A Success Factor Model for Calculus: The Relative Impact of and Connections between Factors Affecting Student Success in College Calculus

Megan Ryals
North Carolina State University

Karen Keene
North Carolina State University

What factors (in terms of the student) contribute to success in college calculus, and what are the relationships between and relative importance of these factors? This study addresses these questions by building on the Academic Performance Determinants Model (Credé and Kuncel, 2008). A new model called the Success Factor Model for Calculus was developed using semi-structured and task-based interviews with fourteen first-semester college calculus students. The data suggests that creative mathematical reasoning and knowing-why are not required for success on college calculus tests. Alternatively, motivation is a determining factor in success in that students can perform well on exams by being motivated to know how to solve specific types of problems. Motivation is decreased by some course-specific factors, such as lack of structure and accountability, and its effect on success is decreased sometimes by a lack of study skills and habits.

Key words: Calculus, Creative Mathematical Reasoning, Success Factor Model, College Success

Approximately 300,000 students enroll in college calculus in the United States each year (Bressoud, 2015). With calculus being a gateway course to science and engineering disciplines, it is concerning that an estimated 25-30% of these students are not successful in the course and that both student confidence and enjoyment of mathematics decreases significantly between the beginning and end of the college calculus course (Bressoud, Carlson, Mesa, & Rasmussen, 2013). To address the challenges of teaching calculus and determine directions for future research, a comprehensive picture of the factors that lead to or prevent student success in the course is needed. As part of a larger study, this research used a comparative analysis of data from fourteen student interviews to address the following research questions:

1. What factors contribute to or hinder student success in college calculus?
2. What are the relationships between and relative importance of those factors?

Framework and Review of Literature

The Academic Performance Determinants Model

Student difficulty in calculus may arise from design aspects of the course, such as the types of questions asked on tests and ultimately what type of reasoning is required to be demonstrated on exams. It also may also be impacted by student-specific characteristics such as background knowledge, level of effort or motivation, or knowledge of how to study appropriately. Credé and Kuncel (2008) suggest that some of these factors mediate the effects of others on student performance. Their model of Academic Performance Determinants (see Figure 1) shows relationships between these determinants and illustrates how some affect performance more directly than others. Their model is not specific to mathematics or any other discipline. It was used in this study as a starting point for developing a similar model that is specific to the learning of calculus.

Credé and Kuncel distinguish between study motivation and study skills and habits. Study motivation refers to a student’s willingness to study and the “sustained and deliberate effort” they exert in studying (p. 428). In Anthony’s (2000) survey of college instructors and students,
both groups indicated that a student’s level of motivation was the most important factor that contributes to student success. However, there is a discrepancy in the amount of practice that is expected from instructors and the amount that students believe is necessary (Cerrito & Levi, 1999, Bressoud, 2015).

The Academic Performance Determinants Model suggests that study skills, habits and attitudes impact performance through the acquisition of certain kinds of knowledge. Note that since this model was not developed specifically for mathematical problems, certain types of knowledge or reasoning may not be accounted for in this model. For example, procedural knowledge means something different in mathematics. To aid in the need to consider mathematical knowledge and reasoning, we utilized two additional sets of definitions described next.

**Types of Knowing and Reasoning**

Building on the work of Ryle (1949) and Skemp (1979), Mason and Spence (1999) distinguish between four types of knowing – knowing-that something is true, knowing-how to do something, knowing-why you do something or why something is true, and knowing-to do something in a particular situation. They explain that while there are certainly connections between the types of knowing, and often one can facilitate another, they are distinct, in that one does not guarantee or precede another. Mason and Spence claim that classroom education focuses on teaching knowing-that, how, and why which amounts to knowing about a subject, but that this does not equate to knowing-to. Knowing-to is often a significant barrier in solving non-routine problems.

Lithner and his colleagues (Lithner, 2006; Palm, Boesen, & Lithner, 2006) have developed a conceptual framing that addresses the issue of solving non-routine type problems. They distinguish between creative and imitative mathematical reasoning. Creative mathematical reasoning (CMR) requires learners to produce a solution method they have never seen or have forgotten. Imitative reasoning, in contrast, is devoid of “attempts at originality” (Palm, Boesen, & Lithner, 2006, p. 6). Imitative reasoning can be further categorized as either memorized or algorithmic reasoning. Memorized reasoning (MR) requires simply recalling information to produce a complete solution, such as a facts, definitions, or even proofs. That is, it requires knowing-that. Algorithmic reasoning (AR) requires knowing-to use a particular procedure and knowing-how to use that procedure. Knowing-to in problems that can be solved using
algorithmic reasoning is simplified by triggers or surface features from previously studied similar problems. In a problem that requires creative mathematical reasoning, knowing-to is significantly more challenging because of the absence of these surface similarities.

