Shape Thinking and the Transfer of Graphical Calculus Images

M. Katie Burden
University of Central Arkansas

Jason Martin
University of Central Arkansas

Shape thinking has previously detailed how students may view function graphs. Students using static shape thinking view a function graph as if it were a wire where learned rules, formulas and quantities appear as a consequence of the perceived shape. This study presents a case study that demonstrates how static shape thinking can be extended to other graphs seen within calculus. Results demonstrate how one first-semester calculus student perceived a "triangular" shape within a function graph. Quantities appeared as a consequence of this perceived shape and his reasoning on multiple related tasks was influenced by his transfer of this perceived shape onto subsequent graphs. Even the the student's reasoning led to inaccurate responses to interview tasks, his reasoning was accurate and consistent within his perception.

Keywords: Calculus, Shape Thinking, Quantitative Reasoning, Transfer, Derivative

Oftentimes students identify solution procedures by the type of question found in particular locations in the textbook, and thereby reduce calculus to a set of disjoint procedures that are conceptually unavailable when removed from the exact setting in which the procedure was presented (e.g., Bezuidenhout, 2001; White & Mitchelmore, 1996). Students' often view their procedural knowledge as irrelevant in "quantitatively complex situations" and have difficulty with reasoning quantitatively when relevant to even seemingly straightforward applications of mathematical concepts (Lobato & Siebert, 2002, p. 88). The quantitative complexity of the concept of derivative at a point is outlined in Zandieh's (2000) work where student understanding is framed within three "layers" of process-objects pairs. Carlson, Jacobs, Coe, Larsen, and Hsu (2002) asserted that students can struggle with derivatives because of an impoverished understanding of function that lacks a coordination of quantities foundational for reasoning about dynamic relationships captured by functions' rules. Carlson et al. (2002) outlined the mental actions needed for productive reasoning about the derivative of a function. In particular, emphasis was placed on Zandieh's ratio layer and the mental actions of coordinating the amount of change in one quantity with changes in the other quantity.

We speculate that interactive images can support the development of mental actions coordinating amounts of change of quantities. Furthermore, research suggests that engaging students in multiple problems from which to generalize can promote a richer understanding of a concept than might otherwise be achieved (Oehrtman, 2008).

This report is part of a larger study investigating the effects of contextual and graphical images of derivatives from multiple contextual problems using virtual manipulatives (VMs). For this paper, focus is placed on an interesting case and we ask the following question:

> What might students transfer while interacting with images graphically modeling similar quantitative attributes of different situations related to the concept of derivative?

A Research-Based Approach to Interactive Image Design

This section provides a review of relevant literature as it relates to the design of the images contained within the VMs. VMs were adopted because they can show continuous change in real time (Castillo-Garsow, 2012) and can be used to aid students in making sense of calculus concepts by highlighting connections between multiple representations, developing quantitative reasoning, and supporting exploration of formal limit definitions (Cory & Garofalo, 2011; Thompson, Byerley & Hatfield, 2013; Thomas & Martin, 2017).

All problems presented in this study (Figure 1) were adapted from Oehrtman's (2008) approximation framework. Oehrtman (2009) found that students' spontaneous reasoning using an approximation and error analysis cognitive model for limit closely resembled the formal structure of limits while simultaneously supporting students in productively engaging quantitatively complex situations. Repeated structured reasoning using quantities and relationships between quantities associated with approximations and error analysis can encourage student generalization to shared structures across similar contextual situations.

Bolt	Sphere	Asteroid	Iodine
A bolt (arrow) is fired from a crossbow straight up into the air with an initial velocity of 49 m/s. Approximate the speed of the bolt at 2 seconds.	Approximate the instantaneous rate of change of the volume of a sphere with respect to its radius when the radius is 5 cm.	NASA has determined that asteroid 1999 RQ36 has a 1 in 1000 chance of colliding with Earth on September 24, 2182. […] Approximate the instantaneous rate of change of the gravitational force between the Earth and 1999 RQ36 with respect to distance when the two objects are 10,000,000 m apart.	The half-life of Iodine-123, used in medical radiation treatments, is about 13.2 hours. Approximate the instantaneous rate at which the Iodine-123 is decaying 5 hours after a dose of 6.4 g is injected into the bloodstream.

Figure 1. Four approximation problems used in each interview.

VMs were created by the second author using GeoGebra 5 (Hohenwarter & Fuchs, 2004). Figure 2 presents an overview of key attributes of a graphical VM indicating the interactive capabilities of the VM within the context of the bolt problem.

