The Common Core State Standards recommend students to decontextualize word problems using symbols and contextualize symbols by defining the meaning of values. We observed pre-service teachers’ difficulties with contextualizing symbols in word problems; hence, we incorporated supplementary word problems’ modeling instructions for an arithmetic course with pre-service teachers. After six weeks of instruction, on a midterm exam, our pre-service teachers started using symbols to present arithmetic word problems. However, many of them still could not clearly define the meaning of the symbols they used. After completing the program, students demonstrated improvement in their reasoning with symbols. We believe difficulties with defining symbols are connected to weaknesses with active scientific vocabulary in terms of measurable attributes. Therefore, we propose mathematics courses for prospective teachers to accentuate scientific vocabulary regarding measurable attributes.

Key words: Algebraic reasoning, symbols, pre-service teachers, word problems

One of the main goals of the Common Core State Standards Initiative (CCSSI, 2010) is to promote quantitative reasoning. Thompson (2011) stated quantification as the “process of conceptualizing an object and an attribute of it so the attribute has a unit of measure, and the attribute’s measure entails a proportional relationship (linear, bilinear, or multi-linear) with its unit” (p. 37). For example, in the statement, Ravi bought 3 kilograms of flour, a qualitative attribute is weight and a unit of measure is kilogram. Here, the measurable attribute (weight) is not specified but can be deduced from the text.

Another aspect of children’s education greatly emphasized by CCSSSI is early development of algebraic reasoning. According to Kaput and Blanton (2005), algebraic reasoning requires students to “generalize mathematical ideas from a set of particular instances, establish those generalizations through the discourse of argumentation, and express them in increasingly formal and age-appropriate ways” (p. 99). Lins and Kaput (2004) recommend developing algebraic reasoning in elementary grades. The National Mathematics Advisory Panel of the U.S. Department of Education (2008) acknowledged this, since research has shown most students struggle in algebra in secondary grades (Kieran, 1992). An early introduction of algebraic reasoning might help in transition to algebra in later mathematics classes. Low algebraic skills are believed to be a gatekeeper to progress in mathematics and science (Greene et al., 2001).

One of the features of algebraic reasoning includes an ability to decontextualize word problems, i.e., “to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents” and contextualize symbols by attending to the meaning of quantities, i.e., “to pause as needed during the manipulation process in order to probe into the referents for the symbols involved” (CCSSI, 2010, p. 6). As a symbol, some choose basic letters (A, B, C, X, Y, Z, etc.) (Davydov, 1975; Dougherty & Slovin, 2004), while others use parameter pointers, letters or words, that point to key words connected with qualitative characteristics of values (Kofman, 2016). The latter are routinely used in physics (Serway & Faughn, 2006) and chemistry (Dingrand et al., 2006). Parameters, such as \( v_i \) is the volume of a gas before expansion or \( m \) is the mass of an object, are used to remind a problem-solver about the symbols’ meaning in challenging word problems, which involve multiple symbols. Shifting gears toward problem-solving (CCSSI, 2010) increased the role of teaching students to use
symbols, which help with solving word problems.

According to CCSSI (2010), children should start developing knowledge regarding measurable attributes as early as kindergarten and be able to “describe measurable attributes of objects, such as length or weight” (CCSS.Math.Content.K.MD.A.1, p.12). A well-developed understanding of measurable attributes helps students to develop an active scientific vocabulary—a basis for communicating math ideas needed to successfully solve mathematics word problems. CCSSI (2010) expects 6th graders “to use variables to represent numbers and write expressions when solving a real-word or mathematical problem” and “understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set” (CCSS.Math.Content.6.EE.6, p. 44).

Since we want our elementary and middle school students to start developing algebraic reasoning by presenting word problems using symbols, we need to ensure teacher candidates (pre-service teachers) are fluent with this material because, according to Patton et al. (2008), pre-service teachers “possess naive conceptions believing that teaching mathematics is only about delivering facts and memorizing mathematic procedures” (p. 494). Patton’s finding corroborate with early studies, which demonstrated undergraduates (non-physics majors) display difficulties with algebraic reasoning, particularly with interpreting variables when solving math problems (Rosnick, 1981). Also, MacGregor and Stacey (1997) have shown novice algebra students do not understand the meaning of symbols and commonly misinterpret symbols as representing objects or words.

