Mathematical Knowledge for Teaching Examples in Precalculus: A Collective Case Study

Erica R. Miller
University of Nebraska-Lincoln

The purpose of this collective case study is to examine mathematical knowledge for teaching examples in precalculus. The instructors involved in the study were experienced graduate teaching assistants who were teaching their course for the third time and were identified as good teachers. Utilizing a social constructivist and cognitive theory approach, I analyzed video recordings of enacted examples. The central question that guided this analysis was: What is the mathematical knowledge for teaching examples in precalculus? The goal of this study is to examine undergraduate mathematical knowledge for teaching from the perspective of practice, instead of relying on existing frameworks. As a result of this study, the author developed a model of mathematical knowledge for teaching examples in precalculus that includes knowledge of representations, students, instruction, specialized content, and connections when enacting high cognitive demand examples.

Keywords: Mathematical knowledge for teaching, undergraduate, precalculus, cognitive demand, examples

Introduction

Mathematical knowledge for teaching (MKT) has been defined as the "mathematical knowledge needed to perform the recurrent tasks of teaching mathematics to students" (Ball, Thames, & Phelps, 2008, p. 395). While MKT has been studied extensively at the elementary level (Ball et al., 2008; Carpenter & Fennema, 1991; Heather Hill, Sleep, Lewis, & Ball, 2007; Ma, 2010) and at the secondary level (Krauss, Baumert, & Blum, 2008; McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012; Rowland, Huckstep, & Thwaites, 2005), research on MKT at the undergraduate level is still a growing field (Speer, Smith, & Horvath, 2010). The goal of this study is to contribute to that field by building upon the link between MKT and cognitive demand (Charalambous, 2010) in order to study mathematical knowledge for teaching examples in precalculus from the perspective of practice.

Problem

Often, it is assumed that earning a degree in mathematics is what initially qualifies ones to teach at the undergraduate level. Historically, undergraduate instructors learned to teach by following the role model of mentors. However, Bass (1997) points out that there is much that cannot be learned through observations alone. To address lack of teaching preparation, many doctoral programs today offer teaching professional development (PD) for graduate teaching assistants, who will make up the future workforce of undergraduate instructors (Bressoud, Mesa, & Rasmussen, 2015; Ellis, 2014). While offering some teaching PD is better than none, the content of what is being taught is an important aspect to consider.

Of course, pedagogical knowledge is a component of teaching and should be included in GTA PD. However, studies have shown that despite their formal mathematical education, GTAs still lack mathematical knowledge that is needed for effective teaching (Kung & Speer, 2009; Speer & Hald, 2008). In these studies, the authors rely on existing frameworks for MKT that where developed at the K-12 level. While it is reasonable to assume that K-12 and undergraduate MKT are similar, Speer points out that there are important differences between K-12 and undergraduate teachers that need to be attended to (Speer, King, & Howell, 2014). Therefore, the goal of this study is to examine MKT at the undergraduate level from the perspective of practice, instead of relying on existing frameworks.

Significance

As previously stated, there is little research on MKT at the undergraduate level. But why is it important to study MKT to start with? First, studies have found that *pure content knowledge* is not a predictor of teaching quality and student achievement (Begle, 1972; Greenwald, Hedges, & Laine, 1996; Hanushek, 1981, 1996). However, studies at the K-12 level have shown that *MKT* is a predictor of teaching quality and student achievement (Hill et al., 2008; Hill et al., 2007; Krauss et al., 2008). This knowledge is not usually taught in content courses, hence why many GTAs seem to be lacking MKT. While no measures of MKT at the undergraduate level exist, it is reasonable to assume that this positive relationship still exists at the undergraduate level. Therefore, if we can identify what MKT at the undergraduate level looks like and integrate it into GTA PD programs, we can have a positive impact on undergraduate education.

The other question that is reasonable to ask is why focus on precalculus? As the number of students needing to take introductory math courses for their degree increases, the teaching burden of math departments increases (Ellis, 2014). Approximately 1,000,000 college students take introductory level math courses each year (Gordon, 2008). Of these, approximately 85-90% are non-STEM intending (Rasmussen, Ellis, Lindmeier, & Heinze, 2013) and success rates are typically around 50% (Gordon, 2008). Even for STEM-intending students, studies have found that difficulty passing introductory-level courses is contributing to the "leaking pipeline" of students leaving STEM (Thompson et al., 2007). Therefore the instructional quality of precalculus has a large impact on undergraduate students.

