The Effect of Self-Efficacy on Student Performance in Calculus

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Self-efficacy is an important variable that has been used to study students' performance at all educational levels and in many content areas. In the report, we discuss the results of a quantitative study considering self-efficacy in college Calculus and its correlation to other variables available in a large scale study. Ultimately, our findings contradict existing findings regarding the effect of self-efficacy on class performance. We add to these results an interesting finding regarding the effect of self-efficacy on student's study habits: while time spent on course homework does not mediate the effect of self-efficacy, more time spent on course preparation by students with high self-efficacy tends to decrease their expected final course grade. Results contribute to math instructors' understanding of their teaching and may help with the construction of more effective instruction.

Key words: Calculus, Self- Efficacy, Class Performance, Homework Hours Spent, Classroom Research.

Introduction

Self-efficacy describes a person's perception of her own potential to master a specific task, and has been shown to have a powerful effect on achievement (Bandura, 1986; Bandura, 1997). Self-efficacy affects behavior by influencing one's choices and actions (Pajares, 1996). Bandura (1997) proposes four sources of self-efficacy: personal experience (referring to prior outcomes), vicarious experience (what one observes), social persuasions (feedback received from peers and others), and psychological state (ones mood), but personal experience and previous performance levels influence self-efficacy above all else (Chen & Zimmerman, 2007). In this research report, Bandura's theory of self-efficacy grounds and supports the work.

Within the context of mathematics, self-efficacy has been shown to be a better predictor of performance than measures of math anxiety or prior experience with math (Pajares & Miller, 1994; Pajares & Miller, 1995), and appears to be tantamount in importance to even intellectual ability (Pajares & Kranzler, 1995). High self-efficacy correlates positively with greater aspirations, greater commitments, and a greater ability to recover from setbacks; high math self-efficacy correlates with greater persistence on long and difficult problems, and greater accuracy of computation (Collins, 1982; Hoffman & Schraw, 2009). Hackett (1985) found a positive relationship between self-efficacy and ACT scores, which agrees with numerous studies showing a powerful link between high self-efficacy and high performance (Fast et al., 2010; Pajares & Miller, 1994; Pajares & Miller, 1995; Pajares & Kranzler, 1995); Peters, 2013). Meece, Wigfield, & Eccles (1990) found that high math self-efficacy corresponds to students valuing math more highly, and expecting to succeed. The study also found that students' performance expectations predict math anxiety, but that math anxiety only indirectly relates to subsequent performance. Performance expectations do, however, predict actual performance. Students who rate math performance as relatively important tend to have lower math anxiety than students who rate math performance as unimportant (Meece, Wigfield, & Eccles, 1990).

A positive relationship has been demonstrated between math self-efficacy and gender (men tend to have higher math self-efficacy) (Hackett, 1985). However, while Hackett (1985) found a positive relationship between gender and math achievement, other studies have found no such relationship (despite confirming that men tend to have higher math self-efficacy than women) (Meece, Wigfield, & Eccles, 1990; Peters, 2013). However, self-efficacy mediates the influence of gender on math performance (Pajares & Miller, 1994). Math self-efficacy has even been shown to mediate the effects of prior math experience on performance, itself the foremost source of self-efficacy (Pajares & Miller, 1994).

Our analysis utilizes the Mathematical Association of America's (MAA's) study of *Characteristics of Successful Programs in College Calculus* (NSF, DRL 0910240) data to revisit these issues by addressing the following two questions:

Research Question 1: Is the effect of student math ability on expected final grade moderated or mediated by student's self-efficacy, such that confidence in ability can overcome lower actual ability?

Research Question 2: Does a student's self-efficacy effect the amount of work he or she does to prepare for class, and does self-efficacy moderate the effect of class preparation on a student's expected final grade?

Methods and Data

As mentioned before, we utilized the *Characteristics of Successful Programs in College Calculus* (CSPCC) data set and conducted factor analysis to construct a measure of self-efficacy by combining students' answers to several questions into a single variable. We then created mixed effects ordered logistic regression models with random intercepts using this measure and other data from the survey. The CSPCC data was collected and made available for reserachers by MAA based upon work supported by the National Science Foundation under grant DRL REESE #0910240 between 2009 and 2015.

The survey data concerns students' experience and performance in Calculus I. The entire survey consists of two parts, a pre-survey administered at the beginning of Calculus I, and a post-survey administered at the end of the Calculus I course. From this data set, we used the expected course grade from the post-survey as the dependent variable. From the pre-survey we use SAT and ACT math scores along with three separate questions that assess self-efficacy as independent variables. One additional independent variable was taken from the post survey: the amount of time spent on homework. As control variables, we used age, gender, mother's education, and father's education from the pre-survey.

