Using Women of Color's Intuitive Examples to Reveal Nuances about Basis

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Research and surveys continue to document the underrepresentation of women of color (WOC) in mathematics. Historically, their achievement in mathematics has been framed in a deficit way. Following the broader call for more research concerning WOC's learning experiences in STEM, we interviewed eight WOC about their understanding of basis in linear algebra. We documented diverse ways that these women creatively explained the concept of basis using intuitive ideas from their everyday lives. These examples revealed important nuances and aspects of understanding of basis that are rarely discussed in instruction. These students' ideas can also serve as potentially productive avenues to access the topic. Our results also challenge the existing broader narrative about the underachievement of women of color in mathematics.

Key words: student thinking, basis, linear algebra, equity

Women of color continue to be underrepresented in most areas of science, technology, engineering and mathematics (STEM), and more research is needed to understand the experiences of women of color in those areas (Ong, Wright, Espinosa, & Orfield, 2011). Their underrepresentation is also situated in the national call for more graduates in those fields in the U.S. (PCAST, 2012). In this study, we centralize the mathematical sense making of these students to counter the common colorblind approach to studying cognition. This focus has the potential to construct counter-narratives about women of color's achievement in STEM (Adiredja, in preparation). Research has historically positioned students of color as struggling or underachieving (Harper, 2010).

Research in post-secondary mathematics education has uncovered useful insights into the process of learning of advanced topics by focusing on students' individual cognition. However, scholars have noted the tendency of cognitive studies to deemphasize equity concerns (Martin, Gholson, & Leonard, 2010). Studies of mathematical cognition often take a colorblind approach in which students' background information is omitted (Nasir, 2013). There is a broader call for research at the post-secondary level to focus on addressing inequities, which include exploring ways that studies of student thinking can engage with issues of equity (Adiredja & Andrews-Larson, under review).

Theoretical perspectives on the nature of knowledge and how it develops directly impact the way we assess students' understanding and their contributions in mathematics. Studies of cognition share the power to determine what counts as productive knowledge, how learning is supposed to happen, and what kinds of students benefit in the process (Adiredja, 2015; Apple, 1992; Gutiérrez, 2013). For example, if we believe that mathematical knowledge can only be built upon prior formal mathematical knowledge, then it would be reasonable to privilege such knowledge in learning. However, one implication of this stance is that students who do not have the requisite formal knowledge would then be positioned as "not ready" or "less able," while students who do are seen as "smart," and are allowed to move forward (Gutiérrez & Dixon-Roman, 2011; Herzig, 2004). This deemphasizes the reality that such knowledge has been more available to some groups than others (Oakes, 1990). Cognitive studies can challenge some of these assumptions, and broaden what counts as productive ways of thinking and who counts as successful learners.

One way this is occurring at the undergraduate level is through research that focuses on building from students' informal knowledge and intuitions. For example, the instructional design theory, Realistic Mathematics Education (RME) (Freudenthal, 1983) has inspired some researchers to design curricula that build from experientially real start points (e.g., abstract algebra, Larsen, 2013; differential equations, Rasmussen & Kwon, 2007; geometry, Zandieh & Rasmussen, 2010; linear algebra, Wawro, Rasmussen, Zandieh, Sweeney, & Larson, 2012). Others focus on students' intuitions or conceptual metaphors to make sense of formal mathematics (Adiredja, 2014; Oehrtman, 2009; Zandieh, Ellis, & Rasmussen 2012). Most of these studies focus on using students' intuitions/ informal knowledge as building blocks for the more formal mathematical knowledge. In this paper, we explore ways that students' intuitive explanations can reveal nuances about a formal mathematical topic.