Background

While research on MKT at the undergraduate level is sparse, there does exist a large body of research on K-12 MKT. While my goal is to examine MKT at the undergraduate level from the perspective of practice instead of using existing frameworks of MKT that were developed at the K-12 level, the two are bound to be closely related. In an effort to situate my study within the existing field of research on MKT and avoid the assumption that I am attempting to study MKT at the undergraduate level in an epistemological vacuum, I will first present a broad overview of existing research on MKT. Also, I chose to study MKT by building upon its relationship with the cognitive demand of tasks. This decision was motivated by Charalambous' (2010) exploratory study, which found that MKT and the cognitive demand of enacted tasks are positively related.

Mathematical knowledge for teaching. Following the studies that showed that subject matter knowledge was not a predictor of teaching quality and student outcomes, Lee Shulman (1986; 1987) proposed that researchers begin studying pedagogical content knowledge. Shulman defined pedagogical content as going "beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching" (1986, p. 9). Shulman situated pedagogical content knowledge in contrast to subject matter knowledge, which is "the knowledge, understanding, skill, and disposition" of a subject matter (1987, p. 8). Since then, math education researchers have begun looking into professional knowledge for teaching mathematics. Hill, Rowan, and Ball (2005) found that elementary teacher's MKT was a significant predictor of student gains. Similarly, Baumert et al. (2010) showed that secondary teachers' MKT was a predictor of student outcomes. In both of these examples, the mathematical knowledge that is specific to the work of teaching is not *usually* taught in general undergraduate mathematics courses. Therefore, using the number of math courses taken beyond calculus is not the same as measuring content knowledge for teaching.

Speer, Smith, and Horvath (2010) conducted a literature review to search for empirical research on the practices of collegiate teachers of mathematics. As a result, the authors identified only five articles, indicating that "collegiate teaching *practice* remains a largely unexamined topic in mathematics education" (p. 100). Since then, more studies have been published specifically on MKT at the undergraduate level (Bargiband, Bell, & Berezovski, 2016; Callingham et al., 2012; Castro Superfine & Li, 2014; Firouzian & Speer, 2015; Hauk, Toney, Jackson, Nair, & Tsay, 2013; Jaworski, Mali, & Petropoulou, 2017; Musgrave & Carlson, 2017; Rogers & Steele, 2016; Rogers, 2012; Speer & Wagner, 2009; Vincent & Sealey, 2015). However, some of these studies utilize existing frameworks for MKT that were developed at the K-12 level, which can be problematic (Speer et al., 2014). Therefore, the purpose of this study is to contribute to this growing body of research by examining MKT at the undergraduate level from the perspective of practice.

Cognitive demand and task unfolding. Smith and Stein (1998) define *lower-level demand tasks* as "tasks that ask students to perform a memorized procedure in a routine manner" and *higher-level demand tasks* as "tasks that require students to think conceptually and that stimulate students to make connections" (p. 269). Stein, Remillard, and Smith (2007) also created a framework to describe the temporal process of task unfolding and factors that contribute to this transformation. In this process, teachers utilize a written task to formulate their intended task, which in turn influences the enacted task. Each phase in this process is motivated by the goal of producing student learning and is influenced by factors, such as teacher's beliefs and knowledge. In 2010, Charalambous found that there was a connection between elementary teachers' MKT and their ability to enact tasks at a high level of cognitive demand. It is this relationship between MKT and cognitive demand that I plan to build upon in this study.

Purpose and Research Question

The purpose of this collective case study is to examine mathematical knowledge for teaching examples in precalculus. I will do this by first examining cognitive demand in order to identify examples that were enacted at a high level of cognitive demand. Building upon Charalambous' (2010) results, I believe that these examples will provide me with fertile ground for examining MKT. While I believe that MKT influences every stage in the process of task unfolding, this report will focus on the final stage of task unfolding. The central question that guides this study is: *What is the mathematical knowledge for teaching examples in precalculus?* To narrow the focus of this study, I will primarily attend to answering the following two subquestions:

1. What mathematical knowledge enables instructors to enact examples at a high level of cognitive demand?
2. How can we characterize this knowledge?

