Reforming the way undergraduate math is taught has been the target of significant research efforts for decades; however, lecture remains the predominant form of instruction. While interest has been primarily focused on entry-level courses in order to recruit and retain STEM-intending students, quality instruction in upper division courses is also important. In a national survey of abstract algebra instructors, we investigated typical teaching practices, beliefs, and constraints that influence pedagogical decisions, and similarities/differences between those who do/not lecture. Of particular interest was exploring whether instructors at Bachelor’s-granting institutions have markedly different circumstances than their counterparts at Master’s- and Doctoral-granting institutions and the effect (if any) this has on their pedagogical decisions.

Key words: Collegiate teaching practices, beliefs, abstract algebra
Johnson & Keller, 2016), it was found that when considering abstract algebra specifically, lecture is overwhelmingly the dominant pedagogical technique both in terms of percentage of instructors using it and percentage of class time devoted to its use.

In our previous study, the target population was mathematics instructors teaching at Master’s- and Doctoral-granting institutions; i.e., ‘research’ universities. The purpose of the present study is to extend this work by investigating the teaching habits, beliefs, and constraints of instructors at Bachelors-granting institutions, traditionally ‘teaching’ colleges, to see if any differences emerge. To that end, we investigate the following research question: Does the way that abstract algebra is taught vary by institution type and can those differences be explained?

**Literature Review**

The literature has cited many reasons why instructors choose to lecture, not least of these is the belief that lecture is the best method and/or necessary for content coverage (as discussed by Roth McDuffie, & Graeber, 2003; Wagner, Speer, & Rosa, 2007; Yoshinobu, 2014). While we do not wish to discount the enormous influence personal beliefs have on instructional decision-making, we must acknowledge that a bevy of other external circumstances can factor considerably when instructors plan their courses. Research has indicated that instructors are reluctant to change practice when serving an administration that provides neither pressure nor incentive to do so and, furthermore, that concerns about promotion and tenure inhibit change when the administration is not perceived to value experimentation in teaching (Roth McDuffie, & Graeber, 2003). Another important factor is departmental and university culture. According to Henderson & Dancy (2007), working with colleagues who either lack knowledge about or withhold support of pedagogical reform inhibits an instructor’s willingness to modify current practice. Finally, departmental resources must be considered as a factor. Instructors who are not offered classroom assistance, release time to plan and prepare tasks and revise syllabi, or financial support for professional development are unlikely to abandon the traditional lecture style in light of the high “start-up costs” demanded by a change in pedagogy (Henderson, Beach, & Finkelstein, 2011; Wagner, Speer, & Rosa, 2007; Roth McDuffie, & Graeber, 2003).

In higher education, a distinction is drawn between what are colloquially referred to as ‘research institutions’ and ‘teaching colleges’. The motivation for this study was rooted in the notion that the culture at teaching institutions might be better suited to adopt non-lecture practices based on the widely-held belief that lack of research demands at these universities affords the instructors time to hone their teaching and thoroughly develop their courses. Additionally, instructors surrounded by faculty and administrators who similarly value teaching might be more likely to have the necessary support for experimenting with new pedagogy. Our literature review showed support for this broad characterization. Differences between teaching and research institutions are real and cannot be attributed to merely academic folklore.

At both types of institutions there are expectations for quality teaching and scholarly research; however, the emphasis can and does vary widely. Professors at research universities are judged and rewarded based on their research; conversely, teaching (or comprehensive) universities “do not effectively accrue status through research” (Henderson, 2009, p. 11). Not surprisingly then, the vast majority of research is produced by faculty at research institutions. A 2003 study of research productivity across higher education found staggering differences in the publication rate of faculty at R1 institutions as compared with Bachelors-granting schools: a 2.04 publication/faculty ratio at the former as compared with a paltry .06 (weighted average across categories) for the latter (Toutkoushian, et al., p. 139). Karen Webber explains this discrepancy:
“especially at research universities, publications and extramurally-funded grants are central to the institutional image and pocketbook, and thus strongly affect individual promotion and tenure” (2011, p. 110). When the most important consideration in tenure decisions is research productivity, it stands to reason that, “teaching is, at best, a secondary obligation” (Vest, 1994). In a 1984 study, Katherine Kasten put forth hypothetical candidates for tenure review with vitaeas reflecting varying degrees of research and teaching quality. The statistical analysis, confirmed by interview data, demonstrated that research is the most important consideration indicating that “excellent scholarship … counterbalances virtually anything else except dereliction of duty” and there exists a “threshold for research below which no degree of teaching…can provide compensation” (p. 506).

