Using Quantitative Diagrams to Explore Interactions in a Group Work and Problem-Centered Developmental Mathematics Class

Martha B. Makowski
The University of Alabama

Sarah T. Lubienski
Indiana University

Despite low success rates and an academically vulnerable population, classes taught at the pre-college (or developmental) level have rarely been examined by mathematics education researchers. Mathematical Literacy, a recent developmental curriculum innovation, aims to better meet the unique needs of developmental students through group work and problem-centered materials. Using a novel quantitative representation of the classroom and descriptive statistics, this study examines the productivity of developmental mathematics students, their engagement with each other and the instructor, and their access to the curriculum in a Mathematical Literacy implementation. We find that students' engagement is regular, productive, and frequently involves the instructor. However, some students have less access to group discussions. Future implementations should focus on early identification of such students.

Keywords: Developmental mathematics, community college, quantitative methodology

Enrolling over one million students (Blair, Kirkman, & Maxwell, 2013), mathematics classes offered at the pre-college level (often called *developmental*) serve those enrolled in academic tracks requiring advanced coursework, but not deemed knowledgeable enough to take college-level, credit-bearing mathematics classes. Most developmental mathematics is taught at community colleges (Blair et al., 2013), which serve students from drastically different stages of life, ranging from fresh out of high school to adults returning to school after many years (Cohen, Brawer, & Kisker, 2013). Problematically, many of the students who start developmental classes never finish (Bailey, 2009; Bailey, Jeong, & Cho, 2010), meaning that developmental coursework serves a gatekeeping function. Thus, examination of the issues surrounding the curriculum and instruction of these classes is critical.

Classroom-level research on developmental mathematics curriculum and instruction is limited, although calls for more high-quality investigations of community college mathematics are common (e.g., Condelli et al., 2006; Mesa, in press; Mesa, Wladis & Watkins, 2014; Speer, Smith, & Horvath, 2010). Most existing work focuses the instructional methods teachers use, rather than on the interplay between the curriculum, students, and instruction (Mesa, in press). Existing work shows that developmental teachers tend to use methods emphasizing skill acquisition (Grubb et al., 1999). Given that many students have previous taken and failed to learn the material (Hoyt & Sorensen, 2001), developmental students may benefit from teaching that uses different approaches.

Mathematical Literacy at Fields Community College

Fields Community College (FCC; all names are pseudonyms), a large community college in a small Midwestern city, has recently become involved in the *Mathematical Literacy* movement (Statway and Quantway are the most well-known and widely implemented of these classes [e.g., Hoang, Huang, Sulcer, & Yesilyurt, 2017]). In addition to supporting student content learning, the designers of *Mathematical Literacy* aimed to (a) make the content relevant to the academic needs of the developmental students, and (b) highlight how mathematics informs students' lives.

To meet these goals, the *Mathematical Literacy* curriculum centers around real-world problem solving facilitated through group work, echoing the calls of the National Council of Teachers of Mathematics' (NCTM; 1989, 2000) push for more problem-centered instruction in the K-12 curriculum. Although similar in intent, *Mathematical Literacy* is tailored to meet the needs of developmental students, who, given their diversity in terms of demographics, life stage, and career objectives (Cohen et al., 2013), create a unique classroom of self-selecting, but often skeptical students. The differences between the developmental and K-12 populations, combined with the focus of *Mathematical Literacy* on group work, raise the question of whether and how these students will engage with the curriculum and their groups.

This study examines a *Mathematical Literacy* classroom, through a theoretical perspective focused on student enactment of the task as implemented (Stein, Grover, & Henningsen, 1996) and their patterns of interaction with the instructor. In particular, we ask:

1. Within their groups, how productive is student engagement with the curriculum materials?
2. Does everyone have equal access to group discussions within their groups?
3. What are students' patterns of interaction with each other and the instructor as they work?

To investigate these questions, we introduce a new method for examining classroom participation and productivity, demonstrating the utility of the method by showing results related to student talk within groups and instructor movement throughout the classroom.

Methods

All the data for this study come from FCC, which implemented a *Mathematical Literacy* curriculum over the 2014-2015 school year. Data were collected in a single classroom taught by an instructor who had participated in the course development.

Sample

The classroom started with 24 students, which adjusted to 22 students (6 men and 16 women) within the first week of the semester. Fourteen were White, six were Black, two were from Asian backgrounds, and one was Hispanic. Nineteen students agreed to be audio recorded in their groups. Because groups shifted throughout the semester and everyone in the group needed to assent to recording, at most 14 students were in audio-recorded groups at any given time.