Methodology

Theoretical Framework

In order to study teacher knowledge, I will utilize a social constructivist lens as well as cognitive theory of the teaching process. A social constructivist lens assumes that "multiple realities are constructed through our lived experiences and interactions with others" (Creswell, 2013, p. 36). Social constructivist researchers believe that reality is shaped by individual experiences, utilize an inductive method of emergent coding, and often collect observational data. Schoenfeld's (1998, 1999) cognitive theory of the teaching process attends to teacher knowledge (as well as goals and beliefs) and how it influences decision-making. The reason why I chose this framework is because it attends to the reasons why a teacher makes certain instructional decisions and what knowledge enables them to do this. Also, it complements Stein et al.'s (2007) task unfolding framework in many ways.

Setting and Participants

For the purposes of this study, precalculus courses are defined to include the College Algebra, Trigonometry, and combined College Algebra + Trigonometry courses. The participants from this study were all instructors at the same large public university in the Midwest. At the university involved in the study, second-year graduate students make up the majority of the instructors for precalculus. Since second-year graduate students are teaching their own class for the first time, I chose to exclude them from my data set and instead only recruited participants who were teaching a precalculus course for at least the third time. The participants in this study included one Trigonometry instructor (Greg) and three College Algebra + Trigonometry instructors (Alex, Emma, and Kelly). All of them were graduate students in their third, fourth, or fifth year who had already earned their M.S. and were working towards their Ph.D. in mathematics. While they all were teaching their prospective course for the third time, they had 2.75 years of collegiate teaching experience on average. Also, all of the participants in this data set were recruited because their department had identified them as good teachers.

Design and Procedures

In order to answer my research questions, I am utilizing a collective case study design (Stake, 1995). In order to examine MKT more generally, I included multiple instructors and collected data on multiple examples. Since I have included a limited number of participants, there is little is known about mathematical knowledge for teaching precalculus, and I seek to propose new theoretical insight into MKT, I chose to utilize an exploratory case study (Yin, 2014). The unit of analysis I am focusing on is the examples enacted by precalculus instructors. Studying teaching from the perspective of practice can be difficult, so I utilized the frameworks of cognitive demand and task unfolding to help make the knowledge the teachers were using more visible. Building upon Charalambous' (2010) finding that MKT and cognitive demand are positively related, I utilized cognitive demand as a way to identify examples that would provide me with rich opportunities to examine MKT. Second, studying teaching through the task unfolding framework (Stein et al., 2007) allowed me to see the instructors' decision-making and examine how their mathematical knowledge enabled them to enacting examples.

Coding proceeded in two stages that concentrated on cognitive demand and then knowledge. In the first stage, I utilize the Task Analysis Guide (Smith & Stein, 1998) to code the cognitive demand of enacted example. Examples that were coded as enacted at a high level of cognitive demand were then analyzed in the second stage, which has two cycles. In the first cycle, I utilized inductive descriptive coding (Miles, Huberman, & Saldaña, 2014) to identify mathematical knowledge that enabled the instructors to enact the example at a high level of cognitive demand. This round of coding would help me to answer my first research question. To answer my second research question, I conducted a second cycle of pattern coding in order to identify emergent themes and relationships between the codes that resulted from the first cycle. A detailed description of this methodology can be found in Author (2017).

Results

Task Unfolding by Cognitive Demand

I will report the results from the first stage of analysis in brief, since the second stage of analysis primarily answers the research questions. In total, there were 39 examples included in the full data set. Of those, 13 examples were either included in the written lesson guide but not used by the instructor or included in their lesson plan but not enacted during class time. While these examples still involved the teacher utilizing their mathematical knowledge to make instructional decisions, this paper focuses on enacted examples, so they will not be discussed. Of the remaining 26 examples, 14 of them were enacted at a high level of cognitive demand. It is also important to note that all 14 of these examples were coded as procedures with connections tasks (Smith & Stein, 1998).

Mathematical Knowledge for Teaching

In the second stage of coding, four main domains of knowledge emerged: representations, students, instruction, and specialized content. In addition, knowledge of connections between and within these domains was also a prominent domain of knowledge that emerged. For each of these domains, I will describe some of the related sub-codes and give examples of the mathematical knowledge that the instructors used in relation to these categories.

