Comparing Graph Use in STEM Textbooks and Practitioner Journals

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In this study we focus on the use of graphical representations to find similarities and differences regarding how graphs are used in mathematics textbooks and how they are used in STE textbooks and journals. After highlighting the need for our study and summarizing the results of related studies, we present our methods. We then present key preliminary findings comparing how a selected pre-calculus textbook and certain textbooks and journals in various STE fields use graphical representations. We conclude with preliminary implications and questions.

Keywords: Graphs and graph use; STEM; Precalculus

The changing nature of the global market has highlighted the need for improving STEM education in the U.S. In order for the U.S. to compete with other nations, U.S. students need to enter STEM fields. However, currently U.S. STEM education is surpassed by other nations at the elementary and secondary levels (Holdren, Lander, & Varmus, 2011). Further, mathematics often plays the role of a "gatekeeper" for students' continued study and future success in STEM fields (Crisp, Nora, & Taggart, 2009; Gasiewski, Eagan, Garcia, Hurtado, & Chang, 2012). Given this role, we focus on college level mathematics, particularly precalculus and calculus. These levels are particularly important because more than one third of students intending to pursue a STEM major in the U.S. enroll in mathematics remediation (e.g., precalculus) (Radford, Pearson, Ho, Chambers, & Ferlazzo, 2012) and students interested in STEM majors are more likely to declare a non-STEM major after introductory calculus (Bressoud, Carlson, Mesa, & Rasmussen, 2013).

In the larger study we aim to explore the mathematics presented at these levels as it connects to the demands of science, technology and engineering (STE) fields. Our research question is, "How are graphical representations used in precalculus and calculus textbooks similar to and different from the graphical representations used in STE textbooks and practitioner journals?" Specifically, we focus on nuances of graphical representations of two covarying quantities in mathematics textbooks and in STE textbooks and journals. We note that at the time of submission we have examined one precalculus text and have not had the opportunity to examine any calculus textbooks, although this is our intention.

Literature Review

Rybarczyk (2011) and Roth, Bowen, and McGinn (1999) analyzed several textbooks and research journals in biology and ecology respectively, examining every visual representation in these sources (e.g., diagrams, photographs, graphs, tables, etc.). The researchers identified a mismatch in the types of visual representations used in science textbooks compared to journals. For instance, journals used graphical models to represent statistical data more frequently than textbooks. Because these researchers focused broadly on the different visual representations used across science textbooks and journals, they did not address nuances in how these sources represent two covarying quantities. Such nuances can impact students' interpretation of graphs (e.g., if graphs follow conventions commonly maintained in school mathematics) (Moore, Paoletti, Stevens, & Hobson, 2016; Moore, Silverman, Paoletti, & LaForest, 2014). Hence, in this study, we attempt to close this gap by producing a fine-grained analysis of how STE textbooks and journals use graphs to represent two covarying quantities and compare these to introductory college level mathematics textbooks.

Methods

To address our question, we have begun to gather data from textbooks and journals in STEM fields to explore how graphical representations are used in these different sources. To date, we have analyzed the five sources described in Table 1. We used the Open Syllabus Project (OSP, opensyllabusproject.org) to determine which textbooks were frequently used in STE courses.

Table 1

The Source Title, Author or sub-journals and reasons for including the source, by source.

Source Title (Short name)	Author or sub-journals	Reason for including
Glencoe Precalculus (Precalculus text)	*Glencoe Precalculus* 2014	Precalculus textbook from a major publisher.
Chemistry: The Central Science (Chemistry text)	Brown, LeMay, Bursten, Murphy, and Woodward (2012)	3rd ranked textbook in OSP under search for "chemistry".
Engineering Mechanics: Statics (Statics text)	Hibbeler (2013)	1st ranked textbook in OSP under search for "statics".
IEEE Journals and Physics Today (IEEE/Physics)	IEEE Electron Device Letters IEEE Network IEEE Communications Magazine IEEE Photonics Journal IEEE software IEEE Journal of Selected Topics in Quantum Electronics Ten IEEE "Transactions" journals Physics Today	IEEE is the world's largest technical professional organization for engineers and scientists. They publish a variety of journals and magazines aimed at providing a venue for these professionals to share their knowledge. Physics Today, with a circulation over 100,000 is the membership magazine of the American Institute of Physics.
Journal of the American Medical Association (JAMA)		The most widely circulated medical journal in the world, JAMA publishes original research, editorials and reviews within the biomedical sciences.

With respect to the three textbooks, we focused on the graphical representations that the authors emphasized. As such, we analyzed all graphical representations that were included in the body of the text and did not analyze the graphs in the problem sets as it is up to instructor to assign these problems. With respect to the journals, for JAMA, we started with the most recent issue available through our university library (July 5, 2016) and backtracked through May 2016, identifying all articles with at least one graphical representation. For IEEE/Physics, we identified journals whose stated purpose seemed to align with informing practitioners. We then scoured these journals for articles with at least one graphical representation. For every article we found, we coded any graphs we observed in the article. As our coding of the graphs is both a method and result, we elaborate on how we coded each graph in the results section.

