Analysis of Teachers’ Conceptions of Variation

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The CCSSM emphasize statistical concepts for grades 6-12. A key factor in thinking statistically is to reason about variation and variability. This paper will present the analysis of survey questions and tasks given to in-service middle school teachers. The paper will attempt to answer the following question: “To what extent do middle school math teachers consider variation and variability when thinking about statistics and reasoning through statistical tasks?”

Key words: Statistics, Professional Development, Teacher Education

Organizations such as the American Statistical Association [ASA] (GAISE, 2005) and researchers (Gal, 2003; Gould, 2004; Davidian & Louis, 2012) have discussed the need for and importance of citizens’ statistical literacy. Since 2010, many states across the United States have adopted the Common Core State Standards for Mathematics [CCSSM] (National Governors Association, 2010). These math standards explicitly contain statistics standards for each grade band starting with the sixth grade. This emphasis on statistics in CCSSM is much greater than in previous standards (Tran, Teuscher, Dingman, & Reys, 2014). As a result, math teachers in the middle grades must now attend to statistical concepts as well as solely mathematical concepts.

However, peoples’ prior experiences with statistics in formal educational settings are usually limited to ideas of center where ideas of variation are not emphasized until later grades, if at all (Pereira-Mendoza, 1986; Shaughnessy and Pfankuch, 2002; Noll & Shaughnessy, 2012). In order for teachers to teach statistics in a manner their students might find useful, the teachers themselves must first have productive beliefs, meanings, and ways of thinking about statistics. These productive meanings, beliefs, and ways of thinking about statistics must, by necessity, include productive ideas about variation and variability (Garfield & Ben-Zvi, 2005).

In this paper the researcher will answer the following question: To what extent do middle school math teachers consider variation and variability when thinking about statistics and reasoning through statistical tasks? This paper will include a preliminary analysis on the beliefs and conceptions that middle school math teachers participating in a professional development program hold about ideas of variation and variability. The beliefs and conceptions will be analyzed through the use of open-ended survey questions, and free-response statistical content questions.

Literature Review

Salisbury (1996) discusses variability and variation in data as the differences among people, among environments, and among things. Cobb and Moore (1997) describe statistics as a methodological discipline that arises from the omnipresence of variability. According to the GAISE (2005) framework, statistical thinking “must deal with this omnipresence of variability; statistical problem solving and decision making depend on understanding, explaining, and quantifying the variability in the data” (p. 6). Though the CCSSM (National Governors Association, 2010) does not formally define statistics, there is an implicit theme of variability

1 Statistics used in this paper will mean the discipline of statistics, unless otherwise stated.
that is essential to this idea of statistics. The first 6th grade statistics standard in the CCSSM is for students to develop an understanding of statistical variability. A person who understands statistical variability will be able to recognize that statistical questions anticipate variability in data and considers its role in the answers (National Governors Association, 2010).

Several researchers have studied students’ and teachers’ conceptions of statistics as they relate to variability and variation (Shaughnessy et al, 1999; Torok & Watson, 2000; Saldanha & Thompson, 2002; Liu, 2005; Noll & Shaughnessy, 2012). For example, Saldanha and Thompson (2002), Liu (2005), and Noll and Shaughnessy (2012) discussed productive meanings for individuals to possess about sampling that involved individuals being able to envision variability of sample attributes between outcomes from repeatedly sampling from a population. Thus, the first 6th grade statistics standard is a large part of the foundation for a much more sophisticated statistical concept.

Methodology and Framework

The data collected for this study were gathered through the efforts of a large-scale professional development and research program. This program focused on middle school teachers in a Southwestern state in the United States. Each teacher in the program was asked to participate in professional development activities for two years. The project focused on promoting the mathematical and pedagogical development of its participants.

At the beginning of the second year, the researcher gave two assessment instruments to 50 teachers prior to formal professional development on statistical ideas. The first instrument was a set of mathematical tasks, both multiple choice and free-response, that related to statistical content the teachers were expected to teach in middle school (grades 6-8). The second instrument was an open-ended beliefs survey with seven questions about statistics and statistics teaching.

The researcher performed an initial examination of the teachers’ survey responses by conducting several passes through the data. During each pass, the researcher examined the response for each question from each teacher before moving on to the responses for subsequent questions. The researcher created themes in teacher responses while using the lens of the GAISE (2005) framework in conjunction with CCSSM (2010) for statistics. Using this lens, the researcher examined the two survey questions and two content tasks where teachers had the most opportunity to consider variability as they thought about statistics and reasoned through statistical tasks.

Questions and Task Description

The focus of this paper is on the analysis of two of the beliefs survey questions and two of the tasks. The researcher analyzed the following two questions. Q1: Briefly give a definition for statistics. What do you take this to mean? What comes to mind when you see or hear the word statistics? Q2: To you, what are the differences, if any, between statistics and mathematics? What are the similarities, if any, between statistics and mathematics?

In addition to these survey questions, two tasks were analyzed: The Sampling Task and The Calorie Intake Task. The researcher designed The Sampling Task (Figure 1) to determine what teachers believed to be important aspects of sampling, as well as to determine which aspects of sampling that teachers would give evidence of noticing in their arguments for or against one of the choices.
Each of the choices in The Sampling Task has a designed strength and weakness pertaining to sampling methodology. Chi’s method is the only sampling method that includes random sampling. However, the number of classes in the school are not given in the prompt, thus the size of Chi’s sample may be too inadequate to accept her conclusions. Kendra’s method is the only sampling method that accounts for gender in the sample. However, Kendra’s sample may not be representative of the students at the school due to the existence or composition of certain sports. Diego’s method has the potential for having the largest sample. However, Diego’s sample, while large, may not be representative of the population at the school due to how he picked the classes. From the prompt, the teacher had to determine that Diego’s method is not a census.