Representations. Since procedures with connections tasks are "usually represented in multiple ways" (Smith & Stein, 1998, p. 348), it is not surprising that representations emerged as a main domain of knowledge. Several instructors depended on *knowledge of representations that reflected student thinking*. For example, Alex introduced exponentials by having students compare simple and compound interest. After letting her students work on the problem for a while, she noticed that many students were working calculating compound interest recursively, so she drew a table that organized their calculations by year. Emma, on the other hand, recognized that her students were struggling to connect verbal descriptions of function transformations to their final graphical representations, so she drew the associated graph for each individual transformation. In teaching her students about the long-term behavior of polynomials, Kelly utilized *knowledge of accessible representations that still capture complexities* (e.g., $y = x, x^2, x^3$) in order to strip away unnecessary distractions and help her students focus on the important features.

Students. Instructors relied upon their knowledge of students in varying ways. Greg used *knowledge of common student struggles* and removed the goal statement from the written lesson guide in order to force his students to make connections between the problem and the content they had previously learned. Both Alex and Kelly applied their *knowledge of students' abilities* and designed their examples around tasks that students would struggle with, but were within reach. This also required the instructors to have *knowledge of student understanding.* Instructors also utilized *knowledge of appropriate questions to ask*, *knowledge of how to probe student thinking, knowledge of how to interpret student thinking,* and *knowledge of how to respond to student thinking* as they collaboratively worked through examples with the input of students. Emma also had to interpret and respond to student thinking, although she did so in the context of reviewing student quizzes and selecting an example that addressed a common mistake many students made. Another general sub-code that was categorized as knowledge of students was *providing explanations to students*.

Instruction. The two most common sub-codes that fell under the domain of knowledge instruction were *knowledge of instructional sequences* and *knowledge of problem scaffoldings.*

To help her students construct an exponential equation, Alex sequenced instruction so that students worked informally with concepts before they were formally defined, utilized familiar problems to reintroduce ideas, and provided motivation for topics. She also scaffolded their inquiry by introducing a table. Emma scaffolded problems by building connections between algebraic and graphical representations and sequenced instruction by first utilizing familiar, but inefficient, methods before introducing new, but more efficient, methods. Also, Greg utilized *knowledge of how to guide instruction towards the mathematical point* by choosing to not pursue a student suggested idea that might detract from the main goal of the example.

Specialized Content. While knowledge of course content influences all of the domains, some sub-codes related primarily to specialized content knowledge that goes beyond the content covered in the course. For example, instructors had to rely on their specialized *knowledge of reasonable and appropriate examples*. While some of this was planned, other times it was something that instructors had to do on the spot. For example, Alex initially introduced function compositions generally. However, she decided to make the example more concrete and constructed functions that were reasonable and appropriate. In order to come up with accessible representations that still captured complexities, Kelly drew upon her *knowledge of critical and non-critical features* of functions and their long-term behavior. In explaining why a certain answer was incorrect, Emma utilized *knowledge of how errors impact the final solution*. While these may all be examples of content knowledge that the instructors would like their students to develop, they were not part of the intended learning outcomes for the course and therefore make up specialized content knowledge that the instructors drew upon when teaching.

Connections. Given that all of the examples were coded as procedures with connections tasks, connections emerged as another main domain of knowledge. However, this domain is different from the others in that it is not independent, but rather captures knowledge of relationships between and within the other four domains. Instructors relied upon their knowledge of connections in a variety of ways. For example, Kelly drew upon her *knowledge of related topics* in order to illustrate how the multiplicity of zeros relates to the behavior of a polynomial function at its zeros. In order to help students understand the purpose of an example or a single step, Alex and Emma relied on their *knowledge of connections between mathematical computations and problem-solving goals.* In many cases, instructors combined their knowledge of connections and their pedagogical skills in order to build *knowledge of how to help students make connections*.

Discussion

In analyzing the data, I found that knowledge of representations, students, instruction, specialized content, and connections enable instructors to enact examples at a high level of cognitive demand. Since knowledge of connections is really knowledge of how the other domains are connected, I represented this model as a pyramid (Figure 1) with specialized content as the base and connections as the edges. In addition to making connections to different domains, knowledge of connections can also be used within a single domain. Finally, knowledge of students, instruction, representations, and connections are all situated within and build upon knowledge of course content, but I chose to not focus on this type of knowledge in my model.