We explore students' explanations about the concept of basis in linear algebra using everyday ideas. Linear Algebra is a critical course for engineering and mathematics majors, and the concept of *basis* is a central topic. There has not been much research done about the concept of basis, though some researchers suggest that it is challenging for students (Stewart & Thomas, 2010). Stewart and Thomas (2010) found that students struggle in identifying span and linear independence in their description of basis. Moreover, they also struggle to define each of those terms, and tend to explain those concepts in terms of procedures. However, the authors found that students who received instruction that emphasized "geometry, embodiment and linking of concepts" (p. 177), were much better in describing the concepts compared to those whose instructions solely focused on symbolic algebra and isolated concepts. Students from the former group were also able to draw richer concept maps about basis than students from the more traditional class. These findings further support the approach of building from experientially real starting points (geometry and embodied ideas) to support students' understanding.

In this paper, we want to explore the diversity of ideas used by eight women of color to describe the concept of basis. We are less interested in identifying students' struggles with the concept of basis. Later we argue for the importance of adopting an anti-deficit perspective in analyzing students' ideas. In particular we explore the following research questions:

1. What everyday contexts do these women use to explain the concept of basis?
2. What do their explanations reveal about nuances in the concept of basis?

In this paper, we position these women as informants into their mathematical thinking and what it reveals about the nuances of the concept of basis.

Conceptual and Theoretical Frameworks

The "Anti-deficit Achievement Framework" from higher education research (Harper, 2010) guides the design and analysis of this project. Instead of perpetually focusing on examining deficits or struggles of students of color in STEM, Harper's framework focuses on understanding the success of these students despite existing inequities. For example, instead of asking the question, "Why are Black male students' rates of persistence and degree attainment lowest among both sexes and all racial/ethnic groups in higher education?" This deficit-oriented question can be reframed with the Anti-deficit Framework as, "How did Black men manage to persist and earn their degrees, despite transition issues, racist stereotypes, academic underpreparedness, and other negative forces?" (p. 68). The Anti-deficit Framework focuses on challenging a particular narrative about underachievement of students of color in STEM, which is also attached to women *of color* (e.g., Why are women dropping out of computer science?). While the author's work focuses on inequities in STEM higher education, some researchers in undergraduate mathematics education share similar perspectives in their study of students (e.g., successful Black mathematics majors, Ellington & Frederick, 2010).

In this paper we focus on countering the narrative of underachievement of women of color in mathematics, particularly with regards to their participation. The principle of "centralizing without essentializing" experiences (Bell, Orbe, Drummon, & Camara, 2000) provides an alternative to *essentializing* experiences of women of color (e.g., "all women of color struggle with basis in this way"). Centralizing instead leads to a focus on understanding the richness of their knowledge and sense making without attributing particular ways of sense making to all students in these groups.

The two authors of this paper come from different theoretical traditions, which guide our joint analysis. The first author comes from Knowledge in Pieces (KiP) (diSessa, 1993) perspective, and the second author has used RME (Freudenthal, 1983) and conceptual metaphors (e.g., Lakoff & Johnson, 1980) to productively analyze student thinking. The commonality of the different perspectives is that they view students' knowledge in an anti-deficit way. These perspectives highlight the use of intuitive ideas to explain mathematical and scientific ideas, and ways that they can be productive in building on students' existing understandings. In laying out our perspectives we position ourselves as being open to students' intuitions and non-normative language.

Methods

Given the desired depth and detail of analysis, this study favored the use of a small number of research subjects and videotaped individual interviews (diSessa, Sherin, & Levin, 2016). Participants were 8 undergraduate female students of color at a large public research university. The university's mathematics advising center shared contacts of mathematics majors and minors who identified as women of color. We invited students via email, and through personal contacts of the authors of this paper. The breakdown of racial and ethnic backgrounds and their past mathematics courses are presented in Table 1. This information was drawn from a student background survey that was administered at the end of the interview. With the exception of one student who is a Biomedical Engineering major (Morgan), all the other students were mathematics major or minor. All pseudonyms were selected to reflect the origin of students' names.

Each interview lasted for 90 minutes. We developed the protocol to explore student understanding of basis. Students started the interview by solving four linear algebra problems that did not mention basis but for which basis could be relevant. We then asked them about basis, which included the way they would define it, and everyday ideas that were useful to explain the concept. In this paper, we focus specifically on students' discussion of everyday examples. These occurred most often in response to questions Q2a and Q2b below, but also sometimes in response to other questions later in the interview.