The mental actions of conceiving of, creating, and making inferences with covarying quantities are a "foundation from which the student can reflect upon to develop mathematical understandings and reasoning" (Moore, Carlson, & Oehrtman, 2009, p. 3). Covariational reasoning is defined as "cognitive activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other" (Carlson et al., 2002, p. 354). Carlson et al. (2002) went on to describe developmental levels of student's images (Thompson, 1994) of covariation based on the accrual of mental actions progressively supporting more sophisticated covariational reasoning in the context of derivative. In particular, Levels 1 and 2 involved coordinating changes in one quantity with changes in the other quantity. Levels 3 and 4 included the same mental actions as from Levels 1 and 2 and additionally involved coordinating amounts of change of one quantity with changes in the other quantity.

Students typically read the problem prior to interacting with a VM. When they first interacted with a graphical VM, the axes and the play/pause button (Figure 2 A) were the only visible attributes of the VM. We anticipated that the inclusion of the play button might help support developmental Levels 1 and 2. After the animation played, students could reveal (Figure 2 C) depictions of amounts of change. For the graphical VMs, these depictions of amounts of change initially appeared as attributes of multiple "triangles" along the curve (not depicted in Figure 2). Once amounts of change were depicted, the student could adjust (Figure 2 D) the amount of change of the independent quantity and highlight (Figure 2 G) particular intervals. After an interval had been highlighted, the student could reveal a secant line for the highlighted interval (Figure 2 H). The VMs' abilities to reveal, adjust, and highlight depictions of amounts of change were included to support Levels 3 and 4. For the purpose of our research, it was up to the student to conceive of and coordinate measurable attributes of the VM that coincided with

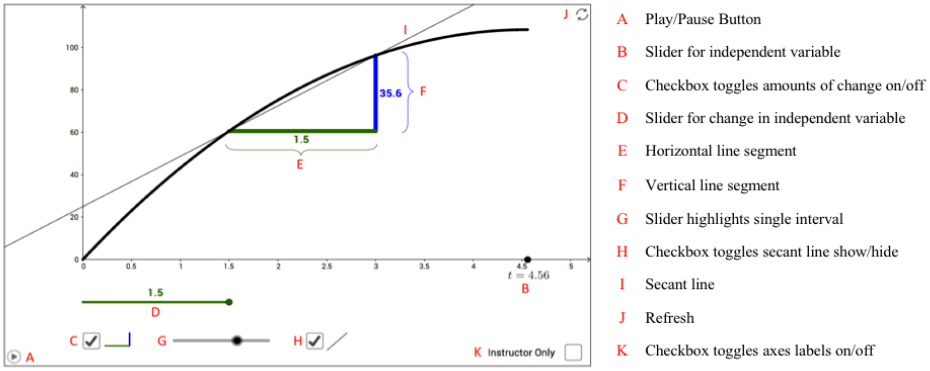

*Figure 2. Interaction points for a graphical VM where particular amounts of change have been highlighted. *Note: The letters A-K and corresponding descriptions on the right did NOT appear on the original VM.*

approximating the requested instantaneous rate. In particular, when interacting with a graphical VM, students needed to conceive of the *lengths* of the horizontal and vertical attributes of "triangles" as representing the amounts of change.

Theoretical Background: Shape Thinking and Transfer

For this study we focus on one of two modes of thinking based upon the extent to which an individual engages in quantitative and covariational reasoning while reasoning about graphs.

Static Shape Thinking

Static shape thinking entails a view of a function's graph as "an object in and of itself, essentially treating a graph as a piece of wire (graph-as-wire)" (Moore & Thompson, 2015, p. 784). The mental actions and operations that students engage during static shape thinking are rooted in Piaget's (2001) figurative thought "based in and constrained to sensorimotor experience (including perception)" (Moore, 2016, p. 324). Thus, a student exhibiting static shape thinking relies on the most salient perceptual cues of shape. Equations, function names, rules and properties of the function appear as consequences of shape. This view of the graph of a function may serve the student well in particular situations, such as function translations; however, static shape thinking obfuscates student's ability to view functions as emergent through covarying quantities. Static shape thinking does not mean students lack quantitative reasoning, but that when quantities appear, they appear as a consequence of the shape.

To better describe a student's meanings and ways of thinking while engaging in shape thinking, Moore and Thompson (2015) drew upon Thompson, Carlson, Byerley and Hatfield's (2014) definitions of *understanding*, *meaning*, and *ways of thinking*.