Data Sources

Data come from field notes and classroom audio, collected during weeks one, seven, 13, and 15 of a 16-week semester. This report focuses on the data from week seven. The full paper will include data from other weeks. Class met three times a week for 110 minutes and the first author was present for the entire class period. During observations, the instructor was audio recorded for the entire period. Student groups were audio recorded using table microphones. Recording started when the class transitioned to small-group work and stopped after the groups were gone. Field notes were taken.

Analysis

The first two research questions examine how students engaged with the curriculum during group work and the equality of their participation. Using descriptive statistics and quantitative representations of the groups, we examine observed patterns of behavior within groups. For the third question, we perform a similar analysis, but look at patterns of engagement at the classroom level, focusing on instructor location and talk. All our results rely on (a) descriptive statistics on participation patterns, and (b) diagrams created from coded and timestamped transcripts of the classroom. Examples of these diagrams occur in Figures 1 and 2 in the results section. The methods for extracting descriptive statistics and generating the diagrams are described in the next few sections.

Preparation of transcripts. All group audio was transcribed in full. Turns were timestamped and began when a new person started talking or there was an extended gap in students' conversation. Transcripts were coded for:

1. Whether the instructor was present for each speaker turn.
2. The activity of the students in their groups.

To code the activity of students in their groups, we included isolated, non-related comments of individuals that were not remarking about something that others at the table were talking about. This decision reflects the fact that we were interested in group, not individual, behavior. At times, separate conversations occurred simultaneously. In such instances, turns were coded separately for the activity, and an indicator identifying that two conversations were occurring was added to the transcript. Group activities were coded as one of four main categories, which are elaborated on in Table 1:

1. Problems from the workbook, coded at the problem level.
2. Homework assignments, coded at the assignment level.
3. Class-related activities not directly related to a graded assignment.
4. Off-task talk.

Table 1. Group activity sub-codes.

Code	Description
Problem From Workbook	Students work towards a solution on a problem from the workbook. These were coded at the individual problem level.
Homework Assignment	Students work on one of the two group homework assignments. These are coded at the assignment level (review or project).
Helping Other Groups	When students from other groups visited the recorded table and engaged in discussion with one or more group members about a problem the table had already solved.
Planning and Other Class-Related Talk	Negotiations about which assignment they should work on, group roles, times they could meet outside of class, check-ins to see if anyone needed help, organization talk as they transition to a new assignment, or discussions about mathematics not directly related to their work for the day. Classroom related discussions not related to completing the workbook problems or group homework assignments.
Off-task	Students talking about non-mathematical issues that did not directly contribute to their classroom assignments or the nature of mathematics.

Computation of times. We computed the total time spent on each activity within groups by adding turn times together. This method of time calculation assumes that the group activity during periods of silence remained the same until the next utterance. Although this may not always be accurate, there is no reason to expect bias in favor of any activity.

Creating group- and classroom-level diagrams. Diagrams were created from timestamped and coded transcripts. All diagrams plot time against classroom or group activities and use colored regions to indicate different classroom-level activities (e.g., group work time, lecture). Group-level diagrams have two additional layers of information:

1. The *y*-axis lists the curriculum-level activities described in Table 1, with on-task activities lower and off-task group activities higher in the diagram.
2. Markers in the diagrams are linked to individuals to keep track of people through time as they move between group activities. Each color represents a unique speaker.

For all diagrams, markers indicate the start time of an individual's speaking turn, corresponding to the curriculum activity of the group at that time.

To visualize the whole class, we created diagrams that included multiple groups on a single plot (Figure 2 is an example). To minimize the overall complexity, three main modifications were made from the group-level diagrams:

1. Individuals within the same group are assigned the same color, rather than separate colors.
2. The instructor is tracked by her presence at the table rather than by her individual contributions using a larger, black marker.
3. Problem numbers are not explicitly labeled. To separate on- and off-task utterances, a dashed line cuts through the region for each group.

After creation, the diagrams were examined for participation patterns and deviations from the patterns. The diagrams are in some ways similar to the Chronologically Ordered Representations of Discourse and Features Used (CORDFU) diagrams that Luckin (2003) proposed. Our diagrams improve on CORDFU diagrams by using color strategically to separate individuals from each other, organizing the *y*-axis strategically, and clumping individuals around group activities to allow quick comparison of individuals.