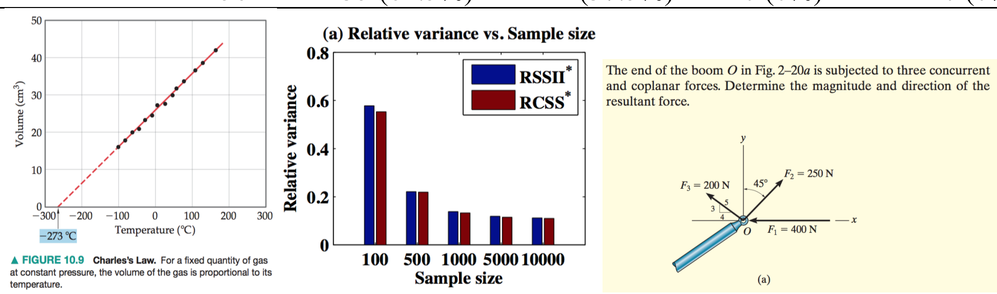
Results

We coded a total of 850 graphs across the five sources (Table 2). Our initial goal was to code the extent to which these sources used graphs to represent relationships between covarying quantities (Covarying Quantities in Table 2, Figure 1a). However, because certain sources used graphs for other purposes, we also coded the number of graphs in each source that were used for these purposes. For instance, and as reported by Rybarzyk (2011) and Roth, Bowen, and McGinn (1999), several sources often presented statistical graphs (Statistical Graphs in Table 2, Figure 1b). Other sources (specifically the Statics text) overlaid a coordinate system over an object or phenomena to help mathematize the situation (Imposing Axes in Table 2, Figure 1c). Table 2 presents the number of each type of graph we observed in each source. We highlight the prevalence of graphs representing two quantities in the precalculus text, chemistry text and IEEE/Physics journals. Statistical graphs played a smaller but still significant role in journal articles when compared to textbooks. Further, we note that the Engineering Statics text almost exclusively used graphs to impose axes on a given object or phenomena.

Table 2

The total number of graphs (N) as well as the number of graphs representing two quantities, statistical graphs, imposing axes, and imaginary planes versus the source.

Source	N	Covarying Quantities	Statistical Graphs	Imposing Axes	Imaginary Plane
Precalculus text	299	273 (91.3%)	23 (7.7%)	0 (0%)	3 (1.0%)
Chemistry text	74	74 (100%)	0 (0%)	0 (0%)	0 (0%)
Statics text	255	3 (1.2%)	0 (0%)	252 (98.8%)	0 (0%)
IEEE/Physics	166	144 (100%)	22 (13.3%)	0 (0%)	0 (0%)
JAMA	56	35 (62.5%)	21 (37.5%)	0 (0%)	0 (0%)



(a) (b) (c)

Figure 1. An example of a graph representing (a) two covarying quantities from Brown et al. (2012, p. 390), (b) statistical information from Li, Yu, Mao, and Jin (2016, p. 13) and (c) imposing axes on a situation from Hibbeler (2013, p. 37).

Because our main focus was on how these sources use graphs to represent the relationship between covarying quantities, all further analysis focused on graphs that fit this subcategory. We were interested in how frequently graphs represent two decontextualized quantities (typically x and y), one contextualized and one decontextualized quantity, or two contextualized quantities (e.g., Figure 1a) (Table 3). We note the significant differences across these sources in regards to using graphs to represent contextualized quantities. Unsurprisingly, STE textbooks and journals almost exclusively used graphs to represent two contextualized covarying quantities. In contrast, graphs in the precalculus textbook rarely represented two contextualized quantities. We found the lack of contextualized examples in the precalculus textbook surprising, and conjecture that we may obtain a different result when we examine calculus textbooks and possibly precalculus textbooks from other publishers.

Table 3

The total number of graphs representing two covarying quantities (CQ), the number of graphs representing two decontextualized, one contextualized one decontextualized, and two contextualized quantities versus the source.

Source	CQ	Two decontextualized	One contextualized one decontextualized	Two contextualized
Precalculus text	273	270 (98.9%)	1 (0.4%)	2 (0.7%)
Chemistry text	74	2 (2.7%)	1 (1.4%)	71 (95.9%)
Statics text	3	0 (0%)	0 (0%)	3 (100%)
IEEE/Physics	144	0 (0%)	0 (0%)	144 (100%)
JAMA	35	0 (0%)	0 (0%)	35 (100%)

Another aspect of graphs representing two quantities that we examined across these sources was the frequency with which conventions with respect to the location of the intersection of the coordinate axes was maintained (Table 4). We coded graphs representing two quantities as either having axes intersect at (0,0), as having axes intersecting at a value other than (0, 0) (e.g., Figure 1a), or if the graph had no scale and we were unable to infer the coordinate values of the intersection of the axes. We note that all graphs in the precalculus and chemistry textbooks had axes that intersected at (0, 0). However, practitioner journals followed this convention with less frequency; it was typical for the intersection of the axes in these sources to not be at (0, 0).

