of students generating, composing, and inverting matrices in the context of increasing the height and leaning a letter "N" placed on a 2-dimensional Cartesian coordinate system (Wawro et. al., 2012). I analyzed two classroom implementations and noted how instructors introduced and formalized mathematical language and notation in the context of this particular instructional sequence, and then related that to the ways that language and notation were subsequently taken up by students. This work was conducted in order to enable me to build theory about the relationship between student learning and the ways in which language and notation are introduced.

[Paper](#)

**What do students attend to when first graphing in R3?**

Allison Dorko

This poster considers what students attend to as they first encounter R3 coordinate axes and are asked to graph functions with free variables. Graphs are critical representations, yet students struggle with graphing functions of more than one variable. Because prior work has revealed that students' conceptions of multivariable graph are often related to their conceptions about single variable functions, we used an actor-oriented transfer perspective to identify what students see as similar between graphing functions with free variables in R2 and R3. We considered what students attended to mathematically, and found that they focused on equidistance, parallelism, and coordinate points.

[Paper](#)

**Support for mathematicians' teaching reform in an online working group for inquiry oriented differential equations**

Nicholas Fortune

There is more need for research on how mathematicians can alter their teaching style to a reform approach (Speer, Smith, & Horvath, 2010), especially if they have always been teaching the same way (Speer & Wagner, 2009; Wagner, Speer, & Rossa, 2007). One particular area that needs more work is investigations of support structures for mathematicians hoping to reform their teaching practice. This poster focuses on supports designed to aid in the reform of teaching practice and specifically discusses the Teaching Inquiry-oriented Mathematics: External Supports (TIMES) project and one online working group (OWG) used as a mode of support in the project. Results indicate that facets of the OWG are successful support structures for mathematicians who desire to align their practice to an inquiry oriented (IO) approach to undergraduate differential equations (Rasmussen & Kwon, 2007; Rasmussen, 2003).

[Paper](#)

**Students' understanding of mathematics in the context of chemical kinetics**

Kinsey Bain, Alena Moon and Marcy Towns

This work explores general chemistry students' use of mathematical reasoning to solve quantitative chemical kinetics problems. Personal constructs, a variation of constructivism, provides the theoretical underpinning for this work, asserting that students engage in a continuous process of constructing and modifying their mental models according to new experiences. The study aimed to answer the following research question: How do non-major students in a second-semester general chemistry course and a physical chemistry course use mathematics to solve kinetics problems involving rate laws? To answer this question, semi-structured interviews using a think-aloud protocol were conducted. A blended processing framework, which targets how problem solvers draw from different knowledge domains, was used to interpret students' problem solving. Preliminary findings describe instances in which students blend their knowledge to solve kinetics problems.

[Paper](#)

**A comparative study of calculus I at a large research university**

Xiangming Wu, Jessica Deshler, Marcela Mera Trujillo, Eddie Fuller and Marjorie Darrah

In this report, we describe the results of analyzing data collected from 502 Calculus I students at a large research university in the U.S. Students were enrolled in one of five different versions of Calculus I offered at the university. We are interested in (i) whether the different characteristics of each version of the course affect students' attitudes toward mathematics and (ii) how each course might affect students' intentions of pursuing science, technology, engineering and mathematics (STEM) degrees. We examine data related to these two issues gathered from students via surveys during one semester in Calculus I.

[Paper](#)

	<p><b>Active learning in undergraduate precalculus and single-variable calculus</b></p> <p>Naneh Apakarian and Dana Kirin</p> <p>The study presented here examines the active learning strategies currently in place in the Precalculus through single variable calculus sequence. While many lament the lack of active learning in undergraduate mathematics, our work reveals the reality behind that feeling. Results from a national survey of mathematics departments allow us to report the proportion of courses in the mainstream sequence utilizing active learning strategies, what those strategies are, and how those strategies are being implemented.</p> <p><a href="#">Paper</a></p>
	<p><b>A Proposed Framework for Tracking Professional Development Through GTA's</b></p> <p>Hayley Milbourne and Susan Nickerson</p> <p>There are several different models of graduate teaching assistant programs in mathematics departments across the nation (Ellis, 2015). One particular public university has recently reformatted their calculus program toward a peer-mentor model and this is the first year of implementation. In the peer-mentor model, there is a lead TA who serves as a support for the other TAs in the program. Because of this, the professional development in which the TAs are engaged is formally directed by both faculty and a peer. We are interested in discussing a framework known as the Vygotsky Space as a methodology for tracking the appropriation and sharing of pedagogical practices among those responsible for calculus instruction.</p> <p><a href="#">Paper</a></p>

**How Calculus students at successful programs talk about their instructors**

Annie Bergman and Dana Kirin

The CSPCC (Characteristics of Successful Programs in College Calculus) project was a 5-year study focused on Calculus I instruction at colleges and universities across the United States with overarching goals of identifying the factors that contribute to successful programs. In this poster, we draw from student focus group interview data collected from schools that were identified by the CSPCC project as being successful. The analyses we will present in this poster will characterize the ways in which calculus students talk about their instructors in an attempt to understand how their perceptions shape their experience.