**Methods**

**Participants and Data Collection**

This report outlines one piece of a larger study that examined the impact of AP Calculus on students who repeat Calculus 1 in college. Ninety-minute interviews were conducted in the fall semester of 2012 between mid-October and early December. Fourteen interview participants were chosen from two small private and four large public universities in two states in the southeast. These six universities were on the semester system and students were recruited anywhere from 8-14 weeks into the 16-week semester. Participants were first-year students enrolled in a Calculus I course who had taken AP Calculus and taken the AP Calculus exam. Participants were required to have made a C- or lower on a recent exam.

A semi-structured, conversational interview process (Drever, 1995, Smith, 1995) was used for the first 45 minutes. The interview protocol consisted of 17 open-ended questions and was influenced by the Academic Performance Determinants Model. The second portion of the interview was task-based (Kelly & Lesh, 2000) and was diagnostic. Its purpose was to determine whether imitative reasoning (see above for description) could have been used to solve each missed test problem. To make this determination, test problems were compared to problems from other course resources, such as examples from class notes or the textbook or assigned homework problems. Participants were asked to identify whether they were able to solve the similar problems prior to the test and also to attempt the test problem in the interview after reviewing how to solve similar problems from the resources.

**Analysis**

Analysis of Part I of the interviews led to the emergence of 17 themes relating to students’ experiences in calculus. We used open coding to identify experiences among participants and axial coding to group codes into themes (Strauss & Corbin, 1997). A theme was eliminated if it was not mentioned by at least three participants.

Analysis of the second portion of the interviews involved writing summaries for each student’s attempts at solving missed test problems and determining what type of reasoning could have been used to solve these problems. After reading each interview transcript, we wrote a 1-2 paragraph description of the student’s problem solving process for the attempted test problems and similar missed problems. These descriptions included the following three items: 1) whether the participant was able to solve the similar problem without assistance, 2) whether the participant was then able to solve the test problem after learning to solve the similar problem, and 3) if not, what barrier(s) the student encountered in solving the test problems (such as insufficient pre-requisite mathematical knowledge). Item 1 spoke to the student’s preparation for the test, while item 2 revealed whether imitative reasoning could have been used to solve the problem. Item 3 for each problem was compared across participants and commonalities were noted.

The summaries from the Interview Analysis Part II were combined with some of the results from Interview Analysis Part I to develop a Success Factor Model for College Calculus. The model captured all observed factors affecting success on Calculus exams for the fourteen participants and the relationships between these factors. Themes from Part I were included only
if we saw a consistent relationship between the factor and student success. For example, while participants consistently claimed that increased course structure led to greater motivation and subsequently greater performance, smaller class size (while desirable) did not present such a consistent impact.

We initially used certain success factors and terminology while leaving out others that we had no data to support or evaluate. We developed the initial draft of the Success Factor Model after examining individually and comparing the summaries of four participants. As we examined additional transcripts and summaries, we began to change the proximity of some factors to success, add additional factors and remove others, as well as add and remove connections between factors. This was an iterative process that involved going back and forth between the overall model and the individual participants’ experiences.

Once the model was complete with its factors and pathways, we set out to determine the relative importance or impact of each factor using a meta matrix and subsequently used the meta matrix to create case-ordered matrices (Miles & Huberman, 1994). That is, we sorted the participants by certain variables and then looked across the other categories for similarities amongst cases with the same characteristic, to determine which variables might be associated with others. We grouped students by their relative success in college calculus. Doing this allowed us to identify which factors were consistent for successful students and which factors varied, or were not essential. This process illuminated the different pathways students could take to success in Calculus (on exams) and the common factors that hindered other students’ progress.

Results

The Success Factor Model below depicts the factors of success and the relationships between the factors. Results also include the relative impact of the factors on student success in calculus.

![Success Factor Model for College Calculus](image)

Figure 2. Success Factor Model for College Calculus.