Results

Group Engagement with Curriculum Materials

The average number of turns per individual suggests active, sustained conversations over the class, although individual participation varied greatly (Table 2). Emilia, in Group B, spoke an average of 38 times each day (about 3.4 minutes), while Craig, in Group D, spoke an average of 424 times a day (about 45 minutes). Groups spent an average of about 15 minutes off-task (Table 3), which includes a take a 10-minute break they were encouraged to take each class. Note that these averages reflect only table-level talk. Classroom observations documented students often texting or examining their phones during class. While these results only speak to off-task *talk*, when students were talking within their groups they generally discussed class-related issues. Thus, groups were productive in completing the materials.

Access to Group-level Discussions about Material

The descriptive statistics suggest that student participation was not even within groups, but do not provide information on whether individuals had the opportunity to participate in group discussions about the material. To examine this question, we examined group-level diagrams. Several groups, in their diagrams, showed a closely working unit, with all members of the group either working on problems or going off-task together. However, this was not always the case. Figure 1 shows a group that clearly split into two overlapping sub-groups to cover the material. Closer inspection shows that in that group, although a student named Tyrone talked frequently (Table 2), it appears that he was supported through the material primarily by Sarah, rather than working through the material with his entire group. For example, Tyrone did not participate in completing the written task with the group on that day. This assignment, which will be elaborated on in the full paper, was one of the few opportunities for students to reflect on the mathematical content of the lesson.

Table 2. Average number of turns and length of group contributions by individual per day.

Table	Individual	Days present	Average number turns (*SD*)	Total time (min) speaking (*SD*)
	Dave	3	199(41)	12.2(2.4)
	Felicia	3	140(29)	9.3(2.9)
	Sarah	3	231(36)	21.1(7.1)
	Tyrone	3	371(71)	30.4(8.0)
	Beth	3	260(38)	40.5(13.8)
	Carrie	3	240(18)	30.5(11.3)
	Emilia	3	38(22)	3.4(2.3)
	Henry	3	189(21)	17.4(3.3)
	Gabby	3	116(22)	9.3(3.2)
	Jen	3	253(11)	44.3(7.0)
	Carley	3	231(82)	15.4(15.4)
	Craig	3	424(33)	45.3(2.6)
	Fiona	2	223(35)	15.1(3.5)
	Helen	2	206(131)	14.3(9.1)
Total[a]	14		223(96)	22(13.8)

[a] This row reports the average contribution for each student, weighting each student equally.

Table 3. Average time spent within groups on different group activities.

	Average[a] time (*SD*)		
Activity	Total	Without instructor	With instructor
Workbook	49.5 (5.9)	41.8 (4.5)	7.7 (1.5)
Reflections	2.2 (1.9)	2.9 (1.5)	0.6 (0.6)
Group homework	15.5 (13.7)	11.9 (7.8)	0.4 (0.3)
Planning and other class-related talk	7.9 (2.4)	6.0 (1.9)	1.7 (0.6)
Helping	1.9 (3.4)	2.5 (3.0)	0.0 (0.1)
Off-task	15.8 (7.6)	13.9 (6.8)	1.9 (1.7)

[a] Time was measured in minutes and averages were calculated by first computing the average time spent on each activity by group over the three days for which we have detailed records. We then averaged over the four groups.

Students' Patterns of Interaction

Inspection of the classroom-level diagrams allow for easier comparisons of groups within a single day and more clearly show the instructor's patterns as she moved between groups. Figure 2 presents the class-level diagram for the same day shown in Figure 1. The diagram shows the instructor stopping at each group several times over the course of the period, although she spends markedly more time with Group A than the other groups. Interestingly, much of her time with Group A on the day shown in Figure 2 was off-task, rather than related to the curriculum materials. The graph also shows the instructor seems to arrive at groups while they are still actively solving a problem, rather than at the end of the problem (to help them finish the problem) or when they are off-task. There was not a time for whole-class discussion.

