Using learning trajectories to structure teacher preparation in statistics

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As a result of the increased focus on data literacy and data science across the world, there has been a large demand for teacher preparation in statistics. Project-SET constructed two hypothetical learning trajectories for teacher learning and subsequently used the hypothetical learning trajectories to structure a professional development curriculum. We illustrate how the utilization of learning trajectories to design professional development allowed participating teachers to develop several aspects of Statistics Knowledge for Teaching (Groth, 2013).

Key words: Learning trajectories, statistics, professional development, teacher preparation

Large-scale research provides some indication of the key characteristics of effective teacher training (Doerr, Goldsmith, & Lewis, 2010; Garet, Porter, Desimone, Birman, & Yoon, 2001; Heck, Banilower, Weiss, & Rosenberg, 2008). These include a focus on content knowledge, opportunities for active learning, and coherence with other learning activities (Garet et al., 2001).

The purpose of this article is to report on how using teacher learning trajectories (TLTs) to design teacher training offers a structure to develop teacher content knowledge in deep and meaningful ways. This study is based on the implementation of Project-SET, a project funded by the National Science Foundation to develop professional development curriculum to enhance teachers’ content knowledge of two statistics topics – sampling variability and regression. Project-SET first developed TLTs for these two topics and then designed a professional development curriculum around the trajectories. In exploring the outcomes of the implementation, we recognized the important role that the TLTs played in creating opportunities for teacher participants to achieve different aspects of Statistical Knowledge for Teaching (SKT).

We aim to answer the following research question: How did the use of learning trajectories to design professional development curriculum support the development of teachers’ statistics knowledge for teaching? Our findings suggest that TLTs offer a promising structure for aiding professional development design.

Learning Trajectories

The idea of a learning trajectory (LT) was initially introduced by Simon (1995) as a way to conceptualize how students might progress through a learning sequence. He stated that a LT included “the learning goal, the learning activities, and the thinking and learning in which the students might engage” (pp. 133). Originally grounded in constructive theory, LTs connect students’ thinking and learning for specific mathematical content with a conjectured pathway to move students through a developmental progression (Clements & Sarama, 2004). While all learning trajectories essentially organize learners’ thinking and learning, how a learning trajectory is built and the scope for which it is used differs in the literature.

For example, Confrey and Maloney (2010) describe LTs for a several of topics as “a researcher-conjectured, empirically-supported description of the ordered network of constructs a
student encounters through instruction (i.e. activities, tasks, tools, forms, of interaction and methods of evaluation), in order to move from informal ideas, through successive refinements of representation, articulation, and reflection, towards increasingly complex concepts over time” (Confrey, 2008; Confrey et al., 2008, 2009). Clements and Sarama (2004) instead focus on early childhood mathematics and describe LTs for narrow sequences of topics.

The LTs constructed by Project-SET differ that those discussed in the literature as they are focused on teachers. In other words, the Project-SET LTs were build and designed to conceptualize teacher content knowledge in statistics. Project-SET adapted the definition of a learning trajectory as that of Clements and Sarama (2004): “learning goal, developmental progression of thinking and learning, and sequence of instructional tasks.” The learning goal for the trajectories was to address teacher content knowledge on the topics of sampling variability and regression. This goal was coupled with a progression and instructional tasks.

**Theoretical Framework**

The Project-SET teacher learning trajectories (TLTs) served to guide and develop a professional development for teachers. In applying the LTs to teachers, we intersect LTs with frameworks of teacher knowledge. In particular, we draw upon the Mathematical Knowledge for Teaching (MKT) framework and the Statistical Knowledge for Teaching (SKT) framework (Groth, 2013).

Figure 1 SKT Framework (Groth, 2013, pp. 143)

Groth (2013) identified Key Developmental Understandings (KDUs) as landmarks in the teachers’ development of subject matter knowledge. Building from the work of Simon (2006), Groth describes KDUs as significant conceptual shifts. According to Groth, these landmarks or conceptual shifts can occur in each of the three types of subject matter knowledge in his framework (common content knowledge, specialized content knowledge, and horizon content knowledge). Groth’s SKT framework also incorporates ideas outlined by Silverman and Thompson (2008) regarding the development of pedagogical content knowledge. In particular, Silverman and Thompson assert that teachers’ development of KDUs with regard to subject matter knowledge are a necessary, but perhaps insufficient, first step with regard to improving student learning.
The purpose of the Project-SET professional development was to develop teachers’ content knowledge. These TLTs served as guides for the structure of the professional development. Figure 2 represents the relationship between teacher learning trajectories, professional development, and SKT. This study focuses on understanding how this process might work.