We hypothesized pre-service teachers might exhibit similar difficulties. Although, the research regarding prospective teachers’ difficulties with generalizing patterns (Brown & Bergman, 2013; Hallagan, Rule, & Carlson, 2009) was undertaken recently, future educators’ active vocabulary with implicitly stated attributes has not been studied, yet.

Based on this, we closely monitored pre-service teachers (our students) performance on contextualizing symbols for simple arithmetic word problems. Observing their difficulties with defining the meaning of symbols in word problems, we undertook this research to analyze how prospective teachers contextualize symbols in word problems. To meet students’ needs, we strongly emphasized developing scientifically valid active vocabulary, which ensured fluent communication in terms of symbols, measurable attributes, and units of measure. Since high-level communication skills are a necessary condition for developing thinking (Vygotsky, 1978), we expected by increasing pre-service teachers’ abilities to communicate using correct terminologies, we could deepen their understanding of word problems; hence positively affect developing communication skills of their future students. Thus, the second stage of the research was to provide specific instructions to improve pre-service teachers’ abilities to identify measurable attributes and clearly define symbols used for presenting arithmetic word problems and acquire preliminary estimation of its impact.

**Theoretical Framework**

Our theoretical framework is based on the schemata approach. A schemata approach subdivides simple addition word problems into logical categories (Combine, Compare, Change, etc.) (Jitendra et al., 2007; 2015) and proposes schema for solving each type of word problem. We believe the schemata approach works because it helps students to concentrate on one learning dimension at a time, while allowing some variations. According to Marton and Pang (2006), namely ‘dimension variation’ elevates quality of the learning process. In the schemata approach, students learn to solve and present one type of problem (one learning dimension), while choosing between arithmetic operations (variation). This approach allowed us to concentrate on teaching students one dimension at a time—students solve one type of problem, while creating algebraic and visual word problems’ models specific for each type.
Contrasting variations were applied in the following aspects:

a) Problems were assigned that can be solved using addition or subtraction in each type of problem (Combine, Compare, and Change problems);

b) Students were taught to continuously differentiate between symbols (capital letters) and units of measure (lower case letters) by denoting them in different ways;

c) Students were taught to define parameter-pointers using precise definitions based on naming attributes (e.g., M is the distance Marta walked), which created a contrast with parameters’ associative meaning (e.g., M is Marta).

Using the schemata approach combined with modeling and defining parameter-pointers allowed us to apply the dimension variation method when creating materials designed to improve pre-service teachers’ knowledge how to interpret symbols they use.

**Methodology**

**Participants and the word problem-solving (WPS) Treatment**

Seventeen pre-service teachers from a western research university enrolled in an arithmetic course for elementary teachers participated in this study. The lead author taught the arithmetic course. This course was the first content course in a two-course sequence for elementary pre-service teachers. This course provided pre-service teachers with a deeper understanding of the real number system and arithmetic operations for whole numbers, fractions, and decimals for Grades K-6. The course textbook was written by Beckmann (2014). Mainly, chapters 1 through 9 were covered during the semester.

Along with standard course materials, supplementary materials on word problem-solving (WPS) were used throughout the semester. These supplementary materials (available upon request) were independently developed by the second author and were chosen because no books concentrating on developing teachers’ vocabulary regarding measurable attributes and defining symbols for arithmetic word problems exist to the best of these authors’ knowledge. These supplementary materials were self-explanatory workbooks, consisting of 13 chapters. Pre-service teachers read each of the chapters, each chapter comprised of multiple sections, worked on the examples, and completed the assigned problems for each section. The exercises involved presenting arithmetic word problems using multiple models and then solving the problems. The WPS workbooks focused on the following list of measurable attributes: number of objects, amount of money, length-type characteristics (depth, width, distance, height, and length), volume, weight (in terms of mass), temperature, and time.

The instructor provided feedback after pre-service teachers handed their assignments to the instructor during each class period. The instructor spent five-ten minutes in class discussing pre-service teachers’ work and ideas from each section of the WPS materials. Pre-service teachers were given chances to correct their assignments until 100% fluency was shown with the assignments. When teaching the WPS supplementary program, the instructor constantly monitored pre-service teachers’ progress, mistakes, and misconceptions, and focused on extending and sharpening students’ mathematical vocabulary. The dimension variation approach was used to teach problem-solving topics. It includes materials teaching identification of the meaning of symbols (Table 1).

