Variations in Precalculus Through Calculus 2 Courses
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In this paper we analyze variations in the structure of courses designed for the Precalculus through Calculus 2 (P2C2) sequence. We examine the nature of such variations, frequency nationally, and how DFW rates and instructional approach compare to the standard courses. While most identified variations in course structures have on average lower DFW rates when compared to the national average, a comparison within institutions indicates that these alternative course structures have higher DFW rates when compared to the standard P2C2 sequence offered at the respective institution. In addition, we observed that course variations which allow for increased instructional time have greater amounts of active learning techniques as part of the instructional format. Results from these findings along with their implications for the next phase of the Progress through Calculus project are discussed.

Keywords: Calculus, Precalculus, Curriculum, Course Variations

There is abundant literature highlighting the importance of student success in introductory undergraduate mathematics courses, often pointing to how such courses are a hurdle for students intending to continue into STEM majors and future related careers. Even for those students who do not choose to major in a STEM field, success in entry-level undergraduate mathematics courses such as calculus can impact a student’s overall persistence in postsecondary education. Research examining students’ success in introductory mathematics courses consistently indicates that students are not learning the intended curriculum (Tallman, Carlson, Bressoud, & Pearson, 2016; Thompson, 1994), resulting in lower preparation for subsequent courses (Carlson, 1998; Selden & Selden, 1994) and a marked decrease in their desire to pursue a STEM degree (Bressoud, Mesa, & Rasmussen, 2015; Ellis, Fosdick, & Rasmussen, 2016; Seymour & Hewitt, 1997). These students often cite the poor instructional experiences in introductory level courses as the primary reason for their departure from the STEM fields.

Institutions of higher education and departments of mathematics are becoming increasingly attuned to the particular challenges faced by these students and are seeking new ways of supporting student success in introductory mathematics courses. Rasmussen et. al (2016) found that most departments are aware and value characteristics of more successful calculus programs, yet they are not always successful at implementing these features at their institutions. As such, departments have responded in a number of ways to support student success in the P2C2 sequence. Departments are utilizing local data to inform placement procedures, using active learning pedagogy techniques, creating course coordination systems, and developing or improving graduate teaching assistant training. Another departmental response, which is the focus of this report, is to offer alternative P2C2 course structures. An alternative course structure reimagines the “standard” P2C2 sequence, which we take to be three, one semester courses consisting of Precalculus, Calculus I, and Calculus 2. An example of an alternative course structure is to offer, in addition to the standard sequence, a “stretched out” Calculus 1 course that takes two semesters and infuses additional algebra and trigonometry in the context of the calculus. In this report we address the following research questions:
1. What variations to the standard P2C2 course sequence are currently in place and how common are they nationally?

2. What are the relationships between variations to the standard P2C2 course sequence, instructional approach, and student success?

Data for this analysis comes from a national census survey focused on the P2C2 course sequence. This census survey is Phase 1 of a five-year project which began in 2015, and was distributed to all mathematics departments that offer a graduate degree in mathematics. These institutions were selected because they produce the bulk of STEM graduates while often struggling to find a balance between the demands of research and teaching. Phase 2 will consist of longitudinal case studies of selected institutions.

**Theoretical Background**

We address our research questions and frame alternative P2C2 course offerings in terms of the intended, enacted, and assessed curriculum (Kurz, 2011; Porter, 2006; Webb, 1997). The intended curriculum refers to the knowledge and skill targets for students (Porter, 2006), in this case the course objectives for precalculus and mainstream calculus. In addition, this includes variations of the intended curriculum designed as specialization for service disciplines (engineering, biosciences, etc.). The enacted curriculum refers to the knowledge and skills delivered during instruction which varies based on instructional time (allocated time for instruction), content coverage (amount and variety of academic standards), and instructional approach (Kurz, 2011). For example, a stretched out Calculus 1 course might be configured to allow for more active learning, but this is an empirical question and one that we address in this report. We also investigate how institutional response to the alignment between the intended and enacted curriculum affects the assessed curriculum. We use DFW rates (grades of F, drops and withdrawals) as a proxy for the assessed curriculum.

Figure 1 captures how we make use of the intended, enacted, and assessed curriculum in this study. In particular, the survey data allows us to detail intended curriculum aspects such as course objectives and intended audience, and the enacted curriculum aspects such as instructional time, content coverage, and instructional approach.