Q2a. Can you think of an example from your everyday life that describes the idea of a basis?
Q2b. How does your example reflect your meaning of basis? What does it capture and what does it not?

Table 1. *Students' Racial/Ethnic Background and Mathematics Course History*

Student	Racial/Ethnic Background	Linear Algebra Completion	Grade	Other Mathematics Courses
Leonie	African American	Spring 2016	A	Calculus I, II, and III

Morgan	Asian/Asian American	Spring 2016	A	Calculus I, II, and III, and Differential Equations
Annissa	Hispanic/Latin@	Fall 2014	B	Calculus I and II
Eliana	Hispanic/Latin@	Spring 2014	C	Calculus I and II
Nadia	Hispanic/Latin@	Fall 2015	A	Calculus I, II, and III
Jocelyn	Hispanic/Latin@	Spring 2015	B	Calculus I, II, and III
Stacie	Hispanic/Latin@	Spring 2016	C	Calculus I, II, and III
Liliane	Hispanic/Latin@/White	Fall 2015	B/C	Calculus I, II, and III

We transcribed the interviews following guidelines from Ochs (1979). Transcripts were organized by turns, marked by changes in speaker. Transcripts use modified orthography (e.g., wanna, gonna, cus) to stay close to the actual students' utterance. Our analysis first focused on identifying the everyday context and the details associated with that context (e.g., how does the student think about a vector, a vector space, or scalar multiple?). We then differentiated between utterances that had to do with characteristics of the basis vectors, and those that had more to do with roles of the basis vectors in relation to the larger space. The next step of the analysis is developing codes through open coding (Strauss & Corbin, 1994) to capture nuances of students' understanding of basis.

Results

Students' Everyday Examples

We found that the majority of the students discussed at least one everyday context to explain the concept of basis. Table 2 provides a summary of the different contexts. We elaborate on the details of some these contexts in a later section.

Table 2. *Everyday contexts used to explain basis and vector spaces*

Student	Context (for basis and vector space)
Leonie	friendship
Morgan	driving in a city (on a grid), Legos, cooking, groups of pens
Annissa	set of solutions (no actual everyday example)
Eliana	least amount of myself I need to cover the space of the room, storage room, dimension, skeleton, outline of a paper
Nadia	floor, universe and earth, syntax in programming
Jocelyn	fashion, recipe, art sculpture, collage
Stacie	walking to places in a room, floor as a plane, marching band
Liliane	religious teachings

Characteristics and Roles of Basis Vectors

Students discussed characteristics of the basis vectors in their everyday examples. While many of the ideas they brought up can be associated with the notion of linear independence, their ideas reveal nuances about linear independence and its role in defining basis. The first set of codes below capture these characteristics of basis:

1. Minimal/maximal focuses on the required number or amount of vectors needed for the basis. Minimal focuses on the fact that the basis is the least amount of vectors necessary. Maximal focuses on the need to include all the basis vectors and that more would lead to redundancy.

2. Essential focuses on the quality of the vectors being the core and necessary.

3. Representation focuses on naming or identifying the smaller set as the structure or representation of the larger space.
4. Non-redundant focuses on not wanting extraneous information in a set, or the act of reducing or removing the extraneous information.
5. Different/sameness focuses on comparing items (vectors) based on their difference/similarity for the sake of keeping or removing items from the basis.

Prior to conducting the interviews we had noticed that basis vectors are sometimes emphasized in mathematics lessons as a way to *generate* a space of vectors. This is an emphasis on the basis vectors as a spanning set. Other situations might emphasize the role of basis vectors to describe the vector space, e.g., the basis for the null space of a non-invertible matrix serves as a short hand to *describe* that space. We asked students directly about this idea of generating and describing through Q5 and Q6, which ask, "Can you see a basis as a way to [generate/ describe] something?" But even before we asked students directly, they spontaneously brought up the idea of generating and describing in their description of basis, suggesting resonance of this idea. Lastly, related to the code of representation, students also brought up the role of choosing the particular vectors to represent the larger space.
1. Generating: To create the larger space from the basis vectors.
2. Describing: To describe the space using the basis vectors.
3. Sampling: To choose the particular basis vectors as representatives of the larger space.
In the next section we share our analysis of one student, Jocelyn to illustrate how we operationalize these codes.