> *Understanding* is an in-the-moment state of equilibrium, which may occur from assimilation to a scheme or from a functional accommodation specific to that moment in time. A *Meaning* is the space of implications that the moment of understanding brings forth—actions that the current understanding implies. *Ways of thinking* are "when a person has developed a pattern for utilizing specific meanings…in reasoning about particular ideas" (Thompson et al., 2014, p. 12). (Moore & Thompson, 2015, p. 784)

Actor-Oriented Transfer

"Actor-oriented transfer is defined as the personal construction of relations of similarity between activities" (Lobato & Siebert, 2002, p. 89). In this study we adopted transfer as actor-oriented “to understand the interpretative nature of the connections that people construct between learning and transfer situations” (Lobato, 2012, p. 239). In particular, we desired to better understand the idiosyncratic interpretative nature of the connections that students were constructing as they progressed through the four problems (Figure 1) using the provided VMs (Figure 2). Due to the quantitative complexity of the problem situations, we anticipated that the nature of their interpretive engagement would be supported by quantitative and covariational reasoning. Quantitative and covariational reasoning have been shown to support students in conceiving of relevant mathematical structures within contexts and generalizing to a common mathematical structure shared by multiple contexts (Lobato & Siebert, 2002; Ellis, 2007; Thompson, 2011). In particular, students’ in-the-moment understanding of a VM is influenced through an interaction of prior learning experiences, their interpretation of the problem, and quantities and relationships conceived concerning the problem’s situation and conceived concerning the attributes of the VM. The space of implications that a moment of understanding brings forth can evidence transfer through the quantities and relationships between quantities conceived as similar to quantities and relationships from prior situations and prior VMs.

Methods and Analysis

Five students from a first-semester calculus course at a medium-sized university voluntarily participated in this study. The participants were A-B range calculus students majoring in a STEM field. Interviews occurred after the concept of the derivative and during definite integral instruction using Briggs, Cochran, and Gillett (2015). Each student was assigned to one type of representation (contextual or graphical) and level of interactivity (VM or static) throughout all interviews. This paper focuses on one student, pseudonym Jeremy, who viewed graphical VMs.

Jeremy participated in four 20 to 45 minute individual interviews where one problem from Figure 1 was presented per interview. He began with the bolt problem since velocity is the most commonly used physical example of derivative within calculus (Zandieh, 2000). In the second interview, he was introduced to the sphere problem and so on. All problems presented different situations but shared a common derivative structure and method for obtaining approximations to support transfer. Using a laptop computer, Jeremy interacted with four VMs, each depicting a different graphical image corresponding to the current problem, and each image similar to the image depicted in Figure 2. He was also provided with a calculator and smartpen.

While interacting with each VM, Jeremy was asked to identify on the image what depicted what he was approximating, approximations, and errors when applicable. Furthermore, he was asked how he could improve any approximation he might have produced. This task was intended to support student use of the VM to explore average rate of change over smaller intervals of the independent quantity. To evaluate his developmental level (Carlson et al., 2002) in relation to the current image, he was asked to complete the statement, “For any fixed amount of the change in (time/radius/distance/time), the amount of change in (height/volume/gravitational force/mass) is (increasing/decreasing/neither).” He was also told to compare rates at different instances, such as comparing the speed of the bolt at two and four seconds. Jeremy was repeatedly asked to indicate attributes of the image that supported his responses to interview questions.

Interview data consisted of written notes from his smartpen and audio and video records, including screen capture and one camera capturing gestures toward the screen. Data were analyzed with the intent to reconstruct in-the-moment understandings and meanings based upon the quantities and relationships between quantities conceived upon the situation and VM image. Records of Jeremy’s responses to interview questions were coded for instances of the appearance of quantities and relationships with particular attention paid to amounts of change and his interpretation of how these amounts were represented within the current image. Noting the appearance of quantities together with the current interview question and Jeremy's indicators of attributes of the image corresponding to such quantities, provide evidence for the origin of his reasoning. The analysis of Jeremy's reasoning concerning the bolt problem served as a baseline with which to compare his reasoning on subsequent tasks. To document transfer, special attention was paid to any moment in which Jeremy explicitly applied attributes from previous interview problems even though these attributes might be irrelevant for the new situation.

Results: A Case Study from a Graphical Virtual Manipulative

This section describes Jeremy's reasoning as he progressed through the four problems.

