Students’ Social Adaptation to Mathematical Tasks

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In this study, an advanced undergraduate geometry class taught in an inquiry-based learning setting was observed for social and socio-mathematical norms. Three pairs of students engaged in three task-based, semi-structured interviews: paired, individually, then paired again, solving the Seven Bridges of Königsberg and related tasks. A fourth stimulated-recall interview was performed using episodes from the last paired interview. Classroom observations and interview discourses were open coded for themes, structure, and function to analyze the norms developed within the classroom and by each pair as shaped by their social interactions. Tentative findings include: 1) norms of consensus, autonomy, and argumentation produced within the classroom, 2) varying metaphors across interview contexts, and 3) reliance on empirical strategies rather than structural reasoning. In this preliminary report, evidence from collected data is shared and a brief discussion how these results could help inform IBL teaching methods is included.

Keywords: Active Learning Strategies, Cooperative Learning, Inquiry-based Learning, Socio-Cultural Theory, Transfer

The White House has issued a call to action for incorporating active STEM learning strategies in K-12 and higher education (White House Office of Science and Technology Policy, 2016). Some studies have shown the effectiveness of inquiry-based learning as an active learning strategy for all mathematics students and especially minority groups (Kogan & Laursen, 2014; Laursen, Hassi, Cogan, & Weston, 2014). Almost no literature can be found, however, on the socio-cultural theoretical underpinnings of the transfer of inquiry-based learning to individual work.

The purpose of the dissertation study shared in this preliminary report was to explore the social and socio-mathematical norms of classroom using inquiry-based learning and the reproduction (or non-reproduction) of those norms in an interview setting. The research questions of this ongoing study are:
1. What were the social and socio-mathematical norms of the classroom?
2. How were these norms reproduced (or not) by the students in the interview settings?
3. In what ways did changes in the social context of the interview settings affect the mathematical practices produced by the students?

Methods

The setting of the study was at a four-year university in the Rocky Mountain region in an upper level mathematics course. In this inquiry-based learning geometry course students worked in small groups on projects and submitted written reports, either as a group or individually. The instructor of the course, Dr. Jackson (pseudonym), had taught the course using inquiry-based learning on more than twenty occasions. All class sessions were observed using written notes, video recording, and audio recording. During these observations, the researcher (first author) described the propositions used by students and the instructor, as well as the perceived sources of authority for those propositions. Descriptions of propositions and sources of authority were then open coded for themes.
The students in the course were a mixture of elementary education majors with an emphasis in mathematics, and mathematics majors with emphases in liberal arts, applied mathematics, or secondary education. Six students, Leo, Jemma, Skye, Melinda, Phil, and Grant (pseudonyms) agreed to participate in interviews, which were conducted as a mixture of individual and paired settings, the first three being task-based and the fourth being a stimulated recall interview (See Figure 1 for a full description of the interview structure). Interviews were then transcribed in full and each utterance was coded for structure. Open coding was used to describe the function of each utterance. Examples of functions included expressing beliefs, proffering hypotheses, and making conjectures. In this report, interview analysis from only one pair, Leo and Jemma, is shared.

<table>
<thead>
<tr>
<th>Interview</th>
<th>Setting</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interview 1</td>
<td>Paired, Task-Based</td>
<td>To investigate reproduction or non-reproduction of social and socio-mathematical norms of the classroom while performing a novel task as a pair.</td>
</tr>
<tr>
<td>Interview 2</td>
<td>Individual, Task-Based</td>
<td>To investigate influence of paired setting on individual cognition.</td>
</tr>
<tr>
<td>Interview 3</td>
<td>Paired, Task-Based</td>
<td>To investigate the influence (or non-influence) of individual cognition on discourse of the paired setting.</td>
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<tr>
<td>Interview 4</td>
<td>Individual, Stimulated-Recall</td>
<td>To gain insight into the covert thoughts of participants during Interview 3.</td>
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Figure 1. Stages of the interviews

