Those Who Teach the Teachers: Knowledge Growth in Teaching for Mathematics Teacher Educators

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This theory-based report gives evidence and builds a conceptual framework for a construct called “mathematical knowledge for teaching future teachers” (MKT-FT). Mathematics teacher educators construct MKT-FT as they teach courses for pre-service teachers. Connections to mathematical knowledge for teaching (MKT) are discussed, with an emphasis on the complex relationships between aspects of pedagogical content knowledge in MKT-FT and MKT.

Key words: Mathematical knowledge for teaching, Discourse, Teacher educators

In the 30 years since Shulman’s (1986) seminal speech on the importance of pedagogical content knowledge, a variety of theories about such knowledge have emerged (Depaepe, Verschaffel, & Kelchtermans, 2013). Among the most well known in the U.S. is at the heart of a primary-grades-focused model of mathematical knowledge for teaching (MKT) introduced by Hill, Ball, and Schilling (2008). The subject matter knowledge (SMK) and pedagogical content knowledge (PCK) components of Ball and colleagues’ model of MKT are illustrated in Figure 1.

![Figure 1. Model of mathematical model for teaching (Hill et al., 2008)](image)

In the context of more advanced mathematics, others have explored how the idea of MKT may be productively refined for use in research and development in secondary and post-secondary settings (Hauk, Toney, Jackson, Nair, & Tsay, 2014; Speer, King, & Howell, 2015). Speer and colleagues considered college instructional questions such as: What are the types of specialized, horizon, and common mathematical knowledge for teaching calculus? While Hauk and colleagues have tackled: How does one productively unpack the aspects of PCK – knowledge of content and curriculum, content and teaching, content and students – when the teaching is in a college, the students are adults, the collections of mathematics experiences brought to the classroom are larger, and the sociocultural relationships between student and teacher are quite different from those assumed in the K-8 foundations of the initial framing of...
MKT? Hauk and colleagues (2014) developed an expanded model for college teacher PCK. They used the PCK components in Figure 1 as the vertices of the base of the tetrahedron and added a fourth vertex which they call *knowledge of discourse* (knowledge about the nature of discourse, including inquiry, socio-mathematical norms, and forms of communication in mathematics within and outside of post-secondary educational settings) – see Figure 2.

![Tetrahedron model of PCK (Hauk et al., 2014).](image)

Here we consider a related question: *What is the nature of “mathematical knowledge for teaching” for college instructors who teach mathematics for pre-service elementary teachers?* Such college teachers are teaching adults in post-secondary settings where the mathematical content is in the context of elementary mathematics (rather than advanced) and yet the content is itself linked to K-8 MKT of common, specialized, and horizon subject matter knowledge.

Indeed, Gallagher, Floden and Gwekwerere (2012) note that we know little about what skills are required to be an effective mathematics teacher educator nor do we know much about how those skills develop. Here, by *mathematics teacher educator* we mean anyone who provides guidance, mentoring, or professional learning opportunities to prospective or in-service teachers. The current paper focuses on the subpopulation of mathematics teacher educators who teach mathematics-content-rich courses where the students are pre-service K-8 teachers.

A natural question arises: Why should the Research in Undergraduate Mathematics Education (RUME) community have an interest in examining the knowledge required of mathematics teacher educators to perform their jobs effectively? First and foremost, much of the mathematical preparation for teaching among future K-8 teachers happens in colleges, most at the undergraduate level (Masingila, Olanoff, and Kwaka, 2012). In fact, Masingila and colleagues found that 88% of the teaching of “mathematics for elementary teachers” college courses happens in mathematics departments, with between 27% and 43% of faculty in these departments holding a Ph.D. in either mathematics or mathematics education. Across institution types (2-year, 4-year, and advanced-degree granting) there were more faculty with mathematics Ph.D.s teaching these courses (as opposed to those with Ph.D.’s in mathematics education).

We know that early learning experiences are formative and that children who learn to see themselves as mathematical agents do better in secondary school and beyond (Aud, et al., 2013, Shim, Ryan, & Anderson, 2008; Woodward et al., 2008). We know teaching that supports
children in building skills with mathematical process, practices, and content is socio-culturally rich and responsive to societal as well as community needs (Aud & KewalRamani, 2010; Gay, 2010; Khisty & Chval, 2002; Téllez, Moschkovich, & Civil, 2011). We know that future teachers have greater resources to draw on and are more likely to offer children what they themselves have experienced as learners (including the undergraduate learning experiences that are most proximal to their launch as teachers; e.g. Ball and Bass, 2000; Conference Board of the Mathematical Sciences, 2001; Hodgson 2001). There is a need for mathematics faculty who are prepared to teach mathematics content courses for pre-service elementary teachers (PSETs) in ways that resonate with the kinds of classrooms those future teachers are expected to sustain.

