This work investigates the following research question: How do non-major students understand and use mathematics to solve chemical kinetics problems involving integrated rate laws? Personal constructs, a blend of personal and social constructivism, serves as the theoretical framework for this study. Semi-structured interviews with 36 general chemistry students, 5 upper-level physical chemistry students, and 3 chemical engineering students were conducted using a think-aloud protocol. Audio and written data were collected using a Livescribe pen. The audio data were transcribed, and screenshots of students’ written data were inserted into the transcripts; these transcripts were refashioned into problem-solving maps. Open coding of the problem-solving maps reveals initial themes regarding students’ understanding and use of mathematics when solving chemical kinetics problems. Blended processing was used as a methodological framework to guide the coding process. Through this analysis, distinctive types of blended processing have emerged.

**Key words:** Rates, Problem Solving, Blended Processing, Chemistry, Kinetics

Understanding fundamental chemistry concepts is intrinsically tied to understanding mathematical symbolism and operations, as well as the ability to translate between equations and physical reality. This nature has led researchers, such as Becker and Towns (2012), to investigate students’ understanding and use of mathematics in scientific contexts. These lines of inquiry are providing insight to researchers and practitioners on how to best enhance students’ abilities to interpret and use mathematical expressions with conceptual reasoning.

Multiple studies have shown that students need a rich conceptual understanding of algebra and calculus in order to succeed in the physical chemistry classroom (Derrick and Derrick, 2002; Hahn and Polik 2004; Nicoll and Francisco, 2001). Thompson and colleagues showed that students struggled when writing mathematical expressions to describe a physical process (Bucy, Thompson, and Mountcastle, 2007; Thompson, Bucy, & Mountcastle, 2006). Conversely, students also demonstrated difficulty interpreting physical meaning from mathematical equations. In another study, students seemed to hold an isolated understanding of topics in physics and mathematics (Pollock, Thompson, and Mountcastle, 2007). This was studied further by Wemyss, Bajracharya, and Thompson (2011) who showed that when students are given analogous questions in the context of mathematics and physics, they perform better on the former. This finding is in line with other studies on students’ understanding of mathematics in the context of science (Bassok & Holyoak, 1989; Beichner, 1994; Black & Wittmann, 2007; Christensen & Thompson, 2012; Cui, Rebello, & Bennett, 2005, 2007; Cui, Rebello, Fletcher, & Bennett, 2006; Orton, 1983a, 1983b; Shaffer & McDermott, 2005; Zandieh, 2000).

Problem solving is also a key component in the learning and practicing of chemistry (Bodner & Herron, 2002; Summerfield, Overton, & Belt, 2003). As Maloney (2011) found, there are many problem-solving strategies or steps required to solve quantitative problems. In quantitative problem solving, there is often an initial qualitative analysis to understand what the problem is asking (Reif, 1983). This conceptual reasoning step is not only important and beneficial to students, but also demonstrates problem-solving expertise (Hull, Kuo, Gupta, & Elby, 2013; Kuo, Hull, Gupta, & Elby, 2013; Reif, 1983). Such research has led to the development of
problem-solving strategies to help students mirror expert-like behaviors (Heller, Keith, & Anderson, 1992; Huffmann, 1997; Reif, 2008; Van Heuvelen, 1991). However, examining how students use and understand the equations when problem solving in science contexts has rarely been studied.

Chemical kinetics was chosen as a rich context to study quantitative problem solving among chemistry students because of its highly quantitative nature. Furthermore, this work contributes to two key gaps in the literature. In their recent review, Bain and Towns (2016) revealed that research in the area of chemical kinetics at the undergraduate level are rare, unlike other chemistry content areas. Additionally, the National Research Council reported that research on upper-level students and courses were scarce in discipline-based education research (DBER) (Singer, Nielsen, & Schweingruber, 2012). This study targets these literature gaps by investigating how both introductory- and upper-level students use and understand mathematics in the context of chemical kinetics problems. Therefore, the following question serves as the guiding research question for this work: How do non-major students in a second-semester general chemistry course, a physical chemistry course, and a chemical engineering course understand and use mathematics to solve chemical kinetics problems involving integrated rate laws?

