Assessing Visual Literacy Competency in Undergraduate Mathematics

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We investigated how linear algebra students acquired mathematical knowledge from visualization objects, and to what extent these students exhibited visual literacy standards in higher education. Seven linear algebra students were the subjects of this research project. The data were collected through questions with high visual content and through semi-structured interviews. We analyzed the data by using descriptive and content analysis techniques. Our study found that linear algebra students were not sufficiently competent in using visualization techniques.

Keywords: visual literacy, visualization, linear algebra, assessment.

Introduction

Research into the teaching of undergraduate linear algebra confirms the advantages of using the visual approach when introducing mathematical content, and that visual representations of mathematical notions have a positive effect on students' learning (Hannah, Stewart & Thomas, 2013; Dorier & Sierpinska, 2001; Dubinsky, 1997; Harel, 1989). Guided by the Visual Literacy Competency Standards for Higher Education (ACRL, 2011), we have designed a framework for assessing students' visual literacy competency level in undergraduate mathematics and used this framework to indicate students' use of visualization objects in linear algebra.

The earliest attempt to define visual literacy was in Debes (1969, p.27; as cited in Avgerinou & Ericson, 1997, p.281). Following his definition, visual literacy will "…enable a visually literate person to discriminate and interpret the visible actions, objects, symbols … that he encounters in his environment." As Bieman (1984) noted, Debes's definition tells what a visually literate person can do, rather than what visual literacy is. Researchers in distinct fields have offered various definitions of visual literacy (Bristor & Drake, 1994; Braden, 1996; Burns, 2006). For example, Ausburn and Ausburn (1978) defined visual literacy as a group of skills that will enable an individual to understand and use visualization objects to communicate with others. Hortin (1980) defined visual literacy as the ability to understand and use images, and to think and learn in terms of images. We adopt the definition of visual literacy given by Stokes (2002) as the ability to interpret images, and to generate images for communicating ideas and concepts.

In 2011, the Association of College and Research Libraries (ACRL, 2011) published standards providing tools for educators seeking to measure visual literacy competency (VLC) of college and university students in undergraduate education. ACRL emphasized that standards outlining student learning outcomes have not been articulated in the research on visual literacy. They proposed the following standards:

The visually literate student should be able to:

1. Determine the nature and extent of the visual materials needed.
2. Find and access the needed images and visual media effectively and efficiently.
3. Interpret and analyze meanings of images and visual media.
4. Evaluate images and their sources.
5. Use images and visual media effectively.
6. Design and create meaningful images and visual media.
7. Understand many of the ethical, legal, social, and economic issues surrounding the creation and use of images and visual media, and access and use visual materials ethically.

In terms of assessing VLC, there are a limited number of instruments in existing literature (Avgerinou, 2007; Arslan & Zeren-Nalinci, 2014). However, we could not find assessment instruments for specific disciplines. We develop a framework based on standards for assessing VLC in undergraduate mathematics, with attention to linear algebra.

Adjusted Framework for Assessing VLC in Undergraduate Mathematics

We adjust the Visual Literacy Standards in Higher Education to undergraduate mathematics, to assess students' visual literacy competencies. The adjusted standards are as follows:

The visually literate students in undergraduate mathematics should be able to:

– perceive a given visualization object and recall prior knowledge related to a given visualization object.	PERCEPTION
– understand a given visualization object and make connections between prior knowledge and the given visualization object.	UNDERSTANDING
– analyze the properties of a given visualization object and interpret that given visualization object.	ANALYSIS and INTERPRETATION
– use a given visualization object.	USAGE
– create a meaningful visualization object.	CREATION
– evaluate a given or personally created visualization object.	EVALUATION

Each standard has sub-categories which can be used to assess students' VLCs. The order in which the standards are given should not indicate their significance. Being focused on the mathematical skills and procedures, we did not attend to ethical and social components addressed in ACRL (2011). For a working definition of a "visualization object" we embraced the definition of a visualization object as a physical object that is viewed and interpreted by a person for the purpose of understanding something other than the object itself. These objects can be drawings, pictures, 3D representations, animations, etc. (Philips, Norris & Macnab, 2010 p.26).

