A Course in Mathematical Modeling for Pre-Service Teachers: Designs and Challenges

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The increased status of mathematical modeling in the K-12 curriculum requires teacher preparation programs to adapt. This design experiment examines a course in mathematical modeling for pre-service secondary mathematics instructors that was co-developed and co-taught by a mathematics educator and an applied mathematician. The students in the course, all mathematics majors, experienced growth as well as challenges, some rooted in quantitative reasoning.

Keywords: mathematical modeling, teacher preparation, quantitative reasoning

Mathematical Modeling is one of just six conceptual categories for high school in the Common Core State Standards for Mathematics (CCSSM) and is one of the eight CCSSM mathematical practices which span all of K-12 mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). These standards are adopted in 42 of the United States ("Standards in Your State," 2017), however there is reason for concern about teachers' preparation for implementing mathematical modeling tasks. It is rare for teacher preparation programs to even introduce students to mathematical modeling (Doerr, 2007; Lingefjärd, 2007a). Moreover, programs that do currently include or wish to develop instruction in mathematical modeling may be hindered by the lack of a robust research base about best practices, both for teaching mathematical modeling and for preparing teachers to teach modeling.

The elevated status of mathematical modeling in the curriculum, both as a practice and as a conceptual category, requires many secondary teacher preparation programs to adapt. Herein, we report on a design experiment in which the authors, a mathematics educator and an applied mathematician, co-designed and co-taught an undergraduate course in mathematical modeling for mathematics majors intending to be secondary teachers (N=9). In this first iteration of our design experiment, we were guided by a very broad research question about the nature of students' dispositions for engaging in and teaching mathematical modeling both before and after the course.

Perspective

The Guidelines for Assessment & Instruction in Mathematical Modeling Education (GAIMME) report described mathematical modeling as a process used to answer "big, messy, reality-based questions" (Garfunkel & Montgomery, 2016, p. 7). The process begins with identifying a problem and ends with reporting results. In between, the mathematical modeler makes assumptions; defines variables; refines the original question; develops and implements models; and analyzes the outputs of the model. This all transpires in a non-linear, often cyclic manner. The messiness, openness, and time requirements of authentic mathematical modeling tasks present an array of both pedagogical and conceptual challenges for teachers and for teacher preparation programs.

Some of the challenges for learners of mathematical modeling are documented by Thompson (2011) in his description of mathematical modeling as emerging from quantitative reasoning, which serves as grounding for several nontrivial abilities that are essential to mathematical modeling. Foundationally, the ability for quantitative reasoning allows a student to conceptualize a situation quantitatively. Extending this, covariational reasoning is needed for students to make sense of dynamic situations in which quantities vary in relation to each other. The ability to generalize, in the context of mathematical modeling, allows a student to represent these relationships. Thompson describes a mathematical model as a generalization “of a situation’s inner mechanics—of ‘how it works’” (p. 51).

Doerr (2007) noted that the pedagogical knowledge for teaching modeling is distinctive and she enumerated some specific pedagogical tasks for teachers of mathematical modeling, among them: choosing and adapting modeling tasks; anticipating and evaluating students’ varied strategies; and helping students make rich mathematical connections. This description of a teacher’s role in supporting mathematical modeling is largely echoed in the GAIMME report (2016), which also devotes considerable attention to the challenge of assessing mathematical modeling. Unfortunately, there is a dearth of research which investigates the development of the pedagogical knowledge teachers need for teaching modeling. Indeed, Doerr observed that “how teachers acquire this knowledge… remains an open question for researchers” (p. 77).

Doerr (2007) also describes a mathematical modeling course for pre-service teachers (N=8) which she developed and taught. In her course, students read about the modeling cycle and they engaged in and reflected on the modeling processes. She suggested that pre-service teachers engage in a variety of modeling tasks that require explanations, justifications, and reflection. Zbiek (2016) designed and taught a course for a similar audience. She focused on productive beliefs and corresponding unproductive beliefs about teaching and learning mathematical modeling. For example, it is productive to believe that mathematical modeling is a messy process, as opposed to the unproductive belief that problem solving should follow a clearly determined path. Through modeling and pedagogical tasks, students in her course moved toward productive beliefs, though this was accompanied by some persistent confusion about mathematical modeling. She echoed Lingefjärd’s (2007b) recommendation that modeling be integrated throughout teacher education programs, not just in a single course.