After the mid-term exam, pre-service teachers were asked to define each of the symbols they used when presenting word problems. Also, pre-service teachers were divided into groups of two or three and given word problems to work in class—they presented their solutions to the entire class. Pre-service teachers modeled arithmetic problems using visual and algebraic representations. When presenting problems in the form of algebraic equations, pre-service teachers were prompted to use parameter-pointers. For example, \( T_1 \) is the number of flowers Linda had at first or \( R \) is the length of Rob’s wire.
Table 1

<table>
<thead>
<tr>
<th>Objectives connected with contextualizing symbols in word problems</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dimensions of learning</strong></td>
</tr>
<tr>
<td>Create models presenting word problems.</td>
</tr>
<tr>
<td>Present problems using symbolic equations.</td>
</tr>
<tr>
<td>Define parameter-pointers in symbolic equations.</td>
</tr>
</tbody>
</table>

In the beginning, the WPS workbook contained word problems with easy to define attributes: “Linda has 5 erasers,” and attributes were stated explicitly: “The weight of Linda’s chair is 7 lbs.” Later in the semester and during the final exam, more difficult, implicit (not stated directly) attributes were used in word problems, “Agnes collected 84 liters of rainwater.” In such problems, pre-service teachers had to deduce the name of the qualitative attribute (volume) connected with the given value, 5 liters.

Data Collection and Analysis

The research design was a mixed-methods study. Data were collected from the pre-service teachers’ writing assignments—two pre-tests, WPS workbook, quizzes, a midterm exam, and a final exam and instructors’ classroom observations. A strong correlation in students’ written work (correct usage of scientific vocabulary) and their usage of the vocabulary in oral presentations in the classroom were observed. For qualitative analysis, the pre-tests, mid-term exam, quizzes, and final exam written data were analyzed following an open, axial coding method (Strauss & Corbin, 1998). Each of the two authors read the written works and created a rubric. Then, we met to discuss our rubrics and created a common rubric (Table 2), based on pre-service teachers’ presentations and explanations of the symbols used. For quantitative analysis, we created an excel spreadsheet of the responses for each task in the pre-tests, mid-term and final exam, which allowed us to follow an individual’s progress throughout the semester. We recorded the responses using the rubric in Table 2 and counted the number of pre-service teachers in each category.

Table 2

<table>
<thead>
<tr>
<th>Categories of symbols’ descriptions</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students demonstrate difficulties</td>
<td>Not defining symbols</td>
</tr>
<tr>
<td>Definition reflecting misconception</td>
<td>Defining symbols using key words from the text. A means Anna</td>
</tr>
<tr>
<td>Students present conceptually correct definition of symbols</td>
<td>Unclear naming of quality of quantity</td>
</tr>
<tr>
<td></td>
<td>Clear naming of quality of quantity</td>
</tr>
</tbody>
</table>

Results

The pre-tests run on the first two days of the semester demonstrated our pre-service teachers had difficulties to explain the meanings of the symbols they used in the word
problems. After observing pre-service teachers’ difficulties with CCSSI K-6th grade materials, we used the supplementary WPS materials for six weeks. The WPS sections regarding measurable attributes were thoroughly discussed in the classroom.

After six weeks into the semester working on the WPS materials, we found pre-service teachers continued to struggle with the concept of contextualizing symbols. More than half (9 out of 17) did not contextualize the symbols in the mid-term exam. For a Combine addition task, one pre-service teacher wrote “$P =$ Pearls” (Figure 1a) instead of writing “$P$ is the weight of the box with pearls.” Another wrote “$F =$ First box” instead of writing “$F$ is the weight of the first box.” It is interesting to note, the same students appeared under the same categories for the two tasks (Table 3). It is noted, the materials were also reviewed a day prior to the midterm exam.

The final exam contained word problems with implicitly stated quantities (i.e., attributes are implied in the word problem, not directly mentioned) (see problem in Figure 2). In spite of this, we saw an increase in the conceptually correct definitions of symbols. The definitions reflected students’ understanding of symbols, but were still somewhat unclear with precise defining measurable attributes (e.g., Figure 1b). We classified such answers as unclear, but in terms of symbol definition, conceptually correct. Across four different tasks in the final exam, 16 (15 in tasks involving length) students defined the symbols correctly or at least conceptually correctly (see Table 3).