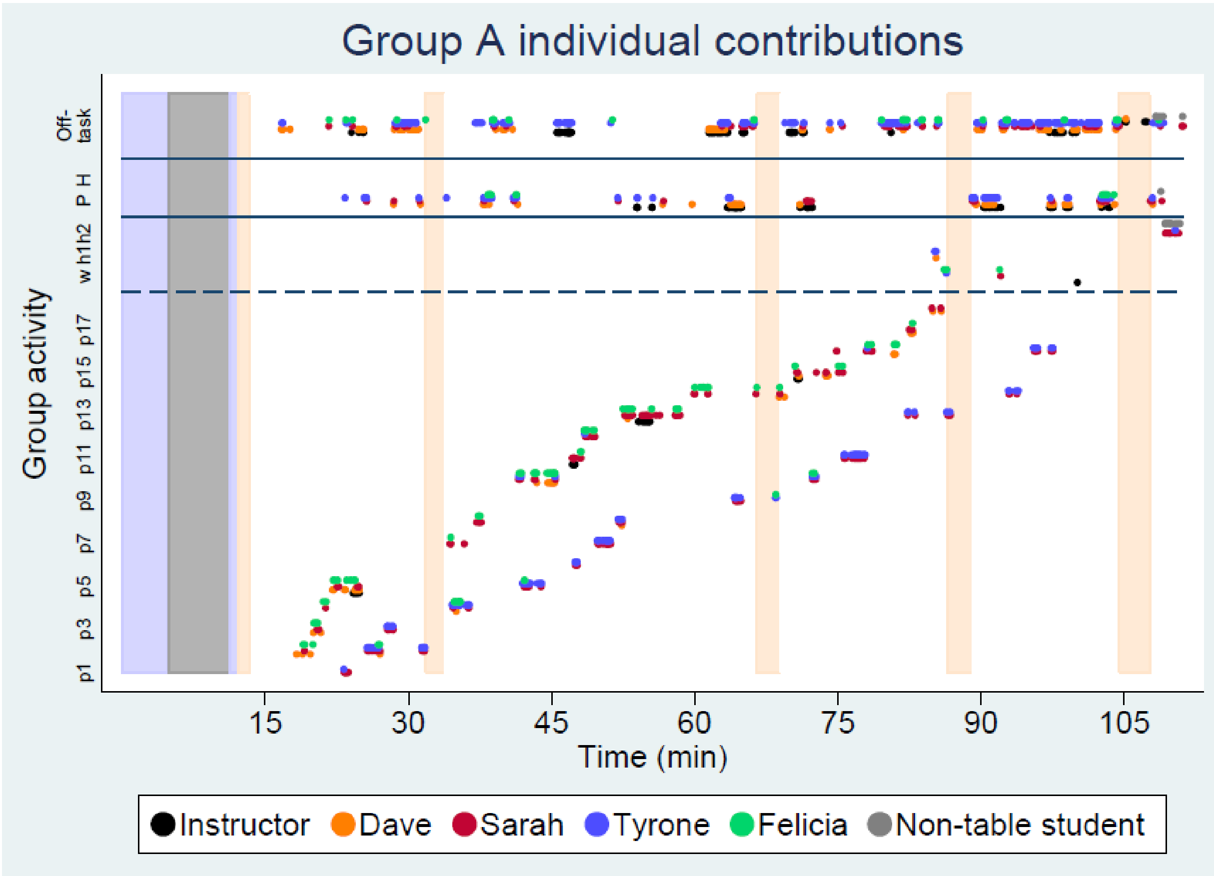

Figure 1. *Individual contributions to group activities for Group D. Each dot indicates when an individual started a new speaking turn. Recording of individuals started when the instructor opened the classroom up for group work (here, at minute 18). Light blue regions indicate when the instructor was returning assignments and checking in with students. Grey regions are when the students were engaged in a quiz or an activity related to the study. Light orange regions are when the instructor was lecturing to students and white regions are when the class was expected to be working in groups. The Group activity codes mean, in ascending order: problems 1 through 17 (skipping even problems) in the workbook, written reflection task (w), group homework assignments (h1 and h2), group planning and class-related talk (P), helping other groups (H), and off-task discussion.*

Discussion & Conclusions

The *Mathematical Literacy* curriculum used at FCC was structured to provide students with multiple opportunities to engage in real-world problem solving. Student enactment of this curriculum showed that (a) students were regularly on-task and engaged with the curriculum materials, and (b) that student engagement with the curriculum was primarily within their groups. Whole-class discussions of the material were rare to non-existent but the instructor provided ample time for students to engage with the problems while the she monitored and supported their progress through regular visits. The instructor’s dramatic move away from lecture, the instructional method that dominates many developmental classrooms (Grubb et al., 2009), demonstrates that such an instructional shift is possible within developmental mathematics.