Arrows connecting factors A and B indicate an impact of factor A on factor B. Bold arrows indicate a direct relationship; an increase in the factor A positively impacts factor B. A dashed arrow indicates an inverse relationship; that is, an increase in factor A results in poorer results for factor B. Distance of a factor from success on the diagram does not determine relative impact of that factor; larger distance merely shows that there is a mediating factor that determines how much of an impact that factor will have on success. Due to space restrictions, only selected factors, relationships, and their relative importance will be discussed.
The Relative Impact of the Direct Factors of Success

We first discuss the direct factors of success, displayed on the right-hand side of the Success Factor Model, and their relative importance for students’ success on their exams. While any one of these three factors may be required to solve a particular problem, some were more frequently required and/or posed more frequent barriers for the participants than others.

One of the direct factors of success is knowing-to using CMR. Ten of the participants needed to apply some degree of CMR to successfully complete at least one of their discussed test problems, but no student had enough CMR required on his or her test so as to prevent passing the test without it. This result is consistent with that of Bergqvist (2007) who found that 15 of 16 Swedish calculus tests could be passed using only imitative reasoning.

Additionally, very few of the problems required CMR for calculus material; rather, most elements of problems requiring CMR dealt with pre-requisite knowledge or skills. For example, when Blake (all student names are pseudonyms) was asked to find the absolute minimum and maximum of a function, he was able to take a derivative and knew to set it equal to zero, but was then unable to solve the equation. In a limited number of cases, the element requiring CMR came at the beginning of the problem and did prevent the student from being able to continue, thereby costing the students all or most of the points for the problem. These cases were limited, however.

As with knowing-to using CMR, difficulties involving knowing-that were almost all limited to pre-requisite knowledge. While some participants did lose points for not knowing-that on one or two of the discussed problems, this was not a primary issue for any of the participants.

Twelve of the 14 participants were able to make more progress on at least one of their two discussed test problems during the interview than they had made during the test after first learning to solve a similar problem from their resources. Very little variation was found in the directions for the problems from the tests and class resources; therefore, recognizing the problem type did not prove to be problematic for the majority of participants. The implication is that students who missed test problems that could be solved by knowing-to and knowing-how with AR could have improved their grade on the test by having worked similar problems prior to taking their test. For these students, lack of success stemmed from either inadequate study motivation or study habits, both of which will be discussed.

Knowing-why Is Not a Direct Factor of Success

Knowing-why was not essential for solving any of the problems students had missed that were discussed during the interviews and was certainly not essential for passing the test. Knowing-why did prove to be helpful to some students, while other students opted to bypass understanding by using AR. Two students, Frank and Michael, were unable to solve a test problem that asked for the location of a function’s horizontal tangent line. When initially reviewing this problem in the interviews, Frank and Michael studied the same similar example from the textbook. Frank focused on the textbook’s discussion rather than the solution. After approximately thirty seconds, he quickly attended to the explanation that the derivative is equal to zero when the tangent line is horizontal. He did not proceed to study the provided solution and therefore did not need to use AR. Frank demonstrated an understanding of the connection between the concept of a horizontal tangent line having a slope of zero and the process of setting a derivative equal to zero. He no longer needed an association with a particular type of problem or set of instructions to be able to solve his test question; he now knew-to because he knew-why.

In contrast, when Michael looked at the textbook, he focused on the solution rather than the
discussion. He explained that in the provided solution, “they would set it equal to zero and get their $x$ values. They then plugged those back into the original equation to get the $y$’s.” When asked why they did that, he replied, “I’m not exactly sure why.” However, this did not prevent him from being able to then solve the corresponding test question. This finding supports the claim of Mason and Spence (1999): understanding is related to, but not dependent on or necessary for knowing-to. “You can ‘understand’ but not know-to act,…you can know-to act and yet not fully understand” (p. 140). This finding raises questions about the type of knowing instructors most value and test.

One important distinction mentioned by several participants was that their college calculus course emphasized knowing-why but their AP Calculus course had not. Katelynn described her college course as significantly more challenging because of this knowing-how component. She stated, “I’d definitely say the big difference I’ve noticed then to now – [in high school] we learned more the how to do things as opposed to why you do them. Like I remember learning like sin $x$ over $x$ equals one, but I had to learn how to derive it this year. That was a lot harder.” Other participants made reference to the material in college going more “in depth.”