The survey data included responses from 13,965 high school students who completed at least one of the surveys. Of these students, 10,506 are eliminated, because they only completed one of the surveys, and the questions we are using to complete the analysis come from both surveys. After using this list wise deletion criteria on just the dependent variable, the final sample of college student respondents is 3,459 from 14 universities. Of those 3,459, 10 did not report a final grade, reducing the sample to 3,449 (in the full sample 6,144 students did not report a final grade). Additionally, the two measures of student math ability, SAT and ACT math score, are also missing across some observations. We converted ACT math scores to SAT scores using the conversion method recommended by The College Board, so as to maximize the sample size of our model. After converting the ACT score, and eliminating observations where respondents did not report either score, the final sample is 2,973. The other independent variables were also missing in some responses. When included in a model together, the final overlapping sample

size(*n)* is 2787. However, the exact *n* of each of our models changed depending on the variables included. Table 1 presents the summary statistics and missing observation counts across all of the variables used in our models.

Table 1: Summary and Missing Data Statistics

	Mean	SD	Min	Max	Missing
Gender	.4531755	.497892	0	1	24
Mother's Ed	3.66272	1.057391	1	5	12
Father's Ed	3.759239	1.132856	1	5	10
Calc I Grade	3.159311	.837746	0	4	10
Last HS Math Grade	3.704399	.6118699	0	4.33	194
Self Efficacy	.0253085	.9761658	-4.654925	1.17197	23
Age	18.81306	1.794104	14	38	5
SAT Math Score	650.3742	78.44545	18	800	477
AP Math Class	1.556512	.4968852	1	2	60
N	2787				

Linear Regression Results for Research Question 1

As mentioned above, previous research found self-efficacy to be a better predictor of mathematics achievement (Pajares & Kranzler, 1995). We reexamined this question and asked whether self-efficacy moderates or mediates the effect of ability on student performance in their Calculus I course. To assess the potential moderating effect, we first constructed a measure of self-efficacy. We used principal component factor analysis to construct a variable that combines three survey questions that evaluate a student's self-efficacy. Factor analysis is a statistical technique for data reduction. It reduces the data by generating linear combinations of "factors" that reconstruct a group of related variables (Hamilton 2013, Ch.11). More specifically, principal component analysis (PCA) conducts an eigen decomposition of the correlation matrix between the selected variables. The eigenvectors represent uncorrelated linear combinations of the variables that capture most of the variance across the variables. In other words, principal component analysis can reduce many variables down to a single variable that captures the most amount of covariance between the variables. We use the first factor loadings to construct a measure of self-efficacy.

The three survey questions that generated the data we use in the PCA are as follows and responses to these questions can range from 0 (strongly disagree) - 5 (strongly agree):

Question 29 Know: I believe I have the knowledge and abilities to succeed in this course.
Question 29 Understand: I understand the mathematics that I have studied.
Question 29 Confident: I am confident in my mathematics abilities.

A histogram of the constructed measure is presented in Figure 1

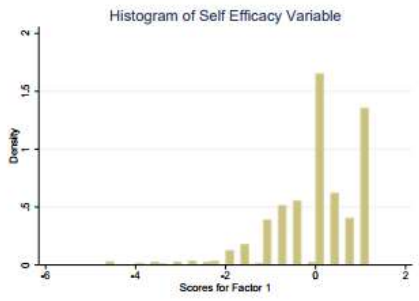


Figure 1: Histogram of Constructed Self Efficacy Variable

With this measure constructed, we interact self-efficacy with SAT math score and students' grade in the last high school math class they completed. Table 2 presents the results of the mixed effects ordered logistic regression models with random intercepts for respondent's university department. The interaction terms are not statistically significant, implying that there is not a strong consistent moderating effect of self-efficacy on course ability.

Table 2: Moderation Analysis for Question 1

	(1) Grade	(2) Grade
Female	0.0975	-0.0750
	(0.0737)	(0.0699)
Mother`s Ed	0.0283	0.0572
	(0.0410)	(0.0388)
Father`s Ed	0.000235	0.0727*
	(0.0388)	(0.0356)
HW Hours	-0.00850	-0.0308*
	(0.00763)	(0.00686)
Age	0.0198	0.00324
	(0.0198)	(0.0130)
Self Efficacy	0.365	0.440*
	(0.264)	(0.184)
SAT Math Score	0.00720*	
	(0.000533)	
Last HS Math Grade		0.631*
		(0.0576)
Interaction	0.000424	0.0530
	(0.000412)	(0.0503)
N	2905	3172