In the U.S., the current population of instructors for such courses includes adjuncts, graduate students, and full time tenure- and non-tenure-track track mathematics faculty (Masingila, et al., 2012). Large segments of this instructor population have difficulty teaching courses for PSETs (Flahive & Kasman, 2013; Greenberg & Walsh, 2008). Though instructors in mathematics departments usually have a deep mathematical background, they often face challenges teaching content that is relevant and has utility for PSETs, unaware of the “cognitive and epistemological subtleties of elementary mathematics instruction” (Bass, 2005, p.419).

Given this state of affairs, Masingila, Olanoff, and Kwaka (2012) advocate for the design and implementation of professional development for mathematics teacher educators. Indeed, Masingila and colleagues note that many faculty who participated in their study asked the researchers where they could find professional learning resources! The RUME community includes experts on such matters: any design and implementation of effective professional development for mathematics teacher educators must involve attention to identifying the types of knowledge that faculty use and need in their regular practice of teaching future teachers.

For these reasons, we propose that college instructors possess a specialized constellation of knowledge to be studied: *mathematical knowledge for teaching future teachers* (MKT-FT). We posit that like MKT, MKT-FT is largely individually constructed for mathematics teacher educators, though often socially mediated. Seaman and Szydlik (2007) discussed the necessity but insufficiency of the early model of MKT for college mathematics instruction, particularly in the context of teaching future teachers. Several authors have noted the existence of what we see as components of MKT-FT. Zopf (2010) and Olanoff (2011) argued that effective teaching of future teachers requires mathematical knowledge of the work of teaching K-8 mathematics and awareness of the complexities of K-8 MKT itself.

According to Rider and Lynch-Davis (2006) and Smith (2003), the mathematical knowledge needed for teaching future teachers attends to the fact that one is teaching adult learners who have some familiarity with the mathematics (as opposed to teaching children who may be learning content for the first time). And, we note, there is a perceived autonomy of the learner in the post-secondary setting that is largely absent in K-8 and high school contexts. Smith (2005) has claimed that faculty who work effectively with future teachers have some (perhaps implicit) knowledge of educational theory and K-12 practice, as well as knowledge resources for connecting ideas and concepts in ways that prepare pre-service teachers to review, select, and engage with the wide array of curricular decisions that must be made by a teacher (e.g., decisions regarding which resources, worksheets, texts, and activities to use or avoid). Olanoff (2011) points out that Deborah Ball herself considers MKT to be the analogue of “common content knowledge” for faculty when considering what it might mean for an instructor to have MKT for teaching teachers.
Research and development on the preparation of teacher educators has long assumed a nesting of types of knowledge. One representation of that can be seen in Carroll and Mumme’s work (2007). Figure 3a represents the nesting of mathematical content as subject matter knowledge (orange disk), linked to (future) teacher and elementary student within the larger context of the classroom (yellow disk). Similarly, in Figure 3a, mathematical knowledge for teaching (the stuff in the yellow disk) is linked to (future) teacher and mathematics teacher educator (“leader”) within the larger context of teacher professional learning (green disk).

Now we have a highly multi-dimensional situation. For each disk in Figure 3a there is an associated set of specifications for what counts as the context and for what constitutes the “content” about which one has “pedagogical content knowledge.” Perks and Prestage (2008) made the case that knowledge for teaching teachers operates on several levels with a partially-nested self-similar design for their model of *teacher-educator knowledge* (Figure 3b).

The model for MKT-FT proposed in Figure 5 blends features of the three models discussed above. At each vertex are both K-8 mathematical content in the college class *and* “content” that is Knowledge of Content and Students in K-8 (illustrated for just the KCT vertex as magnified and highlighted, lower right, in Figure 5). We claim a similar cascade of knowledge structures, related to Content & Students, Curriculum, and Discourses in K-8 are related to the PCK aspect of college mathematics instructor MKT-FT (illustrated by similar “mini” tetrahedral at each of the other vertices in Figure 5).