In comparison, at teaching institutions, the ‘publish or perish’ ethos has yet to establish a strong foothold. Henderson (2009) explains that while traditional research holds the highest status, “other forms of publication have always been given credit in the faculty reward system…department heads, deans, and provosts have always been happy when faculty members have published, no matter where or what they published” (p. 18). Scholarship is appreciated, but students and teaching are the central focus. At comprehensive universities, where the faculty is likely to be unionized (Henderson, 2009), the contract clearly defines faculty teaching loads and criteria for promotion. In one such example, the most recent collective bargaining agreement of the PA State System of Higher Education (composed of 14 teaching institutions) outlines the teaching load to be 24 credit hours in an academic year with a minimum requirement of 5 office hours per week, and allows for the assignment of up to three unique academic preps per term (APSCUF, p. 74). This document further lists the categories for performance review and evaluation as effective teaching, continuing scholarly growth, and service – in that order (p. 24). While the NCES (Cataldi, Bardburn, & Fahimi, 2004) has shown the average faculty workload is 53.4 hours/week – a figure that does not differ significantly when considered by institution type – but the sort of work activities that occupy that time do vary significantly. Instructors at Doctoral-granting institutions spend on average 25.3% of their time in research as compared with less than 1% (on average) across all other types of institutions.

We believe that these institutional differences may lead to differences in the students’ educational experience. On the one hand, diminished research expectations coupled with heavy teaching assignments would tend to suggest that these instructors are more likely to reflect upon and experiment with their pedagogical practice as compared with their research university counterparts. On the other hand, comprehensive universities are more likely to have scarce financial resources in part due to their disproportionate receipt of grant monies, which could limit participation in the types of professional development opportunities that introduce new pedagogical techniques or disseminate curricular materials and are less likely to find the time needed to make instructional changes due to the increased course loads common at teaching institutions. Webber (2011) estimates that the time needed to prepare well for class may equate to three to five times the number of hours for each hour spent in the classroom (p. 113), representing a non-trivial increase in time commitment for each additional course an instructor is assigned. Furthermore, faculty who are somewhat distanced from the research community, whether due to interest or circumstance, are perhaps less likely to be up to date on the literature advocating for pedagogical change when this work is being done by instructors who are neither their colleagues nor, in some circumstances, their peers. The purpose of the present study is to investigate whether there is evidence to support these suppositions regarding inclination for pedagogical change in the specific context of abstract algebra.
Data and Methods

Survey Design

In the previous study, we developed a survey designed to solicit information about the teaching practices, beliefs, and situational context of abstract algebra instructors. This survey was informed in part by both Henderson and Dancy’s physics-education survey (Henderson & Dancy, 2009) and the Characteristics of Successful Programs in College Calculus surveys1. Our survey had sections to assess each of the following types of information: basic demographics and course context, teaching practices, beliefs and influences (including perceived supports and constraints), and knowledge of and openness to non-lecture practices. In this study, we wished to obtain the same information from a different population for the purposes of comparison. For that reason, it was methodologically important that the items under investigation remain largely unmodified. We chose the subset of items we wanted to ask and kept the formatting the same as the previous survey. Due to space constraints, sample items are not here, but a version of the survey can be found at the following link: pcrg.gse.rutgers.edu/algebrasurvey.

Participants

In the previous research, the initial sample consisted of 200 institutions from which 126 completed surveys were received. In this follow-up, a random sample of 400 institutions was drawn from the IPEDS list, targeting specifically Bachelor’s-granting schools. From this, we received 112 responses, 91 of which were completed. For the purposes of this paper, all data has been combined into one data set and then disaggregated by terminal degree for all future analysis; 117 Type B = Bachelor’s, 59 Type M = Master’s, and 108 Type P = PhD – these designations will be used hereafter.

Methods

We first calculated basic descriptive statistics appropriate for each item and compiled demographic information. Where indicated, percentages were tabulated on the aggregate and for each identified sub-group. Group measures were compared using inference testing procedures such as ANOVA, Chi-Square test, or the Kruskal-Wallis Test as determined by the type of data under investigation. When appropriate, the Holm-Bonferroni correction was applied to control for the family-wise error rate affiliated with multiple comparisons. Details as applicable to particular tests can be found in the Results section. In general, the objective was first to provide a characterization of the abstract algebra course (as reported by our participants), specifically determining who teaches it, what is being taught, and how it is being presented. The secondary analysis was designed to explain those findings based on instructors’ beliefs, resources, and constraints.