Table 3
Comparison of pre-tests, mid-term, and final exam. The problems’ difficulty in terms of contextualizing symbols was higher on the final exam than on the pre-test and mid-term.

<table>
<thead>
<tr>
<th></th>
<th>Pre-test (beginning of instruction, 16 students; one student enrolled late)</th>
<th>Mid-term (after 6 weeks of instruction, 17 students using the WPS workbook)</th>
<th>Final Exam (after 12 weeks of instruction, 17 students using the WPS workbook)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurable</td>
<td>Not defining symbols</td>
<td>Definition reflecting misconceptions</td>
<td>Conceptually correct definition of symbols</td>
</tr>
<tr>
<td>attributes</td>
<td></td>
<td></td>
<td>Unclear naming of quantity</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All categories</td>
<td>16</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Number of apples</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Weight</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Length</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Volume</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

Note: The numbers above show the number of pre-service teachers under each category for each problem type.

Students were taught to use any letter but “O” when creating symbols to prevent confusing “O” with zero. However, 15 students used “O” as a symbol to represent the weight of onions and only 2 students used other letters. Figure 3 demonstrates how the same symbol, P, has different meanings in a word problem involving volume. Since the measurable attribute was not explicitly mentioned in the problem, four pre-service teachers used the same words used in the word problem, but failed to use the correct attribute name, volume, when defining the symbols.
Elizabeth found two boxes that together weigh 11 lbs. The first box had pearls weighing 7 lbs and the second box had emeralds. How much did the second box weigh?

(a) Coded as “misconception.” (b) Coded as “unclear.”

Figure 1. Symbol definition: a) misconceptions regarding parameters, b) missing attributes.

Figure 2. Defining symbols: (a) definition is unavailable; (b) definition is conceptually correct—there is a misconception regarding attribute; (c) correctly defined symbols.

Figure 3. Responses of three pre-service teachers on the same task for the final exam.

Additionally, one student failed to define symbols for all the tasks in the final exam. This student did not complete the supplementary WPS workbook on her own and did not meet with the instructor to receive additional help.

Discussion and Conclusions

Our findings regarding students’ misinterpretation of letters as objects or words are consistent with other studies (Booth, 1988; MacGregor & Stacey, 1997). When presenting 1–2-step arithmetic word problems in symbolic form, our pre-service teachers demonstrated
difficulties with defining symbols they used. Their work reflected both—misconceptions in regards to the symbols’ meaning (e.g., $G$ is the green paint) and misconceptions regarding defining measurable attributes represented by the values (e.g., $P$ = grams of pepper Agnes bought). Supplementary instruction in word problem solving allowed us to improve pre-service teachers’ precision in defining symbols.

One can argue that the pre-service teachers did not perform well, since they did not understand the task. This can be true for the pre-test. However, this is not true for all later tests. The pre-service teachers were taught throughout the semester using the WPS materials similar to the examples used in this paper. Hence, during the mid-term exam, pre-service teachers were well-aware of the type of questions and expectations for the symbol-meaning tasks.

The data reveal the difficulties demonstrated by pre-service teachers are real in regards to active scientific vocabulary connected with properties of matter measured and/or calculated. About 50% of the pre-service teachers, who already had prolonged instruction on defining symbols by naming the attributes of the values they represent, on the mid-term exam, still demonstrated difficulties with the materials and only additional studies helped them improve their performance. The final exam demonstrated the improvements.

We corroborate the progress in pre-service teachers’ abilities to define symbols is much greater than presented in Table 3 because (1) the word problems on the final exam were more difficult in terms of symbols’ definition than on the mid-term exam and (2) because we compared performance of the pre-service teachers on the mid-term (discarding pre-test results) and final exams. Meanwhile, the performance on the mid-term exam was measured after the students had some WPS instruction regarding defining symbols used to present word problems.

In the future, it would be interesting to examine pre-service teachers’ performances using a redesigned pre-test and compare treatment group results with a control group that does not have supplementary instruction. Based on our results, it can be concluded, mathematics courses for elementary teachers must put greater emphasis on teaching pre-service teachers scientifically-correct vocabulary regarding measurable attributes.

References


Mathematics.