**Figure 2. Project-SET Conceptual Framework**

**Methods**

**Participants**

Nine secondary teachers completed the first implementation of the professional development. Seven of the 9 teachers taught in the local public school district. Two of the teachers taught in a private school within the city. Their average number of years teaching statistics was 2.4.

**Data Sources**

*End-of-Loop Assessment Tasks.* Assessment tasks were completed by the teachers at critical points of the LT in order to measure understanding with respect to the content included in the LT. The scoring of each part was modeled after the AP Statistics scoring of: E (Essentially Correct); P (Partially Correct); or I (Incorrect). The assessment tasks were scored each week by two scorers who were part of the research team but not present during the professional development session. The scorers graded the papers separately and then discussed their scores to come to a consensus on the final scores.

*End-of-LT Assessment.* At the completion of the content of each LT, teachers were assigned as homework an assessment intended to bring together the content of the entire trajectory.

*Video of Class Sessions.* Each class session of the professional development was videotaped. Outlines of the videos were created and portions of the videos were transcribed. The videos provide a means to confirm and elaborate on the observed patterns of teacher learning documented from the teachers’ written work.

**Analysis**

A two-phase process was used to investigate how the use of TLTs supported the development of SKT, with a particular focus on KDUs. The first phase took place during the analysis of teachers’ written work on the End-of-Loop Assessment Tasks and the end-of the LT projects. This analysis provided insight into which ideas were pivotal to teacher understanding thus permitting the research team to identify a preliminary list of KDUs.
In the second phase of the data analysis, these prospective KDUs were then examined through the analysis of classroom interactions. During this phase, the videos were examined in order to determine how teachers’ SKT developed as they progressed through the LTs.

Results

We present illustrative examples of the KDUs that we identified along with supporting evidence consisting of samples of teacher work or transcript segments.

Example 1 Common Content Knowledge KDU: Sample Size and the Sampling Distribution

One of the most persistent ideas that surfaced in teachers’ work and discussion involved the relationship between sample size and the shape and spread of the sampling distribution. We have identified this as a KDU reflecting Common Content Knowledge in Groth’s framework insofar as this is not a concept specific to the domain of teaching.

Teachers repeatedly made statements alluding to the fact that when repeated samples were taken and a sample mean was computed, then the shape of the sampling distribution should become more bell-shaped and the variability of the sampling distribution should decrease. For example, an assessment task for sampling variability asked teachers to compare three different approximate sampling distributions taken with samples of n=5, 15, and 30 according to their shape, variability, and center. There is evidence at this point that the nine teachers developed an understanding of the effect of the sample size on the spread, even if they were not yet clearly articulating the relationship to shape. Three samples of teacher responses are provided:

<table>
<thead>
<tr>
<th>Table 4. Sample Answers to Loop 3 Task</th>
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<tr>
<td>Assessment task question: Compare the three distributions that you constructed. What can you say about the shape of the distribution as the sample size, n, increases? What can you say about the mean? What can you say about the standard deviation?</td>
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<tr>
<td><strong>Teacher Example 1:</strong> As n increases, the data gets more “compact” around the population mean $27,000 – thus the variation decreases. The mean income was closer to the population mean when n=15 than when n=30, but still both close to $27,000. The standard deviation decreases as n increases.</td>
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<tr>
<td><strong>Teacher Example 2:</strong> As the sample size increase, the distributions are becoming less spread out. There is less variability in the distributions as the sample size increases.</td>
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<td><strong>Teacher Example 3:</strong> The shape of the distribution is more unimodal and symmetric, becoming more approximately normal. There is less variation as the sample size n increases. The mean got smaller and then bigger as the sample size increased. However, the mean stayed in the same interval between 25,000 and 30,000. The standard deviation decreased as the sample size n increased. I would expect that trend to continue, but the standard deviation was decreasing at a slower rate as n increased.</td>
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The videos of the class sessions provide further support for the assertion that this was a KDU for teachers that the loop design of the LTs fostered. After investigating these ideas over a period
of time, teachers expressed different “aha” moments around the effect of the sample size. For example, the activity for Loop 3 engaged the teachers in sampling from four populations with vastly different distributions (bimodal distribution, skewed distribution, roughly normal distribution, and scattered distribution). The teachers took samples of size $n=5$, $n=10$, and $n=25$ and generated approximate sampling distributions for each sample size. They compared the sampling distributions of different sizes and noted the similarities in the effect of the sample size on the sampling distribution.