**Theoretical Underpinnings**

The theoretical framework that guides this study is Kelly’s (1955) theory of personal constructs. This theory is a combination of personal and social constructivism that posits that while individual's knowledge constructions may differ, they can be similar to one another because of social interaction. The theory of personal constructs provides an appropriate theoretical lens to investigate how individuals understand and use mathematics to solve kinetics problems in that each participant has their own individually constructed understanding of the content.

Blended processing serves as the methodological framework for this study. This framework builds on the basic tenant of constructivism, knowledge as being constructed in the mind of the learner, by describing how an individual’s different knowledge constructions interact, or “blend” (Bodner, 1986; Bodner, Klobuchar, & Gleelan, 2001). Blended processing is a framework stemming from the field of cognitive science that explores human information integration (Coulson & Oakley, 2000). It provides a way to describe and understand individuals’ mental spaces (knowledge constructions) and their interactions (Bing & Redish, 2007; Hu & Rebello, 2013). When multiple mental spaces are activated by external stimulus, knowledge elements from each space interact and are organized in a “blended space”, allowing an individual to make sense of cognitive input in an emergent fashion (Bing & Redish, 2007; Coulson & Oakley, 2000; Fauconnier & Turner, 1996, 1998, 2002; Hu & Rebello, 2013).
Methods

The methods outlined below were chosen as they are consistent with the theoretical underpinnings guiding this work and allow for the investigation of student problem solving in chemical kinetics.

Data Collection

Semi-structured individual interviews were selected as the primary mode of data collection. A stratified purposeful sampling technique was employed, providing a participant sample of second-semester general chemistry students and upper-level physical chemistry students (Patton, 2002). The upper-level students were sampled from two courses: a physical chemistry for biological and life science majors course and a chemical reactions engineering course. Non-major science, technology, engineering, and mathematics (STEM) students served as the target study sample because they are a larger population both at the introductory- and upper-level. Additionally, because the ability to integrate different knowledge domains when solving a problem demonstrates expert-like reasoning, this exploratory study aimed to explore how students in interdisciplinary fields (like engineering) solve problems.

A pilot study comprising of four second-semester general chemistry students was conducted initially to test the viability of the interview protocol (Table 1). Audio and written data were collected via a Livescribe pen. The pilot interviews were conducted by a team of two graduate-student researchers in order to develop a shared understanding of the interview environment, prompts, and probing style. After preliminary analysis and discussion by the research team, the full study interviews were conducted independently by one the two graduate-student researchers. This data collection occurred over two semesters, yielding 36 introductory-level and 8 upper-level interviews (Table 1).

Table 1

<table>
<thead>
<tr>
<th>Study population</th>
<th>Number of students interviewed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fall 2015</td>
</tr>
<tr>
<td>Pilot study (second-semester general chemistry)</td>
<td>4</td>
</tr>
<tr>
<td>Second-semester general chemistry (non-chemistry STEM majors)</td>
<td>17</td>
</tr>
<tr>
<td>Physical chemistry for biological and life science majors</td>
<td>-</td>
</tr>
<tr>
<td>Chemical reaction engineering (chemical engineering majors)</td>
<td>-</td>
</tr>
</tbody>
</table>

The interview protocol consists of four prompts. Two prompts provide the participants with an integrated rate law equation and ask them to describe it. The other two prompts are chemical kinetics questions that are reminiscent of homework- or exam-style questions. These prompts provide students with data and information about a chemical reaction scenario, asking them to reason about a relevant chemical kinetics quantity.

Data Analysis

The data were transcribed verbatim. Participants’ written work were inserted into each transcript where appropriate. In order to make the data more manageable for analysis, problem-solving (PS) maps were generated for each interview. Discrete problem-solving steps in student responses were identified. These steps and the corresponding transcript and written data were
organized into tables chronologically. Table 2 includes an excerpt from one of the pilot study PS maps.