Conclusions

Given that examples are an important part of teaching, this model can be used in designing teaching PD opportunities for GTAs. In particular, PD should be designed to help GTAs develop knowledge of representations, students, instruction, specialized content, and connections. This model benefits the community of math education by providing a decomposition of the knowledge used by instructors when teaching examples in precalculus. While it is similar to other models of MKT, it is also different in several important ways. First, the domains of knowledge are inherently connected. Second, while knowledge of representations and connections are implicit in many of the other models, they are not explicitly emphasized.

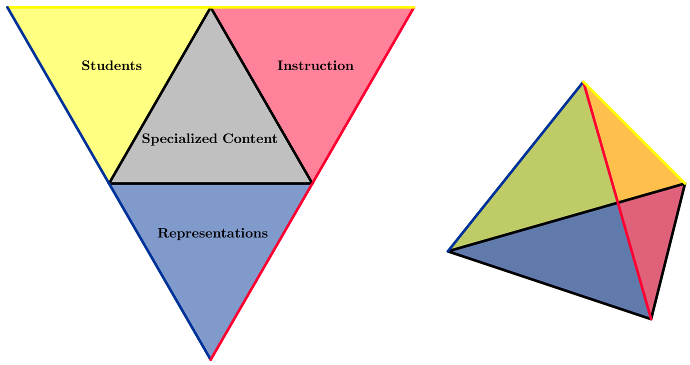

Figure 1 Proposed model for mathematical knowledge for teaching examples in precalculus.

Limitations

First, as noted previously, the five domains of knowledge are not assumed to be independent. From a quantitative standpoint, this is a limitation of the model, but I believe it accurately reflects the interconnected nature of teaching. Second, since all of the high cognitive demand examples were coded as procedures with connections tasks, this model may overemphasize knowledge of connections and representations. However, "doing mathematics" may not be well suited for examples and it may be reasonable to assume that most high cognitive demand examples are procedures with connections tasks. Also, since this study was a collective case study and all of the instructors were graduate students, it may not be generalizable.

Future Research

There is still much work that needs to be done to understand MKT at the undergraduate level, but this study provides a starting point for future investigations. In particular, it would be interesting to extend this study in several different directions. First, expanding the sample size and including instructors with a variety of backgrounds and teaching experience would test whether or not the model could be generalizable. Second, observing enacted examples that are "doing mathematics" tasks (Smith & Stein, 1998) would help further refine the model and test whether or not "procedures with connections" tasks had a large influence on the knowledge domains that emerged. Third, in order to understand post to better understand MKT at the undergraduate level at large, it would be beneficial to collect classroom data that focuses on more than just examples. Finally, my intention is to dig into the entire process of task unfolding and see what knowledge instructors use in the planning stage and utilize pre- and post-observation interview data to dig further into the knowledge used by instructors when teaching precalculus.

Acknowledgements. My deepest gratitude first goes to [faculty] for her support, guidance, and help as my dissertation advisor. I would like to thank all the [research group] members who contributed to discussions concerning the analysis and findings of this study. I would also like to thank the instructors whose participation made this study possible.

References

Author (2017). Details omitted for blind review.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*, 389-407. doi: 10.1177/0022487108324554

Bargiband, J., Bell, S., & Berezovski, T. (2016). Guided reflections on mathematical tasks: Fostering MKT in college geometry. *Proceedings of the 19th Annual Conference on Research in Undergraduate Mathematics Education,* (pp. 414-419). Retrieved from http://sigmaa.maa.org/rume/Site/Proceedings.html

Bass, H. (1997). Mathematicians as educators. *Notices of the American Mathematical Society*, *44*, 18-23. Retrieved from http://www.ams.org/notices

Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., … Tsai, Y.-M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, *47*, 133-180. doi: 10.3102/0002831209345157

Begle, E. G. (1972). *Teacher knowledge and student achievement in algebra.* Palo Alto, CA: Stanford University Press.

Bressoud, D., Mesa, V., & Rasmussen, C. (Eds.). (2015). *Insights and recommendations from the MAA national study of college calculus*. Washington, DC: Mathematical Association of America.