Standard errors in parentheses
$*p < .05$

To analyze whether self-efficacy mediated the effect of their actual ability on their grade in Calculus I, such that actual ability causes self-efficacy, which in turn determines a student's grade, we start by examining whether all variables are statistically significant predictors of expected final grade when included on their own. The results of models (1), (2) and (4) shown in Table 3 show that the coefficients on all variables are positive and statistically significant. Secondly, we tested whether the coefficients on SAT math score or high school math grade lose significance when self-efficacy is included in the model, which would indicate a mediating effect of self-efficacy The results, presented in Table 3 are not consistent with the presence of a mediating effect. Both SAT math score and high school math grade remain statistically significant and positively signed when efficacy is included in the model (models (3) and (5) in Table 3).

Table 3: Regression Model for Mediation Analysis in Research Question 1

	(1) Grade	(2) Grade	(3) Grade	(4) Grade	(5) Grade
Female	-0.0466	0.00704	0.0961	-0.245*	-0.0729
	(0.0717)	(0.0673)	(0.0737)	(0.0678)	(0.0699)
Mother's Education	0.0221	0.0488	0.0285	0.0556	0.0563
	(0.0403)	(0.0373)	(0.0410)	(0.0381)	(0.0388)
Father's Education	-0.00626	0.0818*	-0.000884	0.0891*	0.0725*
	(0.0379)	(0.0343)	(0.0387)	(0.0349)	(0.0356)
HW Hours	-0.0149*	-0.0284*	-0.00880	-0.0397*	-0.0309*
	(0.00741)	(0.00666)	(0.00762)	(0.00668)	(0.00686)
Age	0.0127	-0.0142	0.0205	0.00206	0.00306
	(0.0200)	(0.0115)	(0.0197)	(0.0128)	(0.0130)
SAT Math Score	0.00836*		0.00711*		
	(0.000519)		(0.000528)		
Self Efficacy		0.678*	0.635*		0.630*
		(0.0360)	(0.0402)		(0.0378)
Last HS Math Grade				0.772*	0.623*
N	2922	3357	2905	3191	3172

Standard errors in parentheses
* $p < .05$

Linear Regression Results for Research Question 2

The work ethic of a student is an important determinant of her performance. However, the willingness of a student to invest her time and energy in preparing for the course (as a proxy for this, we used the Homework variable) is certainly impacted by self-efficacy. We included this important Homework variable in our analysis of student performance, and evaluated whether self-efficacy moderates the effect of homework hours invested on student performance, or whether homework hours mediates the effect of self-efficacy. Self-efficacy may determine the amount of time and energy a student decides to invest in her course. To assess the presence of a mediating effect of homework hours on self-efficacy, we first examined whether self-efficacy is correlated with the amount of homework hours students expend. We then evaluated whether homework hours and self perception of ability are statistically significant predictors of expected final grade when included on their own. Finally, we tested whether the effect of efficacy loses significance when homework hours is included in the same model. The results of these models, presented in Table 4, are not consistent with the proposition that homework hours mediate the effect of self-efficacy on student's achievement in Calculus I. In fact, the results imply the opposite direction of causality, but given that self-efficacy is evaluated prior to students completing the course, the causal relationship implied by the model is not plausible.

To evaluate the moderating effect of self-efficacy on homework hours, we create an interaction term between the two variables and again include it in a mixed effects multinomial ordered logit models with random intercepts for university. The results of the model are presented in Table 5.

Table 4: Regression Model for Mediation Analysis in Research Question 2

	(1) HW Hours	(2) Grade	(3) Grade	(4) Grade
main				
Efficacy	-0.495*	0.635*		0.635*
	(0.0866)	(0.0402)		(0.0402)
Gender		0.0961	-0.0466	0.0961
		(0.0737)	(0.0717)	(0.0737)
Mother's Ed		0.0285	0.0221	0.0285
		(0.0410)	(0.0403)	(0.0410)
Father's Ed		-0.000884	-0.00626	-0.000884
		(0.0387)	(0.0379)	(0.0387)
HW Hours		-0.00880	-0.0149*	-0.00880
		(0.00762)	(0.00741)	(0.00762)
Age		0.0205	0.0127	0.0205
		(0.0197)	(0.0200)	(0.0197)
SAT Math		0.00711*	0.00836*	0.00711*
		(0.000528)	(0.000519)	(0.000528)
N	3408	2905	2922	2905