The Interview 1 Task was the traditional Seven Bridges of Königsberg. Pairs were prompted with a map and tasked with providing a path that crossed every bridge once and only once, or provide an explanation as to why no such path existed. The Interview 2 Task was an amendment to the Interview 1 task, in that the individuals were instructed to: 1) construct a bridge such that one could create a path that crossed each bridge once and only once starting in the Red District and ending in the Gold District but it was not possible to construct such a path starting in the Blue District and ending in the Gold District (see Figure 2), 2) construct a second bridge such that one could create a path starting in the Blue District and ending in the Gold District, but no such path could be created starting in the Red District and ending in the Gold District, and 3) construct a third path such that one could create paths that crossed each bridge once and only once, starting and ending in both the Red and Blue Districts.

The Interview 3 Task was to create a continuous curve that crossed each edge of a figure (See Figure 3) once and only once or provide an explanation as to why such a curve was not possible.

Figure 2. Seven Bridges of Königsberg with locations of Red District (southern square), Blue District (northern square), Gold District (central circle), and Grey District (eastern circle).
Figure 3. Figure of the Interview 3 Task

The Interview 4 was a stimulated recall interview using episodes video recorded from Interview 3. Episodes were chosen based on the propositions made during Interview 3. Participants were instructed to explain any thoughts or feelings they had during each episode, and could pause or rewind if they felt compelled to do so. After each episode, the interviewer (first author) would ask follow-up questions if the participant had not already addressed them.

**Preliminary Results**

**Classroom Observation Findings**

Three themes emerged in observing the classroom: *consensus, autonomy*, and *argumentation* (see Figure 4 for a summary of the themes and subthemes). Propositions were sorted into three categories: *beliefs, hypotheses*, and *conjectures*. Beliefs were defined as propositions that were made without prior evidence, hypotheses were defined as propositions made with unverified (by the participants) evidence, and conjectures were defined as propositions made with verified evidence. Categories of sources of authority were *group consensus, prior group consensus*, *authority figure, implicit, preference*, and *logic*. *Group consensus* was evidenced by a small group or the whole class reaching a sense of agreement. *Prior group consensus* was evidenced by a reference to consensuses that had been reached previously. *Authority figure* was evidenced by an appeal to either Dr. Jackson or the researcher (first author). *Implicit* was evidenced by no overt appeal to external authority, implying implicit appeal to the norms established within the classroom such as prior group consensus. *Preference* was evidenced by an appeal to personal preference.

In analyzing these themes and their interactions with one another, I could describe the evolution of the classroom could be described as follows: Students, in general, began the course lacking direction and authority to make and evaluate their own propositions. Dr. Jackson was able to source their authority in group consensus by holding whole-class discussions and building group consensus, as well as referencing these consensuses at later times. This afforded students with the opportunity to transition from lacking autonomy, to tentative autonomy, to free expression. With each new topic or discussion, however, this sense of autonomy could reset and students could revert back to lacking autonomy. By focusing on stylistic and structural aspects of argumentation, Dr. Jackson could then rebuild the students’ autonomy by allowing them to express preferences and validate those preferences with logic. This cyclic process of building-referencing-losing authority continued throughout the course, even after the final project had been completed.
<table>
<thead>
<tr>
<th>Theme</th>
<th>Subtheme</th>
<th>Propositions</th>
<th>Authority Sources</th>
<th>Student Quote</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consensus</td>
<td>Building Consensus</td>
<td>Beliefs, Hypotheses</td>
<td>Group Consensus</td>
<td>“Do you think that’s a good direction? Looking at straight lines in terms of angles?”</td>
</tr>
<tr>
<td></td>
<td>Referencing Consensus</td>
<td>None</td>
<td>Prior Group Consensus</td>
<td>“So we said that Axiom 4’ is saying that there can’t be two lines that share the same two points.”</td>
</tr>
<tr>
<td>Autonomy</td>
<td>Lacking</td>
<td>None</td>
<td>None</td>
<td>“So where do we start?”</td>
</tr>
<tr>
<td>Tentativity</td>
<td></td>
<td>Beliefs, Hypotheses, Conjectures</td>
<td>Authority Figure</td>
<td>“Does this make sense? Is it ok to do this?”</td>
</tr>
<tr>
<td>Free Expression</td>
<td></td>
<td>Beliefs, Hypotheses, Conjectures</td>
<td>Implicit, Preference</td>
<td>“It’s just saying that if we have two triangles that are the same, then the sides and the angles will be the same”, “I guess we decide because it’s our definition”</td>
</tr>
<tr>
<td>Argumentation</td>
<td>Stylistic</td>
<td>NA</td>
<td>Preference</td>
<td>“I don’t think ‘evenly on itself’ is a very clear definition.”</td>
</tr>
<tr>
<td></td>
<td>Structural</td>
<td>NA</td>
<td>Logic</td>
<td>“If we define straight as being the shortest distance between two points, that doesn’t work on the sphere.”</td>
</tr>
</tbody>
</table>