[Paper](#)

**Investigating university students difficulties with algebra**

Sepideh Stewart and Stacy Reeder

Algebra is frequently referred to as the “gateway” course for high school mathematics. Even among those who complete high school Algebra courses, many struggle with more advanced mathematics and are frequently underprepared for college level mathematics. For many years, college instructors have viewed the final problem solving steps in their respective disciplines as “just Algebra”, but in reality, a weak foundation in Algebra maybe the cause of failure for many college students. The purpose of this project is to identify common algebraic errors students make in college level mathematics courses that plague their ability to succeed in higher level mathematics courses. The identification of these common errors will aid in the creation of a model for intervention.

[Paper](#)

**A qualitative study of the ways students and faculty in the biological sciences think about and use the definite integral**

William Hall

In this poster, I share my methods and pilot interview results concerning a qualitative study of the ways undergraduate students and faculty from the biological sciences think about and use the definite integral. In this research, I utilize task-based interviews including five applied calculus tasks in order to explore how students and faculty think about area, accumulation, and the definite integral. Early results from pilot interviews helped me revise the interview protocols and indicate that student reasoning may be affected by experience and context. In presenting this poster, I hope to gain feedback from the community on my research methodology and potential analytical strategies.

[Paper](#)

**Root of Misconceptions – the Incorporation of Mathematical Ideas in History**

Kuo-Liang Chang

The evolution of a mathematical concept in history has been the process of merging different ideas to form a more rich, general, and rigorous concept. Ironically, students, when learning such well-developed concepts, have similar difficulties and make the same misconceptions again and again. To illustrate, despite the well-developed and defined concept of real numbers, many students still have difficulties in comparing fractions or doing basic operations on irrational numbers. In this poster, the incorporation of different ideas to form a general and rigorous mathematical concept in history is examined. Students' struggles and misconceptions in learning the concepts are investigated from the perspective of the incorporation process. Finally, a model for differentiating and validating the variations of a general mathematical concept is suggested for resolving learning difficulties and misconceptions.

[Paper](#)

**Student Experiences in a Problem-Centered Developmental Mathematics Class**

Martha Makowski

Given the large numbers of students who enroll each year in developmental mathematics classes, community colleges have started creating developmental classes that both engage students in meaningful mathematics and provide them with an efficient pathway to the college-level curriculum. This study examines how community college students enrolled in a problem-based developmental algebra class experience the curriculum and instruction. Eight students from a target classroom were interviewed about their experiences in the class, focusing on how they experienced group work, problem solving, and the role of the teacher. Surveys were also used to measure their attitudes towards mathematics along several dimensions at the beginning and end of the semester. Students who liked group work were energized by the opportunity to work with people while learning and valued the multiple opportunities group work provided for them to check their work. Few students saw group work as an opportunity to explore conceptual ideas. The results suggest that implementations of such classes could benefit from structured discussion about group work norms and explicit discussion about the purpose of solving problems.

[Paper](#)

**A collaborative effort for improving calculus through better assessment practices**

Justin Heavilin, Hodson Kyle and Brynja Kohler

Like many institutions across the country, Utah State University's Department of Mathematics and Statistics has embarked on an effort to improve the calculus sequence with the following objectives: (1) improve our students' comprehension and application of key topics, (2) retain/recruit more students into STEM majors, and (3) provide more consistency across sections. After initial planning and preparation in the 2014-15 academic year, new practices were ready for implementation. In the fall of 2015, teams of instructors worked from common guided course notes, and met weekly to discuss instruction and develop common assessments. This poster displays the methodology of test design and item analysis we employed in the Calculus 2 course. While our team is only at the beginning stages of this work, the methods for creating reliable and relevant measures of student learning hold promise for achieving the goals of our reform.

[Paper](#)

**Exploring student understanding of the negative sign in introductory physics contexts**

Suzanne Brahmia and Andrew Boudreaux

Recent studies in physics education research demonstrate that although physics students are generally successful executing mathematical procedures, they struggle with the use of mathematical concepts for sense making. In this poster we investigate student reasoning about negative numbers in contexts commonly encountered in calculus-based introductory physics. We describe a large-scale study ( $N > 900$ ) involving two introductory physics courses: calculus-based mechanics and calculus-based electricity and magnetism (E&M). We present data from six assessment items (3 in mechanics and 3 in E&M) that probe student understanding of negative numbers in physics contexts. Our results reveal that even mathematically well-prepared students struggle with the way that we symbolize in physics, and that the varied uses of the negative sign in physics can present an obstacle to understanding that persists throughout the introductory sequence.