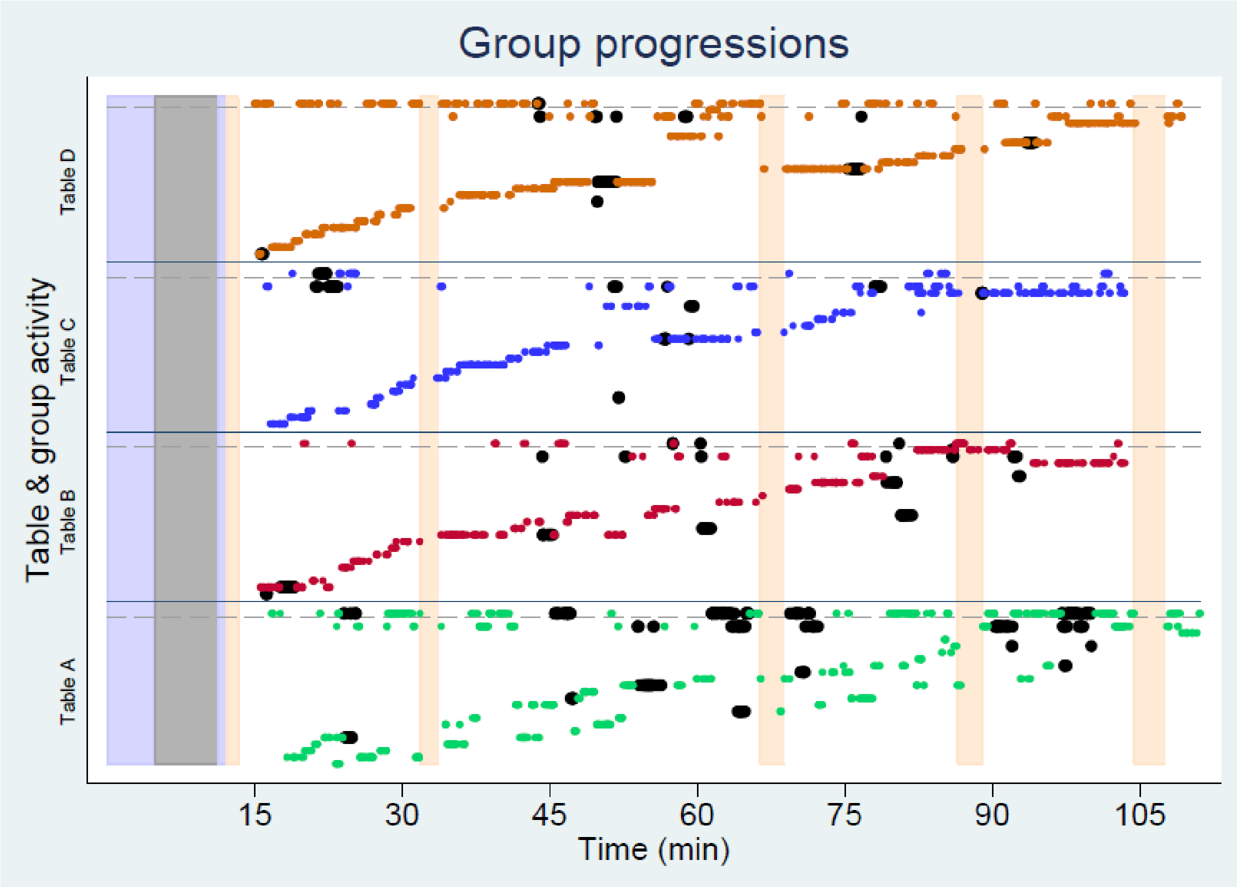

Figure 2. *Progression through group and classroom activities. See note for Figure 1 for an explanation of the shaded regions. Markers indicate that an individual started speaking at that time with respect to a particular classroom activity. Differently colored markers indicate different groups. The larger, black markers always indicate the presence of the instructor at the table, regardless of who spoke. Classroom activities are separated as they are in Figure 1; the dashed lines indicate the location of the off-task line for each group.*

Moreover, the students, although often having had many years of negative experiences in mathematics and only a short time frame to acclimate to group work and problem solving, can and did engage for sustained periods of time with mathematics problems in their assigned groups, a result that runs contrary to similar findings in K-12 mathematics: Wood and Kalinec (2012) offer one of the few looks at a group's time spent on a mathematics problem, and found that approximately 50% of the fourth graders' time was spent in off-task conversation. Adult students have been shown to have highly productive group work sessions in non-mathematical contexts (Barkaoui, So, & Suzuki, 2008). Our results suggest that developmental students in this context seem to exhibit a similar ability to self-regulate, perhaps because they have high achievement goal orientations (Mesa, 2012) and value efficiency in their education (Cox, 2009).

Although the regular engagement of the students and small amount of off-task talk is encouraging, the case of Tyrone and students like him who do not seem to have access to group level conversations about the material means that future implementations of *Mathematical Literacy* should consider ways to better meet such students' needs. Lastly, the use of quantitative diagrams, in addition to providing one method for helping to identify such students, can also be adopted as a research methodology that allows for the blending of qualitative coding with quantitative representations of the classroom.

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