Illustrative example of analysis. We illustrate our analysis and codes with one student, Jocelyn. Asking students to assess the validity of their everyday example turned out to be very informative of aspects of basis to which students were attending. Jocelyn's case illustrates ways that some of the codes emerged in our analysis. Jocelyn described basis in the context of fashion/creating an outfit. Jocelyn saw basis as minimum number of clothing pieces that "allows you to make all those outfits." In this turn, she was explaining what aspects of basis her example captured. We used bold texts to mark the codes in the write up of our analysis.

Interviewer: In the way you think about this sort of outfit idea to describe basis, um, what aspects of your understanding of basis is captured with your example and what part of it is not captured?

Jocelyn: Um it's minimal. To pick one pair of heels and one pair of tennis shoes. So when I think of my idea of a basis, my mind goes to minimal. Um, what doesn't it capture? Well, ok, so it's weird cause I guess you can use one pair of shoes for different outfits. But like if I'm trying to make...it's harder to kind of have like a casual outfit and in a formal outfit there's not a whole lot of like overlap you end up having each piece in each outfit in the basis. So it's like. How do I explain this? I feel like the basis I'm making, all of the pieces aren't as like they're not all the same. Like you have shoes, tops and pants. You can't make an outfit with just shoes. But if you have a basis, you can pick just some of the vectors, combine them and make something and leave all the rest out. Cause you can't just put on shoes and pants. So that's where it kinda...that's one of the ways that doesn't really [captures it].

Jocelyn attended to the **minimal** aspect of a basis. In addition to using the word minimal, she explicitly identified the need for the specific quantity of one pair of heels and tennis shoes. The particular pair of shoes serves as a **representation** for formal shoes and casual shoes respectively, and you end up with "each piece in each outfit in the basis." Earlier in the interview she discussed the necessity of one pair of heels for a formal outfit and one pair of tennis shoes for a casual outfit. It was also important to Jocelyn that the basis vectors were **different** ("they're not all the same"), and as a set was **non-redundant** (lack of "overlap" in the pieces). She also highlighted the **essential** nature of each element of the basis. She explained this in the way that an outfit needs shoes, pants *and* tops. She asserted, "you can't make an outfit with just shoes," or "just put on shoes and pants."

In addition to attending to particular aspects about the basis vectors, Jocelyn also attended to the roles of the basis vectors in the larger space. In particular she attended to the role of the basis vector in **generating** the larger space, and the necessity of choosing or **sampling** the particular basis vectors to represent the larger space. We coded the phrase "combine them and make something" as highlighting the **generating** relationship. We coded "to pick one pair of heels and one pair of tennis shoes" as also highlighting the **sampling** relationship. This is not to be confused with the representation code, which focuses on the basis vectors as representatives of the larger space.

Jocelyn's assessment also revealed a unique concern that we did not observe with other students about the ability to choose some of the vectors in the basis but not all of them. Jocelyn argued that with basis vectors, one could use a subset of them to generate a vector in the space. In the context of fashion, she could take a subset of the pieces to generate an outfit. The fact that she needed a top, a bottom and a pair of shoes meant that her example required that she had one piece from each category of clothing. She saw this as a limitation to her example. In the next section we explore the extent to which students in the study brought up roles of the basis vectors.

A focus on the generating relationship. In the process of analyzing students' everyday examples we found a wide range of examples of generating, a smaller number of examples of describing, and some examples of sampling. We focus now on most common code on the role of the basis vectors: **generating**. Each student had at least one example of what we label as generating.

