Conceptual Understanding of Differential Calculus: A Comparative Study

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The research community shares a concern for students' conceptual understanding of calculus and commonly advocates for student-centered approaches as a way to promote it. In this study, we investigated the effect of different instructional approaches on 151 undergraduate students' conceptual understanding of differential calculus in context-specific, natural settings. We collected data on the pre and posttest of the Calculus Concept Inventory in three classes. In one class, most of the time was dedicated to conceptually oriented problem solving. Another class implemented practice problems for students. The third class was a traditional lecture class. The results showed that there was no difference in students' conceptual understanding of differential calculus controlling for their initial understanding. Thus, our findings do not support the research that advocates for student-centered instruction suggesting that the approaches' implementation and contextual differences may be sources of variation in their effectiveness.

Key words: Calculus, Conceptual Understanding, Active Learning, Instruction, Concept Inventory

Mastery of calculus, a desired and necessary student learning outcome (Sofronas et al., 2011), needs to include not only mastery of procedures but also mastery of concepts (Zerr, 2010). Multiple attempts have been made to identify instructional approaches that lead to greater conceptual understanding of calculus (Laursen, Hassi, Kogan, & Weston, 2014; Rasmussen, Kwon, Allen, Marrongelle, & Burtch, 2006), typically advocating for student-centered instruction. However, those studies either used measures with unknown validity and reliability or aggregated data across classrooms. With our ex post facto study, we aimed to overcome these limitations and investigate students' conceptual understanding of differential calculus (measured by a validated instrument) in three classes with distinct instructional approaches taking contextual factors into account.

The first class was taught in the active learning classroom with most of the class time dedicated to conceptually oriented problem solving (COPS). The second class implemented practice problems (PP) in a lecture hall. The third class, also taught in a lecture hall, utilized primarily direct instruction (DI). The Calculus Concept Inventory (CCI; Epstein, 2007), a measure of conceptual understanding of differential calculus, was administered in all three classes at the beginning and end of the semester. A total of 151 undergraduate students participated in the study.

We conducted an ANCOVA test to determine the differences in the posttest CCI scores between the classes controlling for the pretest scores. The results showed that students in three classes did not differ in their conceptual understanding of differential calculus by the end of the semester controlling for their initial understanding, $F(2, 147)=1.07$, $p=0.35$. The original means on the posttest were 7.33 (SD=3.60; N=49) for the COPS class, 9.28 (SD=3.77; N=64) for the PP class, and 7.29 (SD=3.16; N=38) for the DI class. Our findings suggest that students' conceptual understanding of differential calculus is independent from the instruction type when measured by a validated instrument, considering each class individually. Such an ex post facto design provides a more comprehensive picture of instruction implementation. However, its particulars may lead to different outcomes and be challenging to identify. Future research should explore context specific variations in approaches' implementation to determine commonalities between the effective ones.

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