*Figure 4. Summary of observation themes of the classroom.*

**Interview Findings**
Leo and Jemma began Interview 1 by recalling that they did something similar in Discrete Mathematics, a course they took before, albeit separately. Leo recalled the terms *Euler path* and *Euler circuit*, but was unable to use them on the task. Both recalled that the solution involved the number of even or odd vertices. Jemma then suggested that they “… know it, with like, doors?” They began using a *door-and-room* metaphor, implying that each section of the Task 1 map was a room and each bridge was a door between rooms (see Figure 5). The pair concluded eventually that there was no such path, because there could be at most two odd sections. They also claimed that if there were all even sections, the path would start and end in the same section.

*Figure 5. Leo and Jemma’s picture depicting the door-and-room metaphor.*
During Interview 2, Leo used a diagram that was more similar to a mathematical graph than a *door-and-room* diagram (see Figure 6). Leo eventually concluded, “With exactly two odds, you must start in one and end in the other.” Jemma, however, used a diagram that more closely resembled the map given in the prompt (see Figure 6). Jemma eventually concluded, “If you start in an odd section, you can end elsewhere.”

![Figure 6. Leo’s Interview 2 diagram (left) and Jemma’s Interview 2 diagram (right).](image)

For Interview 3, Leo initially asked if the “outside” needed to be considered. Jemma did not respond, and instead began path-tracing, claiming, “Each of these is like our space [...] and there is a door on every single one of them.” The pair then labeled the degree of each “space” with Leo accounting for three, five-degree sections on the “inside” and Jemma stating that, on the outside, “there would be nine.” They then concluded that the curve was not possible, since they knew “three” parts of the figure had an “odd number of edges, and you can’t have more than two.”

In summary, Leo and Jemma built consensus around their prior understanding of the problem from Discrete Mathematics, including their use of the *door-and-room* metaphor. Individually, neither participant used the *door-and-room* metaphor, instead opting to use other diagrams. For Interview 3, the pair reverted back to using the *door-and-room* metaphor and successfully completed the task. During stimulated recall, Leo lamented that they relied so heavily on path-tracing and in-out strategies and not relying on their conjectures. He also said that he was uncertain initially as to how the previous tasks were related to the Interview 3 task. Jemma also lamented their use of path-tracing. She said that she viewed all the problems as *doors-and-rooms*, and that enabled her to solve the Interview 3 task.

**Conclusion**

The analysis of the classroom indicated the norms of the inquiry-based, geometry classroom as *consensus*, *autonomy*, and *argumentation*. Through the cyclic process of building and referencing consensus, Dr. Jackson was able to help the students develop a sense of autonomy in making choices regarding both stylistic and structural argumentation. During interviews, it seems that the pair of Leo and Jemma reproduced the norm of building consensus by agreeing upon their usage of the *doors-and-rooms* metaphor to solve the task. Both participants, however, did not use their agreed upon metaphor to complete the Interview 2 task. They later used the *doors-and-rooms* metaphor to complete the Interview 3 task, with Jemma feeling fully autonomous to apply the *door-and-room* metaphor, while Leo felt less sure.

These results indicated that while classroom norms may be reproduced in interview settings, they might not necessarily be reproduced when students work individually. As such, further study is required to assess the transfer of inquiry-based learning to an individual’s own work.
References