The Mathematical Modeling for Teachers Course

We co-designed and co-taught the Mathematical Modeling for Teachers course from the joint perspectives of our disciplines, mathematics education and applied mathematics, and with direction from the GAIMME report. Students in the course engaged in the modeling cycle through in-class team activities, homework/exam questions, and a final team project. From the first day of class, we were explicit about the modeling cycle and, after modeling tasks were completed, students reflected on their work as an expression of the cycle.

We did not organize the course as instruction in a sequence of different modeling techniques. The first half engaged students in a variety of modeling tasks in which they relied primarily on their existing algebraic, geometric, and statistical knowledge. This was followed by three weeks of instruction in linear programming, statistical and mathematical simulations, and some useful features of Microsoft Excel (2013) such as visualizing data, using random numbers to do simulations, making predictions with models, and solving linear programming problems. The next three weeks focused on pedagogical content knowledge such as modifying high school textbook tasks, analyzing curricular materials, and evaluating student work; some of this was foreshadowed by similar tasks in the first half of the course. The rest of the course was devoted to final projects by teams of students in which they identified a question, developed a model, reported on the model, and connected their work to the CCSSM. Throughout, we adjusted instruction according to what we perceived to be difficult parts of the modeling process for students. For instance, students struggled with generalization, as described by Thompson (2011), so we focused on this piece of the modeling process with some matching activities; students linked equations to verbal scenarios and linked the structures of equations to scenarios.

Rather than using a textbook, we developed, adapted, and curated tasks for the students. Students did readings from the GAIMME Report and from teacher-focused articles about mathematical modeling. An often used resource was the set of high school textbooks used by the local school district which claimed to be aligned with the CCSSM. The books labeled questions as “Modeling with Mathematics” within each problem set. However, to borrow phrasing from the GAIMME report (2016), the tasks would more aptly be described as “traditional word problems or textbook applications where all of the necessary information is provided and there is a single, known, correct answer” (p.28). This echoes Meyer’s (2015) analysis of two different supposedly CCSSM-aligned textbooks — tasks labeled as “modeling” rarely required students to model. The local textbooks were valuable both as illustrations of some of the curricular challenges our students would face as teachers and as a source of tasks for students to analyze and modify.

Methodology

We approached the development and implementation of the Modeling for Teachers course as a design experiment in which course design and theory development are “iterative and interactive” (Schoenfeld, 2006, p. 198). Herein, we report on the first iteration of the course; our research goals were to: 1. identify emergent themes related to the mathematical modeling preparation of teachers, and 2. generate hypotheses to be tested in future iterations. Our data are comprised of student-generated artifacts from the course (e.g., homework, projects, exams), notes from weekly planning meetings between the researchers/instructors, and an end-of-course survey. Seven of the nine students were undergraduate students in a Bachelor of Science (BS) program in Mathematics, Option in Mathematics Education. The remaining two students had already completed the BS program and were taking the course out of interest. All of the students had at least completed Linear Algebra and a first course in proof.

We are in the process of iteratively coding the data to expose patterns in student work (Coffey & Atkinson, 1996). This initial round of coding is guided by the components of the modeling cycle as defined in the GAIMME report, e.g., “make assumptions and define essential variables” (Garfunkle & Montgomery, 2016, p. 13). Within each of these components, subcodes are based largely on the knowledges and dispositions needed for doing and teaching mathematical modeling that Thompson (2011) and Doerr (2007) enumerated. For example, we are identifying challenges and patterns related to generalization and to types of pedagogical content knowledge. Subsequent rounds will lead to a refinement of the codes.

Preliminary Results

Given the preliminary nature of this report, we will briefly document some emergent themes, some initial results related to the students’ pedagogical and content knowledge of mathematical modeling, and some plans/hypotheses for the next iteration of the course. The end-of-course survey indicated that students found the course to be worthwhile. They reported that their knowledge of mathematical modeling increased and that they were excited to teach mathematical modeling. They also expressed comfort with the openness of the tasks they did in class. They reported that they intend to, as teachers, adapt textbook tasks to engage students in various aspects of the mathematical modeling process though they expressed somewhat less confidence in their ability to do so.