Results

In response to our first research question, Does the way that abstract algebra is taught vary by institution type?, the results indicate that in many regards it does, but there are some key characteristics that appear to be universal. Instructors teaching abstract algebra are generally not new faculty; roughly 78% have been teaching for more than six years (i.e. post-tenure) and this does not vary significantly by institution type; however, the experience with teaching AA is quite varied. On the aggregate, there is nearly an even distribution of experience across the three levels

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1 See www.maa.org/cspcc for more information about the CSPCC project and a copy of the surveys.
though this does not persist (Kruskal-Wallis test: $\chi^2 = 6.667$, df = 2, p = .036) when the data are dis-aggregated as the modal class rises proportionately with the type of institution. At Type P institutions, the majority of faculty have taught the course upwards of 8 times, whereas at Type B institutions the majority of faculty have taught the course only 1-4 times. Across institutions the predominant course structure was an undergraduate (61%) course for math majors; other responses included mixed audience courses and graduate courses. Approximately 9% of respondents reported teaching a “groups only” or “rings only” course, but the majority covered both topics in a “groups first” format and this was consistent across institution type. For nearly everyone surveyed, their algebra course is not taught as a capstone course and has a proof-writing prerequisite, although there are distributional differences by type with Type P being least likely to require this prerequisite course (57% as compared with 79% for Type B and 82% for Type M). The most significant, although unsurprising, differences were found in terms of follow-on courses. Type B schools were much less likely to require a second course (6.9%) than Type M (14%) or Type P schools (18.3%).

Moving away from demographic characteristics, the analysis looked at more subjective and personal measures such as pedagogical style and teaching practices. Using the prompt, Have you ever taught abstract algebra in a non-lecture format?, we coded respondents as either self-identified lecturers (hereafter referred to as “Lecturers”) for responding No or I have in the past, but I currently lecture; or as self-identified non-lecturers (hereafter referred to as “Non-lecturers”) for responding I currently do. Our findings support the notion that lecture is still the predominant mode of instruction in upper-division mathematics courses with 83% of our participants identifying as Lecturers; this represented 78% of Type B, 79% of Type M, and 91% of Type P instructors. On the one hand, it appears that our suggestion that instructors at teaching institutions would be less likely to lecture is supported (Z = 2.23, p = .026); however, a lecture rate of 78% does not indicate that sweeping pedagogical reform has occurred.

In terms of teaching practices, again there were a few noticeable differences, but for the most part institution types were more alike than they were different. When asked to report frequency per term for the following activities: having students present proofs or counterexamples to the class, having students develop their own definitions, having student develop their own conjectures, having students develop their own proofs, and leading discussions in which students discuss why the material is useful and/or interesting, no significant differences in mean frequency were observed between institution types. Similarly, when asked to report frequency per class meeting for the following activities: I pause and ask students if they have questions, I use visual and/or physical representations of groups and group elements, I use diagrams to illustrate ideas, and I include informal explanations of formal statements, there were no significant differences in mean frequency observed between institution types. Only for two activities: I have students engage in small-group discussions or problem-solving ($F = 7.984$, p < .001) and I have students ask each other questions ($F = 4.119$, p = .018) were statistically significant differences found: Type B schools engage in these activities more often than Type P schools. (It is important to note that the average increase in mean frequency was approximately .3 which, given the scale used, translates into roughly one additional occurrence per class, or a difference of engaging in the activity once per class versus every other class.)

In addition to reporting on frequency of certain activities, instructors were also asked to comment on the average percentage of class time devoted to certain pedagogical practices on a scale ranging from 0 (never) to 4 (75-100%) of class time. These included: showing students how to write specific proofs, having students work with one another in small groups, having students
give presentations of completed work, having students work individually on problems/tasks, lecturing, holding a whole-class discussion, and having students explain their thinking. On average, the approximate amount of class time devoted to these techniques was under 25%, across practices and institution type; the notable exception was lecturing which occupied, on average, more than 50% of class time. In support of our hypothesis, Type B/M schools tended to devote more time to student-centered pedagogical techniques with significant differences observed for having students work in small groups (F = 12.075, p < .001) and lecturing (F = 12.493, p < .001). Each time, the mean difference between Type B/M and Type P schools was .5 on average, roughly translating to about 12.5% of class time.

Having found evidence suggesting that instructors at Type B schools self-identify as Lecturers less often and that the percentage of class time engaged in lecture for those who do is less than their Type P counterparts, our secondary analysis was focused on trying to explain these differences in terms of beliefs and resources/constraints. The instructors were asked to indicate degree of belief on a 4-point scale (-2 = disagree, -1 = slightly disagree, 1 = slightly agree, 2 = agree) in the following statements regarding teaching: I think lecture is the best way to teach, I think lecture is the only way to teach that allows me to cover the necessary content, I think there’s enough time for all the content I need or want to teach, When I last taught algebra, I had enough time during class to help students understand difficult ideas, and When I last taught algebra, I felt pressured to go through material quickly to cover all the required topics. The instructors were consistent in terms of pressure and time concerns and really only differed in terms of evaluating the appropriateness of lecture. Type P instructors were significantly more likely on average to indicate that lecture was the only way to cover the necessary content (F = 4.688, p = .01) reporting moderate agreement where the others reported disagreement. Belief that lecture is the best way to teach failed to be statistically significant, but instructors at Type P schools did hold this belief more strongly than those at Type M and Type B schools as was expected.