Table 2

*Excerpt from Trip's PS map (he was prompted to explain the second-order integrated rate law)*

<table>
<thead>
<tr>
<th>Student’s problem solving</th>
<th>PS Steps</th>
</tr>
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<tbody>
<tr>
<td>“Okay, so, this is the second-order integrated rate law.”</td>
<td>Recognizes equation</td>
</tr>
<tr>
<td>“And so, it starts with, you have your rate equation for a second order reaction. And then you, if you integrate both sides with respect to time. dt. Then you end up, and then you rearrange it and you get this.”</td>
<td>Recognizes origin of equation</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>“So, basically, the purpose of this is so you can have a function of concentration versus time. Instead of just concentration versus rate. That way it’s easier to use in like the lab.”</td>
<td>Highlights purpose of equation</td>
</tr>
</tbody>
</table>

Keeping the methodological framework of blended processing in mind, multiple rounds of open coding of the PS maps were conducted (Patton, 2002). Often times codes were assigned to excerpts of data as they were organized as steps in the map, meaning a single problem-solving step received one code. Other times multiple codes were assigned to the data in a single step; further, there were instances in which multiple, consecutive steps received a single code. This coding process is ongoing, where the code book is continually being refined via constant comparison methodology (Patton, 2002).

**Preliminary Results**

Preliminary analysis shows variation in how students integrate knowledge domains to make sense of concepts to solve kinetics problems. A non-blended understanding has the potential to limit students’ problem-solving abilities, while a blended understanding serves to support more productive problem solving. Our data suggests that blended processing is not a binary phenomenon; rather, it is dynamic spectrum. For example, when discussing the purpose of a rate law, a general chemistry participant, Hazel, argued that the equation is useful for solving for a third variable whenever you are given two of the values. This type of understanding could limit a student’s ability to understand and make sense a more authentic problem. In contrast, more blended understandings of concepts, such as those demonstrated by other general chemistry
participants, Trip and Damien, where the rate law is interpreted in terms of relationships and changes between variables, are likely to be more supportive of successful problem solving.

Preliminary analysis has also revealed distinctive types of blending processing in students’ problem solving: mathematics blending, chemistry to mathematics, and mathematics to chemistry. Mathematics blending describes student blending of conceptual and formal mathematical reasoning. The second type of blending describes when students take chemistry concepts/information and translate that to mathematical concepts/symbolism. Alternatively, there is also evidence of mathematics to chemistry blending. This is the most common type of blending demonstrated by the pilot study interview participants. While we have only done preliminary analysis on the general-chemistry-level interviews, we do see similar types of reasoning with upper-level physical chemistry students.

We have also noted a variation in problem solving approaches, varying from very simple approaches utilizing one method with little reasoning to more complex approaches trying multiple methods, often with conceptual predictions and justifications. The latter represents a more sophisticated approach to problem solving, as it draws on multiple types of understanding and explicitly incorporates justification for problem solving steps. We plan to explore the relation of 1) success in problem solving and 2) complexity of problem-solving approaches to blended processing.

Conclusions and Questions

While our preliminary results suggest that there is evidence of blending among our student participants, this evidence is sparse and irregular. Chemistry faculty members want mathematics to be connected to (blended with) the chemistry represented in these problems. Ultimately, meaningfully blending and integrating mathematical understandings to science and engineering concepts and problem solving will help students develop a deeper understanding of STEM disciplines. Therefore, practitioners must explicitly model the cognitive practices of blended processing. Furthermore, they should provide students the time and space to practice blending and assess blended processing on course assessments.

The following questions outline potential avenues for investigation.
1. How is student problem-solving success related to blended processing?
2. How is student problem-solving sophistication/complexity related to blended processing?
3. What is the nature of chemical kinetics assessment in the participants’ courses?
4. How do we foster this blending across disciplines in our classrooms?

References