While the nesting of knowledge structures within others is represented as geometrically self-similar, a fractal structure, the knowledge and thinking represented at each vertex is not identical. Each vertex of the “large” tetrahedron for PCK in the mathematical knowledge for teaching future teachers has a four-to-one mapping. For instance, the MKT-FT vertex for *knowledge of content and students* (KCS) is defined a la Ball as “content knowledge intertwined with knowledge of how students [who are future teachers] think about, know, or learn this particular content” (Hill et al., p. 375). In Figure 5, KCS includes knowledge of how college students engage with learning the MKT they will need in the future as teachers as well as (from the “smaller” tetrahedron) MKT related to K-8 teaching and learning. Knowledge of content and teaching in the MKT model is about teaching moves for working with K-8 mathematics students. In the MKT-FT model, knowledge of content and teaching includes a knowledge of teaching moves for mathematics as a part of teaching the college course for future teachers as well as of teaching moves for including attention to K-8 MKT in teaching that college course.
Teacher education has long noted that task design may be a significant component in the development of knowledge for teaching pre- and in-service teachers. For example, Stylianides and Stylianides (2006) asserted that it is crucial for activities to be teaching-related mathematics tasks, so future teachers learn important school mathematics while at the same time making connections between how learning the content relates to its teaching. For Seaman and Szydlik (2007), well-designed tasks deepen the “mathematical sophistication” of future teachers, which they define as occurring as a result of enculturation into the mathematics community signified by teachers exhibiting the ways of knowing and values of mathematicians as their own. A broader example comes from the Journal of Mathematics Teacher Education issue devoted to the important topic of task design (Zaslavsky, Watson, & Mason, 2007). Papers in this special issue discussed different aspects of good task design in courses for teachers. For instance, according to Chapman (2007), effective tasks facilitate new understandings of familiar concepts and prompt reflection and discourse, while Bloom (2007) argued that quality tasks enhance mathematical habits of mind among college learners who are future teachers.

Yackel, Underwood, and Elias (2007) demonstrated the profound effect that attention to task design and reflection on task implementation can have on MKT and MKT-FT development of those who teach future teachers. One of their instructors commented,

I found it interesting that adult students also go through some of the same progressions that children do. In particular, I often noticed that many students initially needed to use [iconic representations] to perform calculations, such as explicitly drawing boxes, rolls, and pieces... Having never taught young children, I had never seen this first hand. Base 8 gave me the opportunity to experience this part of children’s learning.
[emphasis added]. I think this is valuable to college instructors because most, like myself, will never have an opportunity to work with elementary school children closely. (p. 364)

Hence, we see that task design becomes a part of Knowledge of Content and Teaching for teacher educators: it is part of a teaching move designed to facilitate PSET learning of MKT. This example also reinforces the nonlinearity of our model for MKT-FT, as the instructor above mentions building the Knowledge of Content and Students (both for primary and adult learners) by engaging with their Knowledge of Content and Teaching.

As another example of the pluralistic nature of MKT-FT, consider subject matter knowledge itself: as Ball noted, MKT becomes common content knowledge for mathematics teacher educators. However, MKT-FT specialized content knowledge includes an awareness of and responsiveness to the educational literature as a means of helping PSETs understand why certain mathematical practices or pedagogical practices are favored (e.g., in the Common Core Standards). Horizon knowledge for teacher educators includes recognition of district, state, and national mathematics standards.

And what about the knowledge of discourse discussed earlier? Well, for mathematics teacher educators, MKT-FT knowledge of discourse subsumes the same knowledge of discourse that teachers have, and draws on knowledge of communicating about MKT and the teaching of mathematics in different sociocultural situations.

As Hauk, et al. (2014) point out, the literature on PCK includes both stable and dynamic features. To account for this, they use the edges in their tetrahedron to represent the ways of thinking about teaching mathematics that teachers use in practice. These ways of thinking are enacted in the classroom as teachers adapt to varying sociomathematical and cultural contexts that arise over time. In like manner, effective mathematics teacher educators also possess ways of thinking about teaching mathematics and about teaching MKT that change as the social, mathematical, and cultural climates change in their courses for PSETs. Hence, the edges in our fractal tetrahedron also represent these dynamic ways of thinking for faculty who teach future teachers.

At the Conference

In pursuit of applications of this model in current data analysis and in future research designs, at the conference we will present several examples of both knowledge and thinking as we envision them in the model. We have these questions for RUME participants in the session:
1. What would make a compelling argument for you about the connections among the ideas?
2. What kinds of data provide evidence for each?
3. How might we design a study to focus on a subset of the “small” or “large” tetrahedra?
4. Based on your experience, what connections among the ideas in the model are central?
5. How would knowing the answer to the questions we ask help faculty preparation and development? Or, inform practice of teaching adults who are in- and pre-service teachers?
6. What other questions are coming to mind, now that we have had these questions?

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References