Table 4

The total number of graphs representing two covarying quantities (CQ), the number of these graphs with the intersection of the axes at (0, 0), not at (0, 0) and no scale versus the source.

Source	CQ	Axes intersect at (0, 0)	Axes do not intersect at (0, 0)	No Scale
Precalculus text	273	273 (100%)	0 (0%)	0 (0%)
Chemistry text	74	74 (100%)	0 (0%)	0 (0%)
Statics text	3	0 (0%)	0 (0%)	3 (100%)
IEEE/Physics	144	40 (27.8%)	89 (61.8%)	15 (10.4%)
JAMA	35	19 (54.3%)	16 (45.7%)	0 (0%)

We conjectured that mathematics textbooks used time as a quantity under consideration frequently and wanted to compare how frequently other sources used time. Hence, for all graphs representing covarying quantities with at least one contextualized quantity, we coded if time was represented on the graph (Table 5). Articles in JAMA represented time in a majority of their graphs representing contextualized quantities. The chemistry text and IEEE/Physics journals had significantly more graphs in which time was not a quantity under a consideration.

Table 5

The total number of contextualized graphs, graphs in which time is a quantity under consideration, and time is not a quantity under consideration by source.

Source	Contextualized Graphs	Time is a quantity under consideration	Time is not a quantity under consideration
Precalculus text	3	1 (33.3%)	2 (66.7%)
Chemistry text	72	10 (13.9%)	62 (86.1%)
Statics text	3	0 (0%)	3 (100%)
IEEE/Physics	144	35 (24.3%)	109 (75.7%)
JAMA	35	25 (71.4%)	10 (28.6%)

Preliminary Discussion, Implications, and Future Research

One important preliminary implication of our results is the importance of preparing students who may enter STE fields to use coordinate systems both to represent two covarying quantities and to mathematize a situation or phenomena. Although researchers have focused on students' understandings of representing relationships between covarying quantities (e.g., Carlson, Jacobs, Coe, Larsen, & Hsu 2002; Kozhevnikov, Motes, & Hegarty, 2007; Thompson, 2011) there has been less focus on students' use of coordinate systems to help mathematize an object or phenomena. There have been some efforts to examine the mental operations that students' use when imposing axes onto a situation or phenomena (Lee, 2016; Lee & Hardison, 2016; Piaget & Inhelder, 1967), however, the extent to which the Statics textbook uses coordinate systems to mathematize a phenomena or situation reinforces the need to continue to examine students' understandings of this use of coordinate systems.

A second implication relates to previous researchers' findings in regards to students' interpretations of graphs. There is a large body of research examining individuals' struggles interpreting graphs in both mathematics and the sciences (e.g., Glazer, 2011; Leinhardt, Zaslavsky, & Stein, 1990; Shah & Hoeffner, 2002). For instance, several researchers have indicated students make iconic translations, interpreting graphs intended to represent two covarying quantities as an image of the situation (Carlson et.al., 2002; Leinhardt et al., 1990; Monk, 1992). We conjecture the lack of contextualized graphs in precalculus curriculum may help explain some of the observed struggles students encounter when interpreting contextualized graphs. Consistent with Shah and Hoeffner's (2002) argument, if graphs are presented only abstractly, students are likely to struggle translating this knowledge to graphs in contextualized situations; future research is needed to examine the validity of this conjecture.

A third implication relates to graphing conventions. Researchers (Gattis & Holyoak, 1996; Moore et al., 2016; Moore et al., 2014) have indicated that students often maintain what are conventions to teachers and researchers as inherent aspects of their mathematics. These findings reflect what students experience with their textbooks. If students repeatedly experience graphs in which these conventions are maintained, they may develop mathematical understandings that are constrained by such conventions. Hence, it may be unsurprising when students struggle to make sense of situations in which these conventions are not maintained.

A final implication relates to the extent to which different sources represent time as a quantity under consideration. Entering the study, we conjectured time would be a predominant quantity used in mathematics textbooks but found this was not the case in the precalculus textbook. In JAMA time was a quantity under consideration in a majority of graphs but in the chemistry textbook and IEEE/Physics journals time was a quantity under consideration in less than a quarter of the graphs. The extent to which time is used as a quantity under consideration in various STEM fields requires further examination.

Intended Questions

We intend to examine more precalculus and calculus textbooks. What other sources (both mathematics or sciences textbooks or journals) would be good to consider in this study? Why?
What other data would be worth analyzing within the graphical representations? Why?
What do you see as some other possible implications of a study like this? What could we do to make our implications stronger?

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