Jeremy's Reasoning During the First Interview

In the first interview, Jeremy described getting an approximation by taking the derivative of the function because "the derivative of position is velocity." As the animation played, he said, "It's pretty much showing […] the position with respect to time." He then stated that the vertical axis was showing meters. He noted that he could use the graph to make an approximation, and after he was asked how he would do that, he gave the formula "position divided by time."

The appearance of quantities based on perceived shape. After he was directed toward the "clickable features," Jeremy selected the checkbox that toggled the horizontal and vertical line segments denoting amounts of change. He said, "This makes me think of an area function." He then began talking about integrals and Riemann sums he was studying in class, "I think these shapes represent rectangles, it looks weird because it's in a way that I've never seen it." He then drew his own version of the Riemann sum showing an "overestimate" (Figure 3a), and concluded that the area under the function depicted the distance traveled.

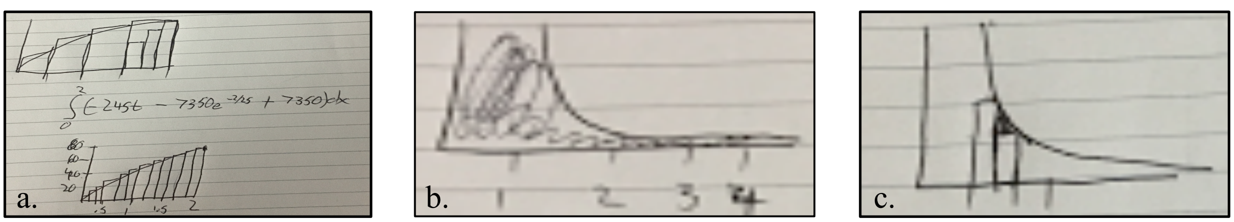

Figure 3. Jeremy's graphical depictions involving areas and rectangles.

After he displayed the secant line, he called it the "tangential." In this moment, he said that the "tangential" indicated the rate of change and moved both sliders to increase the number of "triangles" and to get the secant line to "land on" the point corresponding to $t = 2$. He stated that this method would give him the rate of change at two and repeatedly mentioned using the formula "position divided by time" to obtain an approximation. Even though Jeremy had been talking about "tangential" lines and a rate formula, he went on to say that to get a better approximation he would "take the integral from zero to two." Jeremy was viewing an image with many "triangles" depicted as he made this comment. Jeremy eventually reminded himself that since the image was the graph of position he needed to take the derivative to obtain the velocity. He was asked if there was a way to approximate the rate of change without taking the derivative. He said it was just "geometry" and went back to using the rectangles under the curve.

The previous paragraphs have detailed how Jeremy cued off of his perception of the image in front of him and his remembrances of learned rules and facts. For Jeremy, one of the most salient features of the images were the "triangles." When he cued off of the "triangles" or the "geometry" of the image, Jeremy "saw" rectangles and that reminded him of Riemann sums. When he cued off of the "tangential" lines or the physical situation, he was reminded of derivative and rate of change but neglected related quantitative meanings depicted on the graph.

The appearance of amounts of change. Once given the fixed amounts of change question, he looked at the image and said, "From 0 to 4.5 the height increases." The question was restated with emphasis on the "*amount of change,*" but he continued to neglect amounts of change. When asked to compare the rate of change at one second to the rate at four seconds, Jeremy finally spoke of amounts of change in reference to rates as he gestured over the graph of the function.

> The rate of change is definitely slower. As you can see, the height from zero to one [gesturing over the x-axis from $x = 0$ to 1] is in a difference of forty [upward gesture to the function at $x = 1$], but the height from three to four [gesturing over the x-axis from $x = 3$ to 4] is barely twenty [making the same upward gesture to the function but at $x = 4$].

In this moment, the fixed amounts of change and the comparing rates questions appeared to help focus Jeremy on quantities relevant to rate that were depicted on the image.

Transfer Enabled by Static Shape Thinking During Subsequent Interviews

When presented with the sphere VM, Jeremy stated, "Oh this is just another approximation problem." He then concluded he could produce an approximation by "doing the same thing as last time" and inquired if the image in front of him was depicting, "straight up triangles?" Clearly, Jeremy had engaged in attempting to transfer his way of reasoning about the first problem to the next problem and that this reasoning was influenced by his "triangles."