Standard errors in parentheses
$*p < .05$

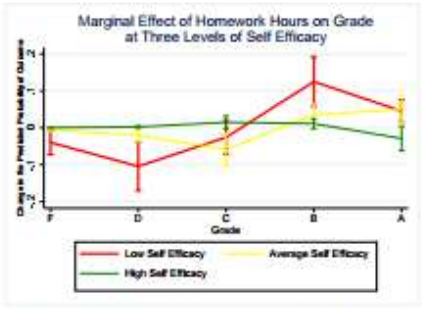
Table 5: Moderation Analysis for Question 2

	(1) Grade
Female	0.104
	(0.0738)
Mother's Education	0.0290
	(0.0411)
Father's Education	0.00288
	(0.0388)
HW Hours	-0.0131
	(0.00765)
Age	0.0232
	(0.0198)
SAT Math Score	0.00709*
	(0.000529)
Self Efficacy	0.799*
	(0.0604)
Interaction	-0.0254*
	(0.00677)
N	2905

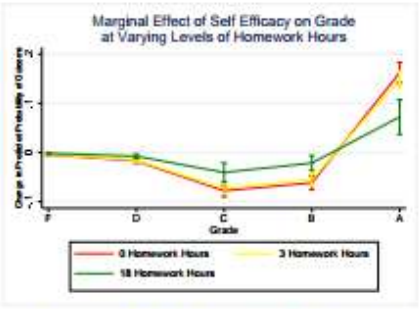
Standard errors in parentheses
$* p < .05$

The interaction term is negative and statistically significant. Examining the marginal effect of homework hours at different levels of self-efficacy produces some surprising results. When we examined the marginal effect of a five hour increase in homework hours at various levels of self-efficacy on a student's expected grade, in Figure 2, we observe that a one hour increase in the amount of time a student with low self-efficacy spends on his or her homework decreases the probability that she receives a D or an F. However, every additional hour of study time increases the probability that she earns a B by much more than it increases the probability that she earns an A. The effect of increased homework hours has a similar effect for students with average self-efficacy, in that the larger amount of study time increases slightly the probability that the student earns an A, and decreases the probability that the student earns a D or an F.

Interestingly, for students with high self-efficacy, every hour of study time decreases the probability that the student earns an A by about .008, but increases the probability that the student earns a B (by .005) or a C (by .003). Increased homework hours has no statistically significant effect on the probability that a student with high self-efficacy receives a C or lower. Implied in this relationship is that self-efficacy has the largest positive impact on grade when a student studies the least. Figure 1 presents the change in the predicted probability of receiving each grade for a one unit increase in self-efficacy at different amounts of homework hours. This figure demonstrates that high self-efficacy increases the probability that a student receives an A in the course, having completed on average zero hours of homework per week, by .162, increases the probability that the student receives a B by .148, and decreases the probability of an F by .004. However, the effect of self efficacy on the likelihood that a student earns an A decreases to .072 when the students spends 18 hours per week on homework. Therefore, high self-efficacy proves more impactful on a student's grade when he or she does not spend a substantial of time studying. For those students with high self-efficacy, putting in more prep time is an indication that they are more like to earn a B than A in the course.



(a) Marginal Effect of Homework Hours on Grade



(b) Marginal Effect of Self-Efficacy on Grade

Figure 2: Marginal Effects from Moderation Analysis for Research Question 2

Conclusion

In this study we were unable to confirm findings of past studies that suggest that self-efficacy is more important than actual ability. Mathematics ability and experience correlate with self-efficacy, but self-efficacy does not overtake the effect of experience and ability. Additionally, we inquired as to whether student's self perception mediates or moderates the impact of their study habits on their course grade. Ultimately, when considered in aggregate, the regression analyses presented in this paper seem to suggests the following conclusions. Students have a fairly accurate perception of their math ability. Those who are are not confident in their ability because their ability is not very strong put in more effort preparing for class, and benefit from increased time spend studying and completing homework. However, this increased effort most greatly increases the odds that they earn a B in the class. Those with very high ability, and who are confident in that ability, do not expect to earn a higher grade as a result of increased time spent on course preparation, but do expect to earn high grades. In fact, students with high math ability and high self-efficacy are most likely to expect to earn an A. For those students with average confidence, applying themselves and putting more time in on homework and class prep tends to increase the likelihood that they earn an above average grade. Finally, self-efficacy has the largest effect on a student's grade when the student did not spend a substantial amount of time preparing for class.

For mathematics instructors, the take home lessons from this study are simple and commonsensical. First, confidence is not a replacement for hardwork, experience, and preparation in mathematics achievement. Apart from especially gifted mathematics students, homework and study time boost exam performance for most students. Over time, sucessful experiences in mathematics courses compound and create a virtuous cycle whereby students become more confident and this confidence, in conjunction with a strong work ethic, in turn contributes to further good performance.

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