Our initial analysis and reflections have made us rethink our decision to begin the course with a discussion of the modeling cycle. Throughout the course, we asked students to reflect on the modeling process and connect it to their work. These reflections revealed that the process began to make sense only after substantial engagement with mathematical modeling. Moreover, there were instances in which students unproductively looked to the cycle for quasi-procedural guidance in the problem solving process. Even after successful completion of a modeling task students had trouble answering, "What is the model?" As a remedy for this discomfort, many students later communicated that they would have preferred to begin the course by watching the instructors demonstrate the mathematical modeling process. Though we are unlikely to honor that request in the next iteration, it may be a sign that the students have greater comfort with more traditional modes of teaching.

By delaying the explicit naming of the components in the modeling process, we can first begin to address some unproductive problem-solving dispositions of students. In particular, we found that we had to encourage the students to approach modeling tasks by first considering specific examples and exploring a "toy model." Without intervention, students often became mired in premature attempts to define appropriate variables and develop an abstract model. Furthermore, their work with abstraction often betrayed a lack of comfort connecting verbal and symbolic representations. In an early linear programming task adapted from a local school district's quarterly Algebra II exam, six of the nine students made errors with units while connecting an inequality to the problem's context. In general, most students experienced some level of difficulty with quantitative and covariational reasoning; ongoing analysis aims to characterize this with more granularity.

We also observed challenges with more pedagogically focused tasks. For example, students' attempts to modify textbook problems to create authentic mathematical modeling tasks often resulted in tasks that were not open enough or were imprecisely stated. As instructors, we sympathized with this as we also experienced the challenge of finding or producing appropriate tasks. Other pedagogical tasks posed challenges that were likely not exclusive to the context of modeling. Of note, on an exam we asked students to make sense of and recommunicate a hypothetical student's linear model for a scenario that our students had already modeled (geometrically) during an in-class activity. In our analysis of their work, we have not yet been able to parse out the sources of difficulty, whether they be related to the nature of their content knowledge or insufficient practice evaluating student work or something else.

We have thus far documented several challenges faced by students. We view these challenges as opportunities to improve the Mathematical Modeling for Teachers course and to frame an examination of students' experience throughout our teacher preparation program. Certainly, as instructors, we faced several challenges that stemmed from our lack of familiarity with and resources for teaching mathematical modeling to preservice teachers. But that is a subject for another paper.

Discussion

Though students were satisfied with the course and we generally viewed it as a success, students and instructors encountered significant challenges that extended beyond those reported above. The existence and persistence of these challenges may give credence to the call for greater integration of mathematical modeling throughout teacher preparation programs, not just in a single course. However, if we accept Thompson's (2011) view that mathematical modeling depends on quantitative and covariational reasoning, our preliminary analyses indicate that increased focus on these foundational abilities is merited and, from a pragmatic perspective, are perhaps more feasible to implement throughout the program. This is not to say that engagement in mathematical modeling cannot be done in service of developing those reasoning abilities (e.g., see Swan, Turner, Yoon, & Muller, 2007), but our experience illuminated that, even with two instructors for nine students, teaching mathematical modeling as envisioned in the GAIMME report and finding (or developing) good modeling tasks requires time and expertise. Furthermore, the challenges students faced related to generalizing (e.g., translating from verbal or specific scenarios to symbolic representations) and quantitative reasoning (e.g., working with units) may not be detected by assessments in more computationally-focused lower-division courses. Building foundational reasoning abilities and mathematical dispositions for mathematical modeling in those courses would yield impactful benefits throughout the students' undergraduate careers and their careers as teachers.

As we continue analysis and interpretation of data from the Mathematical Modeling for Teachers course, we are also planning the content for and research of the next iteration of the course. Input from the RUME community will provide valuable guidance. Audience discussion will be prompted, in part, by the following questions:

1. Given the breadth of our data, we could investigate it with various foci: the teacher educators, the students as mathematics majors, the students as future teachers, the curricular materials, and the teacher preparation program. How can we capture that in our analysis? Or how should we narrow the focus?
2. Widening the scope, what are the broader questions about the nature of mathematics teacher preparation across the curriculum? How can this work contribute to answering those questions?
3. What are particularly salient opportunities for research during the second iteration of the course?
4. What are the broader implications of this study for undergraduates who are not pre-service mathematics teachers or mathematics majors? What attainable goals should we set in designing college-level mathematical modeling courses or experiences at various levels?

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