Implications of triangle reasoning. Throughout all four interviews, Jeremy continued to cue off of the "triangles" and imbue "rectangles" upon the image even though no rectangles were ever depicted. When looking at the triangles in the VM for the sphere he said, "I want to use a rectangle to estimate the area under the curve, and that will tell me the rate of change." He then reproduced the graph from the bolt problem and illustrated a rectangle under it.

As a consequence of Jeremy's "rectangles," he spoke frequently of the area under the curve (Figure 3b) and using the rectangles to approximate the area under the curve (Figure 3a). In addition, his graphical structure based on his perceived rectangles evolved to include quantities related to overestimates, underestimates and error. For example, he described the triangles as depicting error as "area that's not captured" by the area of the rectangles corresponding to an underestimate (Figure 3c). He observed, "the more rectangles I put in, the more accurate the estimation gets," and noted that the VM "program doesn't take the shapes under the curve to infinity." He described the integral as a limit of rectangles with no error, "An integral is pretty much this, but making your shapes [go to] infinity to where there is no error." When asked for detail to explain why area approximated instantaneous rate, he expressed uncertainty but remained steadfast that area under the curve depicted the appropriate rates of change.

"Tangential line" reasoning. Throughout the interviews, Jeremy continued to talk about the displayed secant lines as if they were "tangent lines." For him, his "tangential lines" represented the instantaneous rate of change at a point while simultaneously evolving to "fitting" the "hypotenuse" of his "triangle." Even though he had related rate of change to amounts of change during the first interview, in subsequent interviews he had difficulty describing amounts of change in relation to any rate. For example, in the second interview he drew a picture of the graph with several lines illustrated and compared rates where greater rates corresponded to steeper slopes. When asked to describe what the lines showed, he said, “Oh, the rate of change at that point.” When asked to identify the attribute of the lines that was the rate of change, he faltered. By the third interview, his notion of area was influencing how he compared rates, claiming that a “significantly higher” area corresponded to greater rate. Jeremy detailed how the area under the curve in Figure 3b from zero to one corresponded to a greater rate than from three to four because there was significantly more area under the curve from zero to one.

Learned rules and lower level mental actions. In addition to his graphical attributes that could produce instantaneous rates, Jeremy repeatedly stated his learned rule that derivatives would give instantaneous rates. Even so, when cuing off of the image, he continued to claim that he could take the integral over an interval to produce an instantaneous rate. Furthermore, in all four interviews, Jeremy repeatedly indicated no more than Level 2 reasoning while responding to the fixed amount of change questions. Neglecting amounts of change, he repeatedly described how the one quantity would change with respect to the other quantity.

Discussion

The nature of static shape thinking rooted in figurative thought based upon the most salient features of graphs suggests that Jeremy’s ways of thinking about these graphs likely represents a large population of students. Indeed, every student in our larger study spontaneously mentioned seeing “triangles” when viewing graphs.

Jeremy may seem contradictory at moments, yet he did not come into any observable state of cognitive conflict that ultimately led to any abandonment of approximating instantaneous rates using Riemann sums. Why not? Jeremy’s reasoning becomes very structured and consistent throughout the interviews. Consider Jeremy’s reasoning *as if* the problems in Figure 1 could be solved using definite integrals. Jeremy identifies 1) what he is approximating as area under the curve, 2) approximations as areas of rectangles, and 3) error as the area of “triangles.” In addition, 4) he can make approximations more accurate by including more rectangles and 5) has imagined the definite integral as taking the number of “[rectangle] shapes under the curve to infinity” where there is “no error.” Jeremy’s development and repeated use of this reasoning demonstrates this reasoning as a way of thinking concerning these tasks. This way of thinking was likely reinforced by his ability to answer approximation questions with corresponding graphical attributes.

Jeremy’s way of thinking developed through his repeated transfer of “triangular” shape and the space of implications that such transfer brought forth. It makes sense that static shape thinking would likely enable transfer due to the low cognitive demand of figurative thought. Thus, static shape thinking served Jeremy well by supporting transfer and supporting his development of his way of thinking about these tasks. Keep in mind, Jeremy “saw” rectangles.

The interview protocol focused on students’ perceptions of the images and did not include interventions intended to illuminate the irrelevance of area quantities to instantaneous rate. Thus, this study provides insight into how calculus students might perceive these types of graphs when no interventions are provided. Data from our larger study suggests that interventions designed to bring a student’s attention to the context may support students in moving away from static shape thinking. For example, we have observed notable differences between student reasoning when viewing graphical VMs compared to students viewing VMs depicting contextual images.

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