STUDENTS' THINKING IN AN INQUIRY-BASED LINEAR ALGEBRA COURSE

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This evaluation study compared the mathematical thinking of undergraduate students (as they responded to class work and interview prompts) who participated in an inquiry-based linear algebra course to a comparison group of students who participated in a traditional course.

Keywords: Linear Algebra, Inquiry-Based-Learning

The research literature shows that in traditional linear algebra classrooms students often memorize algorithmic methods that "work" even when not properly understood. Students develop understandings of matrix algebra and solving systems of linear equations using the Gaussian elimination, yet have problems with the more abstract notions of spanning set, linear independence and linear transformation (Stewart & Thomas, 2007). In the language of Sierpinska's modes of thinking (2000), the student dependent upon reducing a matrix to echelon form to determine whether vectors are linearly independent are thinking in *arithmetic mode*, whereas a student who is able to think more generally about objects in a system, by applying a definition or theorem when appropriate, is thinking in *structural mode.*

In this poster session we will share results from an evaluation study conducted in a large public research university, that examined how an inquiry-based linear algebra course supported the development of student mathematical reasoning from actions in the embodied world to formal structural thinking (See Figure1). Our framework, used to guide analysis of student work/interview responses, is adapted from Stewart & Thomas (2009, 2010), incorporates three mathematical worlds (embodied, symbolic and formal) and depicts a progression in the development of mathematical thinking (Tall, 2004).

	Embodied world/Synthetic Geometric Thinking	Symbolic World/ Arithmetic Thinking	Formal World/ Structural Thinking
Action	Adds multiples of two given vectors in R^2 or R^3 to visually determine whether a third is a linear combination of the given vectors.	Tests if a set of vectors is LI by constructing a matrix with the vectors as columns and row reducing it.	
Process	Generalizes this visualization process to two arbitrary vectors v_1, v_2, v_3 in R^2 or R^3.	Thinks about the action above without actually carrying it out.	
Object	Operates on this visualization of LI vectors (eg. Transforming them via reflection, rotation, etc)	Understands process as above and can operate on the resulting matrix (eg. knows that if matrix has a pivot in every column the original set of vectors is LI.	Shows set of given vectors is LI by definition by considering the vectors space that the vectors are in (eg. Gives a dimensional argument)

Figure 1. Framework of Progression of Mathematical Thinking (with Linear Algebra examples)

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