Common verbs for the generating code were variations of "to make" or "to build." For example, Morgan talked about building in terms of Legos, "you're given like the 3 by 2 Lego [pieces] and you have like a 2 by 2 Lego [piece] you can just like *build* on to that to create that I guess space that you have." Nadia spoke about computer programming syntax, "Syntax is like stuff so you can *make* a program that doesn't give you an error." Other variations include "add," "expand", and "come from that," which Liliane used in her description of basis using religious teachings:

> So I'm very religious and so the teachings that I that we share with each other and that we read about and all that stuff. Like, there are a lot of things that you can *add* to and be like here's an application and here's the things, and this *expands* to this and this and this. But there's like the most basic teachings and like it all comes back to that. And this is the basic thing like you have the Ten Commandments. You have the Scriptures and you have like the prophets and you have your connection with God and, like all of the decisions and all of things that *come from that* and you can reach all of the other points with this basis.

These examples illustrate the different ways students brought up the notion of generating, and these women's creativity with the everyday contexts.

Discussion and Implication

The two main components of our results illustrate (1) the creativity and breadth of the everyday contexts used to describe basis by these female students of color, and (2) the nuances in understanding of basis that have come out of our open coding of this student data. The range of examples that students used was particularly interesting and useful. Most of these were not examples we had thought of ourselves prior to beginning the study. We are mindful of not gendering or racializing these examples, which would lead to essentializing the students. Students did discuss basis in the context of fashion, cooking and religion, but they also brought up other contexts like driving, skeleton, and the universe. These contexts are likely inspired by the students' experiences, and not their background characteristics. Future studies can further explore the range of contexts to explain basis, and the details of their differences. One can also investigate if there are shared learning experiences among these women that contributed to their flexibility to come up with these examples. Moskovich (2012) asserted that there is nothing inherently different about the cognitive processes of students of color in mathematics. However, there is a difference in their "conditions of learning" (p. 96). We conjecture that the different conditions for learning might have contributed to the creativity of these students.

These women were also fairly sophisticated in judging their own examples in terms of what aspects of the examples worked well for their understanding of basis and what aspects of the context were harder to line up with their understanding of basis. For example, Jocelyn did not think her outfit example captured the idea that you can create vectors in the space by just using one or two of the basis vectors, whereas in her outfit example, one would need to use all three basis vectors (shoes, tops and pants) to create a wearable outfit. In grappling with what aspects of their context worked well and which did not, the students revealed many nuances of basis that we might not have discovered using strictly formal mathematical questions. Together, these students' creativity and sophistication in assessing their examples challenges the narrative of underachievement of women of color that Harper (2010) has noted.

We argue that this paper makes contributions both to research on student cognition in addition to equity research. From a cognitive point of view, this is the only study that we know of that focuses on students' everyday examples of basis. In fact, there have been very few studies done on student understanding of basis and also few studies on students' ability to create everyday examples of mathematical constructs at the undergraduate level. For these reasons, this paper adds to the literature on student mathematical cognition and reasoning at the undergraduate level. In addition, we argue that the paper adds valuable data to the corpus of research in undergraduate mathematics education in that few studies have been written about the mathematical thinking of women of color. Sometimes this is because women of color have not been included in data sets (perhaps because there were not many women of color in the population from which the data was drawn). Other times we simply do not know whether or not women of color were in the data sets because, as one can see from a review of the papers in recent proceeding of the Conference on Research in Undergraduate Mathematics Education (RUME), it is not common in the RUME community to report data on gender and particularly on ethnicity.

This work may have implications for curriculum design. As an example, consider the experientially real starting points emphasized in the curriculum design framework of RME. Our analysis challenges us to reflect on what counts as an experientially real starting point for our students. Creating these experientially real starting points requires us to know our audience. In our past work we may have focused on certain types of students more than others in imagining what is experientially real to this audience. Making sure to interview and listen to the thinking of students who are not as often interviewed in RUME studies is vital to making sure we are reaching all students in instruction and also with our curriculum design.

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