**The Impact of Study Motivation is Mediated by Study Skills and Habits**

The data suggests that the need for good study motivation and habits may in many cases be circumvented by other factors, but also that having strong study motivation and habits can be the deciding factor in whether some students succeed. Students at four of the six institutions could have been making A’s on tests by learning to solve specific problems from their resources prior to the exam and then using AR. Students at the other two institutions were limited somewhat by the presence of multiple problems requiring CMR. However, even these students could have improved their exam scores to at least a B- by making improvements only with AR.

The impact of study motivation on knowledge acquisition for the participants was mediated by their study habits. That is, the effect of a student’s willingness or effort to study was either amplified or tempered by the student’s ability to study appropriately. Maggie, for example, had very high study motivation, but poor study habits. When asked if she would be able to make at least a C in the course, she declared, “I’ll MAKE it happen!” But when she was asked to describe how she would do this, she referred only to doing more of the same activities she had already been doing that had not led to success. For example, she said “I will probably read my whole textbook, if I have to.” For Maggie, high motivation did not automatically translate into greater knowledge acquisition that produced results.

**Structure, Accountability, and Relationships Impact Motivation**

The participants’ study motivation was impacted by the amount of structure of their course and the amount of accountability provided by their instructors, as well as the relationships they had with their instructors. The most commonly discussed element of course structure was the grading of homework, or lack thereof. Most participants had no regularly graded assignments in high school, but did have this in college, and most claimed it being graded was the only factor that would motivate them to complete it. This was an exception to an important pattern - in most cases, participants described their high school course as much more structured and indicated this was a positive aspect of the high school course. College instructors were not thought to have many expectations of students. Albert explained, “her job is to teach us, not make sure we care about class.” Maggie’s comments echoed this sentiment. She said, “I think it’s like, we’re supposed to do it, be responsible for ourselves. It’s college, so…” Students seemed to understand
and validate the responsibility put on them in the college environment, but they offered no evidence that it made them more successful. In contrast, Katelynn discussed her AP teacher “staying on top of [the students]” and claimed she would not have passed the course if he hadn’t. AP courses tended to provide more regular examination, frequent reminders from teachers, and accessibility of help from teachers.

The relationship between teachers and the participants was one of the most widely discussed differences between high school and college calculus courses. Some participants indicated that their relationship was not just academic but also personal – one student had attended the wedding of his teacher and another described his teacher as “involved and relatable, explaining “I actually went out to dinner with him after I graduated, like me and four friends.” Several participants had taken previous courses with their AP Calculus teacher. One explained that “it was almost like we had become a family because everyone had had [this instructor] for so long.” The impact of these relationships was significant because the students did not want to “let down” their teachers. Multiple participants increased their amount of studying simply because of their relationship with their teacher. Jeffrey and Frank went so far as to say they would have been content with a B in their AP courses, but they did not want their teachers to think they weren’t trying, or that he or she was a bad teacher, so they put it the extra effort to get an A.

There was a glaring void of these kinds of stories from the college courses. Just as the relationship motivated the students in high school, the lack of relationship in the college course was demotivating. Haley explained, “I don’t know him as well [as my high school teacher] so I’m not motivated to do as well because I know him. It’s terrible to get bad grades, but not because I know him and I’ll be embarrassed.” When students had a relationship with their teacher, they were motivated to work harder because of how their teacher would perceive them; when the relationship was absent, so was the extra motivation.

**Conclusions and Implications**

The Success Factor Model presented in this study highlights the need for larger scale studies that assess the relative impact of the success factors and their connections. It also challenges us to ask which of the factors of success are currently being taught or impacted by instructors, and moreover, which *could* be. For example, some have suggested that the best place to teach study skills and habits is in the classroom, rather than divorcing it from content (Taylor & Mander, 2003, Wingate, 2006). This will lead to a discussion of not only what is effective in increasing student success, but how certain recommendations for aspects of course design, such as increased structure or accountability, are perceived by college instructors and how attitudes and resource limitations may challenge such recommendations. Similarly, how do instructors’ beliefs about what is important align with how they test students? For example, our findings suggest that students can be successful in college calculus without knowing-why, but this may not be the instructors’ intent. The ability or inability to work non-routine problems may have little to do with a strong conceptual understanding, and instructors may be testing for one when desiring the other (Selden & Selden, 2013, Tallman, Carlson, Bressoud, & Pearson, 2016). Finally, we need to know the potential impact of emphasizing and developing each particular factor on students’ success in subsequent courses. For example, if calculus courses become more structured, will students find later, less structured courses more challenging, or could the development of CMR in Calculus I lead to greater success in Calculus II?
References