The instructors were also asked to indicate degree of belief on the same 4-point scale in statements regarding students. These included: I think students learn better when they do mathematical work in class, I think students learn better when they struggle with the ideas prior to me explaining the material to them, I think that all students can learn advanced mathematics, I think all students can learn abstract algebra, and I think students learn better if I first explain the material to them and then they work to make sense of the ideas for themselves. Interesting here was that Type P instructors were the most pessimistic about the ability of the students to learn either advanced mathematics in general or abstract algebra specifically (although these differences did not achieve statistical significance). Consistent with the reported lecture practices, Type B/M instructors held significantly stronger belief in the students’ need to do mathematical work in class (F = 5.568, p = .004).

Looking at differences in professional activities, there was little that was unanticipated. When ranking interest on a 4-point scale ranging from 1 (very weak) to 4 (very strong), the following activities were considered: Discussing/reading about how students learn key ideas in abstract algebra, teaching abstract algebra, teaching other advanced classes, doing research in abstract algebra, and doing/reading research that could be considered the scholarship of teaching and learning. Type P instructors had greater interest in algebra research and less

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2 Based on reported means of 0.5 for Type P, 0.3 for Type M, and 0.0 for Type B schools. This resulted in an F value of 3.73 with p = .026 which failed to meet the more conservative p < .0125 threshold for the Holm-Bonferroni correction for multiple comparisons.
interest in education research (reading or doing) and teaching. Significant differences were only observed for the research items ($F = 7.295$, $p = .001$ for algebra research; $F = 4.407$, $p = .013$ for educational research). Probably the most surprising finding here is that all faculty rated interest in teaching nearly a full point higher on average than either type of research activity.

Type P instructors were lecturing at significantly higher rates than Type B/M; however, the lecture rates for all three institution types was quite high. To investigate the propensity for pedagogical modification, the respondents were asked if they would consider teaching in a non-lecture format and were directed to specific follow-up items investigating their reasons why they would not or why they have not. Type P instructors were significantly less likely (48%) to consider not lecturing as compared with Type B/M (65%) instructors ($z = -2.196$, $p = .028$). Those unwilling to consider a change were most likely to cite concerns over content coverage or a belief that it would go poorly (in that order), and surprisingly, this was consistent across all institution types. A very small number of individuals (5) indicated that they would not have departmental support for such a change. For those willing to consider a change, the number one impediment was lack of time to redesign their course, independent of institution type. Secondary concerns were lack of materials and not knowing where to start. Interesting here is that there were 12 individuals who would like to switch but feel that they lack departmental support (6 of whom were Type P instructors).

**Discussion**

Overall, abstract algebra instruction looks fairly similar across institution types, there being many commonalities regarding course structure, class activities, and time use. Virtually all instructors who hold Ph.D.s in mathematics receive degrees from research universities where lecture is the dominant paradigm. It is perhaps to be expected, then, that their instructional decisions – influenced in part by their own experiences as students – might reflect the teaching culture in which they were raised. A difference appears among Type B faculty, though, who self-report as Lecturers at lower rates than Type P (78% versus 91%, respectively) and who spend less (12.5%) class time lecturing than do Type P instructors. Moreover, the fact that using small group discussions and asking students to inquire into one another’s thinking occurs approximately twice as often in Type B classes than in Type P further indicates a distinction in the day-to-day experience of students in these classes. That a clear majority of Type B instructors still report conducting class primarily through lecture, however, indicates that reformers still have a long way to go in spreading student-centered practices in mathematics (at least in abstract algebra), specifically with regard to existing curricular materials.

The current numbers should not detract from what may be a promising outlook: 65% of Type B/M Lecturers reported that they would consider switching to non-lecture pedagogies (significantly more than the 48% of Lecturers at Type P schools). The remaining 35% of Lecturers were most likely to cite concerns over content coverage or their belief that it would go poorly (in that order). It is interesting therefore, that 81% of these same Lecturers also report feeling no departmental pressure to cover a fixed set of topics, making the issue of content coverage largely one of internal orientation. Future directions for this research include examination of the actual content coverage by topic and its variance by institution type, the possible influence possession of a growth/fixed mindset plays in pedagogical decision-making, and researching the potential effect of the interaction between institution type and pedagogical style on the distribution of class time.
References