![Figure 1. Observed components for the intended and enacted curriculum, and their relation on the assessed curriculum.](image)

**Methods**

In the United States there are a total of 330 departments that offer either a Masters or PhD in mathematics. All 330 institutions (178 Doctoral and 152 Master’s) were surveyed yielding a 68% response rate. We designed the census survey to gather information on the implementation of the
features of successful programs identified by the CSPCC project and to gain an understanding of the variety of P2C2 programs being implemented across the country, the prevalence of such programs, and what institutions are doing to improve their programs. The survey consisted of three main parts. Part I asked for a list of all courses in the mainstream P2C2 sequence. Mainstream refers to any course in this sequence that would be part of student preparation for higher-level mathematics courses such as a first course in differential equations or linear algebra. Part II asked about departmental practices in support of the P2C2 sequence. Part III asked for detailed information about each course in the mainstream P2C2 sequence, including enrollment data and details about course delivery.

Survey responses were then cross-referenced and updated to the extent possible by a comprehensive search of publicly-available course catalogs and department websites. This led us to a collection of 1108 courses from 223 institutions, with details supplied for 895 of these courses by 205 institutions. We used a grounded theory approach (Corbin & Strauss, 2008) to code and categorize variations to the standard P2C2 course sequence. Based on this analysis, 11 variants of the standard P2C2 courses were identified and are outlined below. We choose to highlight these variations since they demonstrate the response from institutions to support student of varying preparedness levels and interest.

Results

We present results according to the two research questions. We first describe existing course variations in terms of three main themes: intended audience, instructional time, and content coverage. We then provide a summary table (Table 1) specifying the frequency in which these variations are present. We then address the second research question by analyzing how course variations fare in terms of DFW rates and instructional approach.

Intended Audience

Several institutions have specialized their mainstream calculus sequence to tailor the intended curriculum to service different disciplines. The most common variation is calculus for the biosciences, which is a mainstream calculus course designed explicitly for students in biological or life science majors. Often time the course includes applied topics in biological modeling and investigation of real-life phenomenon. Calculus for engineering is another course variation on the mainstream calculus course that is intended for students in engineering majors. Often this course includes emphasis on physical applications to engineering and computational techniques. Seymour and Hewitt (1997) found that a disconnect between calculus content and intended major to be a major contributing factor in students’ decision to leave a STEM field. Such variations have the potential to address such concerns.

In addition, we identified institutions that offered a mainstream calculus for another subject, which was specifically designed for students in a non-STEM major (e.g. Calculus for Economics). Mathematics departments often have to provide non-mainstream introductory mathematics courses for students in other disciplines, yet these students face a subsequent challenge if they intended to switch to a STEM intending degree. Some institutions have responded to this potential audience by providing a transition to mainstream course variation. This course serves as a bridge between a non-mainstream calculus course and a mainstream calculus course or upper-division mathematics course that has a lower credit load, and does not require the student to retake the entire mainstream P2C2 sequence.

Instructional Time

The most common course variation observed alters the standard curriculum by allowing the course content to be covered over a longer duration of time, which addresses another major
factor that students cite for leaving a STEM major – overpacked courses taught at too fast a pace (Seymour & Hewitt, 1997). For the preparation for calculus courses we observed institutions that offered the choice between a single course covering the requisite skills necessary for enrolling into calculus or a two (or more) course sequence, typically consisting of a course in trigonometry and a course in college algebra. We denoted these options as modular precalculus and modular stretched-out precalculus. We use the term modular since students are able to modularize or choose between the necessary sequence. In addition, we observed institutions that offered only a single course option or only a two course sequence as preparation for calculus. We denoted these options as standard precalculus and standard stretched-out precalculus.

For single variable calculus we observed two unique course variations that allowed for the formal curriculum to be covered over an extended length of time. The stretched-out calculus course takes the traditional one-term course and stretches the content over a two-course sequence, often including additional requisite material when appropriate. This option is usually intended for students who would be at-risk in the traditional calculus sequence and would benefit from a slower paced delivery of the material. The stretched-out Calculus 1&2 is a variation on the stretched-out calculus where three courses, when taken together, are the equivalent to the standard two-course single variable calculus sequence.