Callingham, R., Beswick, K., Clark, J., Kissane, B., Serow, P., & Thornton, S. (2012, July). *Mathematical knowledge for teaching of MERGA members*. Paper presented at the Annual Meeting of the Mathematics Education Research Group of Australasia (MERGA), Singapore.

Carpenter, T. P., & Fennema, E. (1991). Research and cognitively guided instruction. In E. Fennema, T. P. Carpenter, & S. J. Lamon (Eds.), *Integrating research on teaching and learning mathematics* (pp. 2-19). Albany, NY: State University of New York Press.

Castro Superfine, A., & Li, W. (2014). Exploring the mathematical knowledge needed for teaching teachers. *Journal of Teacher Education*, *65*(4), 303-314.

Charalambous, C. Y. (2010). Mathematical knowledge for teaching and task unfolding: An exploratory study. *The Elementary School Journal*, *110*, 247-278. doi: 10.1086/648978

Creswell, J. W. (2013). *Qualitative inquiry and research design: Choosing among five approaches* (3rd ed.). Los Angeles: SAGE Publications.

Ellis, J. (2014). Preparing future professors: Highlighting the importance of graduate student professional development programs in calculus instruction. *Proceedings of the Joint Meeting of PME 38 and PME-NA 36, 3*, 9-16.

Firouzian, S., & Speer, N. (2015). Integrated mathematics and science knowledge for teaching framework: Knowledge used in teaching applied derivative problems. *Proceedings of the 18th Annual Conference on Research in Undergraduate Mathematics Education,* (pp. 524-536). Retrieved from http://sigmaa.maa.org/rume/Site/Proceedings.html

Gordon, S. P. (2008). What's wrong with college algebra? *PRIMUS*, *18*, 516-541. doi: 10.1080/10511970701598752

Greenwald, R., Hedges, L. V., & Laine, R. D. (1996). The effect of school resources on student achievement. *Review of Educational Research*, *66*, 361-396. doi: 10.3102/00346543066003361

Hanushek, E. A. (1981). Throwing money at schools. *Journal of Policy Analysis and Management*, *1*, 19-41. doi: 10.2307/3324107

Hanushek, E. A. (1996). A More complete picture of school resource policies. *Review of Educational Research*, *66*, 397-409. doi: 10.3102/00346543066003397

Hauk, S., Toney, A., Jackson, B., Nair, R., & Tsay, J.-J. (2013). Illustrating a theory of pedagogical content knowledge for secondary and post-secondary mathematics instruction. *Proceedings of the 16th Annual Conference on Research in Undergraduate Mathematics Education* (Vol. 1) *,* 308-322. Retrieved from http://sigmaa.maa.org/rume/Site/Proceedings.html

Hill, H., Blunk, M., Charalambous, C., Lewis, J., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction*, *26*, 450-511. doi: 10.1080/07370000802177235

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *42*, 371-406. doi: 10.3102/00028312042002371

Hill, H., Sleep, L., Lewis, J., & Ball, D. L. (2007). Developing measures of teachers' mathematical knowledge for teaching: What knowledge matters and what evidence counts. In K. F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 111-155). Reston, VA: National Council of Teachers of Mathematics.

Jaworski, B., Mali, A., & Petropoulou, G. (2017). Critical theorising from studies of undergraduate mathematics teaching for students meaning making in mathematics. *International Journal of Research in Undergraduate Mathematics Education*, *3*, 168-197. doi: 10.1007/s40753-016-0044-z

Krauss, S., Baumert, J., & Blum, W. (2008). Secondary mathematics teachers' pedagogical content knowledge and content knowledge: Validation of the COACTIV constructs. *ZDM*, *40*, 873-892. doi: 10.1007/s11858-008-0141-9

Kung, D. T., & Speer, N. (2009). Mathematics teaching assistants learning to teach: Recasting early teaching experiences as rich learning opportunities. In L. L. B. Border, N. Speer, & T. J. Murphy (Eds.), *Research on graduate students as teachers of undergraduate mathematics*. Stillwater, OK: New Forums Press.

Ma, L. (2010). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States* (2nd ed.). New York, NY: Routledge.