Instructor: So, tell us what you’ve got there from Jamal [the bimodal distribution] and what happened with the samples of different sizes? Teacher 6: As you can see, as you increase the sample size, the variability gets smaller and smaller, if you look you are going from 20 to about 110, versus the spread going from 20 to about 80 where for size 10 you are going about 52 to 66, your variability is decreasing.

As the discussion continued, several teachers also compared the sampling distributions to the corresponding population distributions, noting the way in which the distributions with the larger sample sizes behaved in a similar manner, regardless of the distribution of the population. In particular, even when the population had a non-symmetric or bimodal shape, as the sample size increased, the variability of the sampling distribution decreased. During this comparison of the different distributions, one of the teachers, focused on the behavior of sampling distributions of the bimodal population distribution, began to talk through the reasons behind what she had observed.

Teacher 5: No matter what the population looked like, there was a mean. And our data, or our samplings, were samplings of the average. So, they all should have been near the average of the population. No matter what [the population] looked like.

Another teacher builds on this idea and offers an argument for why the variation of the sampling distribution should logically decrease with an increase in the sample size:

Teacher 7: I guess they can’t do this because they are obviously...cards, but if we had done $N=60$, a.k.a. all the cards, it would have just been a straight line at 60... So that like $N = 60$ literally is just 60, 60, 60, 60 [the mean of the sample] over and over again, just a straight line of 60, but that would have been a good thing to compare to $n = 4$, $n=10$, $n =30$. [Note: The example to which she was referring had a population mean of 60 and a population size of 60.]

In this segment, the teachers appear to not only recognize the effect of the sample size on the spread of the sampling distribution (as with their written work, their descriptions of the shape are not as explicit at this point). They also appear to be creating corresponding mental images of why this makes sense, no matter what the shape of the population is.

The significance of this idea as a KDU is reflected in their own comments a few minutes after the observations from Teacher 5 and 7 described above.

Teacher 3: You know...I’m not sure that I ever understood that...I’m serious. Teacher 5: The light did come on, in terms of understanding what was happening in this activity.

**Example 2 Specialized Content Knowledge KDU: Line of Best Fit Counterexamples**

Specialized content knowledge, defined in the SKT framework as knowledge of content needed in the practice of teaching, may include teachers’ ability to comment on student work and strategize ways they can address student errors. One way to illustrate to students their errors would be to provide students with examples for which their solution will not work. The ability to develop such counterexamples is knowledge specific to teaching.
In the regression LT, the teachers were asked to examine a scatterplot of drop heights versus bounce heights of a golf ball and place a piece of spaghetti on the scatterplot in such a way that they believed represented a line of best fit. Teachers also had to explain what their criterion was for the placement of the line and why they chose to place it there. This same activity had been given to 8th grade students. A second component to the teacher activity was then for the teachers to comment on the 8th grade students’ work and, if the work showed a misunderstanding, then provide a counterexample scatterplot that would illustrate to the student that their placement criterion would not be successful in general.

During part one of the activity, all of the teachers created criteria that matched that of the previously collected student data. For example, one teacher asked “do you assume it [the spaghetti] goes through (0,0)?” She noted that in the context of the problem, dropping balls, if you dropped the ball from 0 height, you would get a 0 bounce height. She thus concluded that her line of best fit must go through the origin. This same reasoning was also seen in the student work. Another teacher stated that she placed her line in such a way that “there are 4 dots above and 4 dots below and so it is in the middle.” Again, similar reasoning was uncovered in the student work with a student stating that they wanted to “split” the points.