Content Coverage

While some variations seek to allocate more time for course delivery by spreading the curriculum over extra courses or terms, another set of calculus course variations are designed to provide background content and requisite skills as supplements to course offerings. The most common variation in this category is calculus infused with precalculus, which is a single-term calculus course (typically with more credit hours) designed to include requisite pre-calculus topics covered throughout the course duration. A similar course variation, referred to as co-calculus, is a one-term course taken concurrently with a single variable calculus course that covers selected precalculus topics, coordinated with the content of the calculus course. This course variation is intended for at-risk students who can be identified early in the term through low-course performance and subsequently enrolled in the co-calculus course to provide supports for study skills and the coverage of precalculus content.

While the previous two course variations are typically intended to support students in the first course in single variable calculus (Calculus I), we also observed one course variations intended to specially support students in the second term of single variable calculus (Calculus 2). Accelerated calculus is a calculus course explicitly designed for students who have taken calculus in high school (usually with AP credit). These courses cover mainly material that would be considered Calculus 2, but also include Calculus 1 material that may not have been covered in sufficient depth in an AP course. All variations in the “content coverage” category involve strategies to supplement the mathematics content of regular course while keeping students on-track in terms of time to graduate.

Other

In addition to previously mentioned variations we also observed several unique course offerings that failed to warrant their own code, but were identified as other unique course structures. This category included: courses (or course sections) explicitly designed for students who have not seen calculus before; a course designed to divert less-prepared students mid-term; precalculus courses which include a preview of calculus topics; courses designed for transfer students; applied courses in technology; courses offered only in summer as preparation; etc.
Course Variation Summary

Overall, 125 (56.3%) of the institutions have at least one course variation and excluding the most common variation, for modular precalculus and modular stretched-out precalculus, 75 (33.8%) of the institutions have at least one calculus course variation. The frequency for which we observed each of the course variations at surveyed institutions is presented in Table 1. The remaining results present a descriptive account of the survey responses describing the enacted curriculum (instructional approach) followed by measures of the assessed curriculum (DFW rates) stratified by the observed course variations. The descriptive statistics are only provided for variations which had data from three or more observations.

Table 1. Overview of course variations offered nationally

<table>
<thead>
<tr>
<th>Category</th>
<th>Variation</th>
<th>Institution (n=223)</th>
<th>Percent (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intended Audience</td>
<td>Calculus for biosciences</td>
<td>15</td>
<td>6.8</td>
</tr>
<tr>
<td></td>
<td>Calculus for engineering</td>
<td>14</td>
<td>6.3</td>
</tr>
<tr>
<td></td>
<td>Calculus for other subject</td>
<td>3</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>Transition to mainstream</td>
<td>3</td>
<td>1.4</td>
</tr>
<tr>
<td>Instructional Time</td>
<td>Modular Stretched-Out Precalculus/Modular Precalculus</td>
<td>62</td>
<td>27.9</td>
</tr>
<tr>
<td></td>
<td>Stretched-out calculus</td>
<td>20</td>
<td>9.0</td>
</tr>
<tr>
<td></td>
<td>Stretched-out calculus 1&amp;2</td>
<td>7</td>
<td>3.2</td>
</tr>
<tr>
<td>Content Coverage</td>
<td>Calculus infused with Precalculus</td>
<td>11</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>Co-calculus</td>
<td>3</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>Accelerated calculus</td>
<td>14</td>
<td>6.4</td>
</tr>
<tr>
<td>Other</td>
<td>Other</td>
<td>10</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Course Variation DFW Rates and Instructional Approach

For each of the P2C2 courses provided, respondents were asked to indicate the primary instructional format during the regular class meetings. The response options along with the percentage for which they occurred for each of the course variations is presented in Figure 2. We observed the highest amounts of active learning techniques implemented in course variations that increase the amount of instructional time, with the greatest observed amounts in the stretched-out Calculus 1&2 variation, followed by the stretched-out calculus variation. In addition, we observed greater amounts of computer based instruction in calculus for engineering and the stretched-out Calculus 1&2 course variation.

Figure 2. Percentage of courses within each of the major calculus course variation that are administered with a given instructional approach.
Respondents also indicated the typical drop, fail, or withdraw rate for each of the courses they listed as part of the mainstream P2C2 sequence. We observed that the DFW rates between the variations of the preparation for calculus courses (Figure 3) are markedly similar, with no significant differences.

Figure 3. Box-and-whisker plot depicting DFW rates for the four major course variations of preparation for calculus along with the reported averages: modular two course (n = 60, 27.6%), modular one course (n = 47, 27.2%), standard two course (n = 34, 27.1%), and the standard one course (n = 90, 27.5%).