The histogram presented in The Caloric Intake Task (Figure 2) depicts a collection of calorie (kcal) counts.

Figure 2: Histogram from The Caloric Intake Task

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2 Assuming gender as being dichotomous.
Using the histogram in Figure 2, the teachers were tasked with describing the data, developing questions to ask their students about the data, and explaining the statistical concepts underlying these questions. The researcher designed this task to determine what teachers believed to be important enough aspects of a distribution of data to ask their students. The researcher wanted to examine the ideas and language relating to the data’s shape, center, and variability that the teachers would (or would not) use in posing questions to their hypothetical students.

**Preliminary Results and Discussion**

The new standards unambiguously state variability as a necessary condition for statistical thinking. However, the preliminary data from the teachers in this study suggest that teachers rarely consider the notion of variability when thinking about statistics or engaging in statistical tasks. In the beliefs survey, only one of the 50 teachers responded to Q1 with anything pertaining to variability.

An important difference between statistics and mathematics is that people who do statistics are focused on the variability of data. Thus, Q2 is a natural question to probe teachers’ thinking about variability. Several teachers responded to the question with statements about utility, subjectivity, or practicality of statistics when related to mathematics. Several teachers also responded with the notion of statistics as a subset of mathematics. None of the teachers responded to Q2 by mentioning anything about variability being a key difference between the two, not even the teacher who had mentioned variability in her response to Q1.

Q1 and Q2 provided the best opportunity for teachers to discuss variability’s role in statistics in the survey format. With only 1% of the total responses mentioning variability, the teachers seem to indicate that they do not consider variability when thinking about statistics. To strengthen this argument, analysis of the statistical tasks is presented below.

The Sampling Task and The Caloric Intake Task provided the teachers with opportunities to reason about statistics both personally and pedagogically. Of the 44 respondents to The Sampling Task, 31 selected only Deigo’s method, 12 selected only Chi’s method, and one selected Kendra’s method or Diego’s method. None of the teachers selected only Kendra’s method, in fact, the teachers overwhelmingly selected against Kendra’s method. As Table 1 shows, many teachers picked out the potential for sampling bias in Kendra’s method as a weakness. None of the teachers (including the lone teacher who picked Diego and Kendra) mentioned the designed strength of Kendra’s method.

### Table 1: Teacher Responses to Kendra’s Method

<table>
<thead>
<tr>
<th>Sample of Teacher Responses to Kendra’s Method</th>
<th>Kendra’s method will undoubtedly be skewed because of clubs and especially sports.</th>
<th>Kendra is only athletes which could be biased to tall people.</th>
<th>Kendra’s only samples a certain demographic and is not representative of &quot;normal&quot;</th>
<th>Kendra or Diego because their samples would be random vs biased. You must take into account not only the sample space but the direction the question at hand is leading you to a conclusion.</th>
</tr>
</thead>
</table>

The variability in middle school students’ heights due to gender\(^3\) is familiar to middle school teachers. However, based on the responses, it seems to have gone unnoticed by the teachers that

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\(^3\)The height differences due to gender in middle school are obvious, but these claims can be corroborated by the Center for Disease Control. [http://www.cdc.gov/nchs/data/series/sr_11/sr11_252.pdf](http://www.cdc.gov/nchs/data/series/sr_11/sr11_252.pdf)
Kendra deliberately accounted for gender in her sampling. If some teachers did notice it, accounting for gender did not seem to be important enough for the teachers to give comment. Given this context that is near to the teachers, the researcher speculates that teachers who attend to the necessity of variability for statistics would have had no problem mentioning this strength of Kendra’s design, even if the teacher did not choose Kendra’s method.

The Caloric Intake Task was more open-ended than The Sampling Task. As stated previously, it was designed to allow the teachers to share what they felt might be salient statistical concepts for their students. A surface-level analysis revealed that 19 of the 31 teachers responded to the task with language or calculations that were related to variability. A deeper analysis of the teacher responses showed that only four of the 31 teachers asked meaningful questions about variability. Of these four teachers, two asked questions of their students about why the caloric intake between students could potentially vary. The other two teachers gave examples of how the caloric intake could potentially vary due to students’ socioeconomic statuses, students’ athlete statuses, and students’ health statuses (Table 2).

Table 2: Teacher Responses to The Caloric Intake Task

<table>
<thead>
<tr>
<th>Sample of Variability Language Responses</th>
<th>Meaningful Variability Responses</th>
<th>&quot;Students are between 1800 and 3699 calories which is a range of 1899 calories.&quot;</th>
<th>&quot;What's the mean? What's the range?&quot;</th>
</tr>
</thead>
</table>
| "Range of intake, mean, median, mode, MAD..." | "Why do you think the data varies?" | "What was the variation of the number of calories consumed?" | "Which groups of students might be in different categories? (athletes, poor who wouldn't get dinner at home, etc)"
| "I would ask my students about the spread and shape of the data as well as the skew." | "There is a large range in calorie intake. I wonder how many students play sports - this could affect the needed calorie intake." |

Based on the fact that the variability response rate for Q1 and Q2 was so low, it is safe to assume that the four teachers are atypical of the teachers in the program at-large. Most of the teachers do not seem to consider variability when thinking about statistics or reasoning through statistical tasks.

Future Work

Teachers responded to Q1 and The Caloric Intake Task with language that relates to variability. However, the teachers’ responses to all four assessment items do not indicate that they are considering how variability influences statistical thinking. In the future, the researcher plans to 1) collect task-based interview data to assess the meanings that teachers have when they use variability language and 2) interact with teachers during lesson-planning activities to focus their attention on attending to variability while planning tasks and activities for their students.

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