[Paper](#)

**Classroom observation, instructor interview, and instructor self-report as tools in determining fidelity of implementation for an intervention**

Shandy Hauk, Katie Salguero and Joyce Kaser

A web-based activity and testing system (WATS) has features such as adaptive problem sets, instructional videos, and data-driven tools for instructors to use to monitor and scaffold student learning. Central to WATS adoption and use are questions about the implementation process: What constitutes “good” implementation and how far from “good” is good enough? Here we report on a study about implementation that is part of a state-wide randomized controlled trial examining student learning in community college algebra when a particular WATS suite of tools is used. Discussion questions for conference participants dig into the challenges and opportunities in researching fidelity of implementation in the community college context, particularly the role of instructional practice as a contextual component of the research.

[Paper](#)

**Separating issues in the learning of algebra from mathematical problem solving**

R. Cavender Campbell, Kathryn Rhoads and James A. Mendoza Epperson

Students' difficulty in learning school algebra has motivated a plethora of research on knowledge and skills needed for success in algebra and subsequent undergraduate mathematics courses. However, in gateway mathematics courses for science, technology, engineering, and mathematics majors, student success rates remain low. One reason for this may be to the lack of understanding of thresholds in student mathematical problem solving (MPS) practices necessary for success in later courses. Building from our synthesis of the literature in MPS, we developed Likert scale items to assess undergraduate students' MPS. We used this emerging assessment and individual, task-based interviews to better understand students' MPS. Preliminary results suggest that students' issues in algebra do not prohibit them from using their typical problem solving methods. Thus, the assessment items reflect students' MPS, regardless of possible misconceptions in algebra, and provide a mechanism for examining MPS capacity separate from procedural and conceptual issues in algebra.

[Paper](#)

**Communicative artifacts of proof: Transitions from ascertaining to persuading**

David Plaxco and Milos Savic

With this poster, we wish to highlight an important aspect of the proving process. Specifically, we revisit Harel and Sowder's (1998, 2007) proof schemes to extend the authors' constructs of ascertaining and persuading. With this discussion, we reflect on the original theoretical framework in light of more recent research in the field and draw focus to a critical aspect of the proving process in which the prover generates the communicative artifacts of proof (CAP) critical to shifts between ascertaining to persuading. We also discuss possible ways in which an attention to the psychological and social activities involved in the development of the CAP might inform research and instruction.

[Paper](#)

**On the variety of the multiplication principle's presentation in college texts**

Zackery Reed and Elise Lockwood

The Multiplication Principle is one of the most foundational principles of counting. Unlike foundational concepts in other fields, where there is uniformity in presentation across text and instruction, we have found that there is much variety in the presentation of the Multiplication Principle. This poster highlights the multiple aspects of this variety, specifically those with implications for the combinatorial research and education community. Such topics include the statement types, language and representation of statements, and mathematical implications.

[Paper](#)

**Assessing students' understanding of eigenvectors and eigenvalues in linear algebra**

Kevin Watson, Megan Wawro, and Michelle Zandieh

Many concepts within Linear Algebra are extremely useful in STEM fields; in particular are the concepts of eigenvector and eigenvalue. Through examining the body of research on student reasoning in linear algebra and our own understanding of eigenvectors and eigenvalues, we are developing preliminary ideas about a framework for eigentheory. Based on these preliminary ideas, we are also creating an assessment tool that will test students' understanding of eigentheory. This poster will present our preliminary framework, and examples of the multiple-choice- extended questions we have created to assess student understanding.

[Paper](#)

<p><b>6:30 – 9:00 pm</b>  <i>Grand Ballroom Salons 2-4</i></p>	<p><b>Using adjacency matrices to analyze a proposed linear algebra assessment</b></p> <p>Hayley Milbourne, Katherine Czeranko, Chris Rasmussen and Michelle Zandieh</p> <p>An assessment of student learning of major topics in linear algebra is currently being created as part of a larger study on inquiry-oriented linear algebra. This includes both the assessment instrument and a way to understand the results. The assessment instrument is modeled off of the Colorado Upper-division Electrostatics (CUE) diagnostic (Wilcox &amp; Pollock, 2013). There are two parts to each question: a multiple-choice part and an explanation part. In the explanation part, the student is given a list of possible explanations and is asked to select all that could justify their original choice. This type of assessment provides information on the connections made by students. However, analyzing the results is not straightforward. We propose the use of adjacency matrices, as developed by Selinski, Rasmussen, Wawro, &amp; Zandieh (2014), to analyze the connections that students demonstrate.</p> <p><a href="#">Paper</a></p>
<p><b>8:35 – 9:05 am</b></p>	<p><b>DINNER AND PLENARY</b>  <i>Sean Larsen</i></p>