The DFW rates for course variations in mainstream calculus (Figure 4) however do show major differences when compared to the national average. We computed the average DFW rate for any standard calculus course (24.1%), but isolated those courses intended for honors students, as these courses are typically intended for high-achieving students. We observed that each of the course variations had on average lower DFW rates compared to the standard course structure, except stretched-out calculus which had a similar yet higher DFW rate (25.3%). Indicative of our qualitative descriptions the lowest DFW rates occurred in course variations intended for high achieving students with strong prior backgrounds (Stretched-out Calculus 1&2, and Accelerated Calculus).

Figure 4. Box-and-whisker plot depicting DFW for the major course variations of calculus along with the reported averages: Standard (n = 383, 24.1%), Standard (honors) (n = 68, 9.6%), Accelerated Calculus (n = 8, 9.12%), Calculus for engineering (n = 16, 23.6%), Calculus for biosciences (n = 19, 22.0%), Calculus for other subjects (n = 3, 14.6%), Calculus infused with Precalculus (n = 14, 18.9%), Stretched-out Calculus 1&2 (n = 17, 11.0%), Stretched-out Calculus (n = 38, 25.3%).

In addition to comparing DFW rates against the national average, we also analyzed the DFW rates for a given course variation against the DFW rates for the standard course sequence offered at that given institution (Table 2). While accelerated calculus has the same trend in comparison to the national average, the other course variations present an entirely different picture with regards to the DFW rates. We observed a larger amount of comparisons in which the alternative
course variation had higher DFW rates compared to the equivalent standard course sequence at that particular institution.

Table 2. Institutional comparison of DFW rates for alternative and standard course sequences in single variable calculus.

<table>
<thead>
<tr>
<th>Course Variation</th>
<th>Avg. DFW rate for variation</th>
<th>Avg. DFW rate for standard</th>
<th>Institutional comparisons where DFW rates were higher for:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accelerated Calculus</td>
<td>8.2</td>
<td>20</td>
<td>Variation 3 Standard 1 Identical 1</td>
</tr>
<tr>
<td>Calculus for other subjects</td>
<td>10.2</td>
<td>8</td>
<td>2 2 2</td>
</tr>
<tr>
<td>Calculus for Engineering</td>
<td>25.3</td>
<td>26.1</td>
<td>11 3 2</td>
</tr>
<tr>
<td>Calculus for Biosciences</td>
<td>22.1</td>
<td>23.6</td>
<td>11 6 2</td>
</tr>
<tr>
<td>Calculus infused with Precalculus</td>
<td>19.0</td>
<td>13.1</td>
<td>7 3 1</td>
</tr>
<tr>
<td>Stretched-out Calculus 1&amp;2</td>
<td>12.8</td>
<td>11.6</td>
<td>5 0 0</td>
</tr>
<tr>
<td>Stretched-out Calculus</td>
<td>22.4</td>
<td>19.6</td>
<td>7 3 2</td>
</tr>
</tbody>
</table>

Conclusion

Our project presents the first description of the variety (and frequency of) nonstandard curricular structures for P2C2 courses across the nation. In particular, we are able to paint a picture of how institutions across the country are attempting to address concerns about student success in introductory mathematics courses by addressing the intended and enacted P2C2 curriculum. We note that relatively few departments across the country are experimenting with these innovations, but our results indicate that these innovative structures have some potential at impacting the assessed curriculum. Given that this information comes from Phase 1 of a multi-year project, we find ourselves with the unique opportunity to further investigate the implementation of alternative course structures during Phase 2 of our case study analysis. The in-depth institutional analysis will provide a richer understanding of the surrounding programs and context in which such structures support student success, and to explore more nuanced measures of success and student understanding.

While Phase 2 of our research will reveal much more about the nature of alternative course structure, we can begin to comment on some of the potential advantages of these non-standard offerings. First, we note that many variations involve increasing the time dedicated to courses – be it stretched over extra terms or supplemented with co-requisite units. This increase in contact hours may free up class time in order to leverage active learning while alleviating concerns about content coverage. Secondly, many alternative course structures are aimed at supporting students who are underprepared for calculus. While the DFW rates in these courses tend to be higher than their non-standard counterparts, we expect this is related to the proportion of underprepared students in each – though further exploration is needed to conclude this with certainty. As failure in calculus is akin to an exit from STEM, supporting these students and bringing them up to speed with their peers may be an important piece of stopping the leaking pipeline.
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