McCrory, R., Floden, R., Ferrini-Mundy, J., Reckase, M. D., & Senk, S. L. (2012). Knowledge of algebra for teaching: A framework of knowledge and practices. *Journal for Research in Mathematics Education*, *43*, 584-615. Retrieved from http://www.nctm.org/publications/journal-for-research-in-mathematics-education

Miles, M. B., Huberman, A. M., & Saldaña, J. (2014). *Qualitative data analysis: A methods sourcebook* (3rd ed.). Thousand Oaks, CA: SAGE Publications.

Musgrave, S., & Carlson, M. P. (2017). Understanding and advancing graduate teaching assistants' mathematical knowledge for teaching. *The Journal of Mathematical Behavior*, *45*, 137-149. doi: 10.1016/j.jmathb.2016.12.011

Rasmussen, C., Ellis, J., Lindmeier, A., & Heinze, A. (2013). Who is switching out of calculus and why. *Proceedings of the 37th Conference of the International Group for the Psychology of Mathematics Education, 4*, 73-80.

Rogers, K. C. (2012). *The proof is in the practice? Graduate teaching assistants and future teachers* (Doctoral Dissertation). Retrieved from ProQuest Dissertations & Theses A&I. (1038835959)

Rogers, K. C., & Steele, M. D. (2016). Graduate teaching assistants' enactment of reasoning-and-proving tasks in a content course for elementary teachers. *Journal for Research in Mathematics Education*, *47*, 372-419. doi: 10.5951/jresematheduc.47.4.0372

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education*, *8*, 255-281. doi: 10.1007/s10857-005-0853-5

Schoenfeld, A. H. (1998). Toward a theory of teaching-in-context. *Issues in Education, 4*, 1-94. doi: 10.1016/S1080-9724(99)80076-7

Schoenfeld, A. H. (1999). Models of the teaching process. *The Journal of Mathematical Behavior*, *18*, 243-261. doi: 10.1016/S0732-3123(99)00031-0

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*, 1-23. doi: 10.17763/haer.57.1.j463w79r56455411

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-14. doi: 10.3102/0013189X015002004

Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School*, *3*, 344-350.

Speer, N., & Hald, O. (2008). How do mathematicians learn to teach? Implications from research on teachers and teaching for graduate student professional development. In M. Carlson & C. Rasmussen (Eds.), *Making the connection: Research and teaching in undergraduate mathematics education* (pp. 305-217). Washington, DC: Mathematical Association of America.

Speer, N. M., King, K. D., & Howell, H. (2014). Definitions of mathematical knowledge for teaching: Using these constructs in research on secondary and college mathematics teachers. *Journal of Mathematics Teacher Education*, *18*, 105-122. doi: 10.1007/s10857-014-9277-4

Speer, N. M., Smith, J. P., III, & Horvath, A. (2010). Collegiate mathematics teaching: An unexamined practice. *The Journal of Mathematical Behavior*, *29*, 99-114. doi: 10.1016/j.jmathb.2010.02.001

Speer, N. M., & Wagner, J. F. (2009). Knowledge needed by a teacher to provide analytic scaffolding during undergraduate mathematics classroom discussions. *Journal for Research in Mathematics Education*, *40*, 530-562. doi: 10.1007/s11858-015-0738-8

Stake, R. E. (1995). *The art of case study research*. Thousand Oaks: SAGE Publications.

Stein, M. K., Remillard, J., & Smith, M. S. (2007). How curriculum influences student learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 319-369). Charlotte, NC: Information Age Pub.

Thompson, P. W., Castillo-Chavez, C., Culbertson, R. J., Flores, A., Greeley, R., Haag, S., … Rutowksi, R. L. (2007). Failing the future: Problems of persistence and retention in Science, Technology, Engineering, and Mathematics (STEM) majors at Arizona State University. Tempe, AZ: Office of the Provost. Retrieved from http://pat-thompson.net/PDFversions/2007FSICfinalreport.pdf

Vincent, B., & Sealey, V. (2015). Connecting research on students' common misconceptions about tangent lines to instructors' choice of graphical examples in a first semester calculus course. *Proceedings of the 18th Annual Conference on Research in Undergraduate*

Mathematics Education, 1039-1043. Retrieved from http://sigmaa.maa.org/rume/Site/Proceedings.html
Yin, R. K. (2014). *Case study research: Design and methods* (5th ed.). Los Angeles, CA: SAGE Publications.