When teachers were given the student results to analyze, they were asked to evaluate whether the criteria the students used to place their line was one that would work for any data set. If not, then the teachers were to give an example scatterplot for which their student criteria would not work. This proved difficult for the teachers. For example, one student had the origin criteria similar to one of the teachers. Looking at the student work, she stated: “I think that is a good idea.” However, teacher 8 responded by saying “in this case, it [going through the origin] is ok but not all the time.” At this point, a conversation emerged as to whether the criterion the student applies must work for any set of linear data or just the golf ball data in front of them. After a short deliberation about what defining a criterion means and how it should be applied, it was accepted that a criterion must work for any set of data. Then, the teachers created the counterexample of a data set that had a negative association and thus would require a line to have a negative slope so it would not enable the line to go through the origin. Although the teachers were able to develop counterexamples to help guide student misunderstandings in the context of the line of best fit, the work was non-trivial.

Discussion & Conclusion

As noted by Simon (2006), for someone to develop a KDU, one must have repeated exposure to the concept. Additionally, according to Simon, students without a KDU “do not tend to acquire it through explanation or demonstration” (p. 362); instead a KDU must emerge through discovery. In this way, a person would be able to shift their understanding and gain a Key Understanding. We gathered evidence to show that the Project-SET LTs offered a platform for teachers to develop KDUs by scaffolding more complex ideas and repeatedly looping for each topic. Due to limitations in space, only two examples were presented above. We assert that the design of the Project-SET activities to progress teachers through the TLT facilitated the development of SKT and the emergence of KDUs. In addition, the TLT also allowed for the conceptual unpacking necessary to develop teachers’ knowledge.

The TLTs’ mapping created clearly-defined conceptual boundaries that allowed us to recognize when inadequate connections were begin made to horizon content knowledge. Thus,
although this is a small study with nine teachers, we see promise in the use of TLTs in the design of teacher preparation curriculum to support the growth of teachers’ knowledge. The SKT construct asserts that teacher statistical knowledge for teaching consists of both content knowledge and pedagogical knowledge. We found evidence that building professional development using TLTs can help teachers advance both their content knowledge and their pedagogical knowledge. In addition to the development of subject matter knowledge, we evidenced the translation of this subject matter knowledge into pedagogically powerful ideas.

The goal of this paper was to analyze the affordances of TLTs in the design of professional development. In particular, we sought to understand how the use of TLTs might support the development of SKT, with a particular focus on the KDUs that emerged. Doerr et al. (2010) have identified this type of small-scale study as making important contributions to our understanding of professional development. In particular, through the description and analysis of the “critical elements” (Borko et al., 2008) of the program, it is possible to better understand the teacher learning process and the potential of the program for sustained success.

We view the findings in this paper contributing to the advancement of knowledge and literature base in three ways. First, this study provides small-scale evidence that learning trajectories can not only be used to map student curriculum and learning, but also can be used as maps for teacher curriculum and learning.

A second contribution of this study is the connection of learning trajectories to existing teacher learning constructs such as SKT, KDUs, and Pedagogically Powerful Ideas. This study provides evidence that TLTs offer a means to observe and develop such constructs with teachers.

The analysis of the use of TLTs as a “critical element” of a professional development program suggests that TLTs can offer similar structures for teacher learning that mirror those previously documented for student learning. In particular, the TLTs offered a framework for identifying and achieving KDUs and making instructional decisions based on the KDUs. The TLTs gave the research team a way to see how KDUs were directly related to the development of SKT. Furthermore, the TLTs provided a means for teachers to achieve KDUs due to their repeated exposure while moving through the loop structure of the TLTs. By construction, the TLTs provided scaffolding for KDU development. The repeated exposure illustrated when cognitive shifts were occurring in teachers’ knowledge. In addition, this repetition allowed teachers to transform KDUs into pedagogically powerful ideas.

While Project-SET has a specific focus on teachers’ statistics knowledge, we submit that the implications for mathematics teacher training are broader than this teacher population. In particular, by focusing on a “critical element” of Project-SET— the use of LTs for teacher knowledge— we assert that the model has potential for other content within mathematics teacher professional development. The use of LTs for teacher learning offers a potentially powerful strategy for developing teachers’ knowledge of other concepts within the larger mathematics curriculum.

References