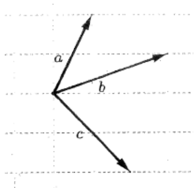
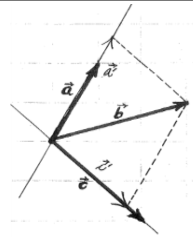
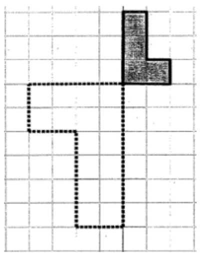
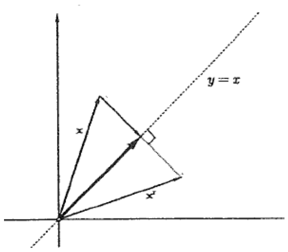
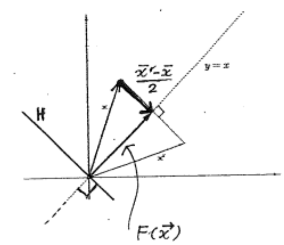
We specifically focused our research on the following adjusted standard: Use a given visualization object (usage standard). To gain insight into the extent linear algebra students' usage of visualization object, our main research questions are as follows:

- How do linear algebra students use a visualization object in the problem-solving process?
- To what extend do linear algebra students exhibit usage standard in the problem solving-process?

The Method

We adopt the qualitative-interpretative paradigm (Lodico, Spaulding & Voegtle, 2006, p. 264) applied to a holistic single case study (Yin, 2003, p. 39). Seven undergraduate linear algebra students were selected via the purposive sampling technique (Cohen, Manion and Morrison, 2007, p. 114). Collected data consisted of students' responses to three linear algebra questions with high visual content (Table 1). These questions were given to students at different occasions as test questions. Webb's (2009) Depth of Knowledge model was used to identify questions' complexity level; six mathematics professors assisted with classifying questions' complexity level. We also conducted five semi-structured interviews with student volunteers that were recorded and transcribed. Obtained data was analyzed and interpreted using percentage frequency distribution (Shapiro, 2008, p. 292) and content analysis techniques (Cohen, Manion and Morrison, 2007, p. 475).

Table 1. Questions and their complexity levels

Level	Questions	Possible Solutions
1- recall and reproduction	 Determine if the vectors given in the figure below are linearly independent or linearly dependent without computing. Please, use the figure to justify your answer.	 One possible answer is to construct a parallelogram in which $\vec{b}$ is a diagonal and $\vec{a}$ and $\vec{c}$ are on adjacent sides. Then (following the figure) $\overrightarrow{a'} = \alpha\vec{a}$ for some $\alpha > 1$ and $\overrightarrow{c'} = \gamma\vec{c}$ for some $0 < \gamma < 1$. We have $\vec{b} = \alpha\vec{a} + \gamma\vec{c}$.
2- skill and concepts	 Assume T: $\mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation that is a composition of two transformations: reflection with respect to the origin ($\boldsymbol{x} \mapsto -\boldsymbol{x}$), followed by scaling by factor 2. Write the standard matrix of the inverse transformation. Please justify!	Note that $T = S \circ R$ where $R(\vec{x}) = -\vec{x}$ and $S(\vec{x}) = 2\vec{x}$. So $R^{-1}(\vec{x}) = -\vec{x} = R(\vec{x})$ and $S^{-1}(\vec{x}) = \frac{1}{2}(\vec{x})$. Therefore $T^{-1}(\vec{x}) = (S \circ R)^{-1}(x) = R^{-1}\left(S^{-1}(\vec{x})\right) = R\left(\frac{1}{2}(\vec{x})\right) = -\frac{1}{2}\vec{x}$ and standard matrix of T^{-1} is $\begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}$
3- strategic thinking	 Assume that the mapping $F: \mathbb{R}^2 \mapsto \mathbb{R}^2$ maps each vector $\boldsymbol{x}$ into the vector $\boldsymbol{x} + \frac{\boldsymbol{x^r} - \boldsymbol{x}}{\boldsymbol{2}}$ where $\boldsymbol{x^r}$ is the reflection of vector $\boldsymbol{x}$ through the line $y = x$. Find the subset $H \subseteq \mathbb{R}^2$ such that $F(\boldsymbol{x}) = \boldsymbol{0}$ for every $\boldsymbol{x}$ in H. Sketch the subset H.	 Note that if $\vec{x} = \begin{bmatrix} a \\ b \end{bmatrix}$ then $\overrightarrow{x^r} = \begin{bmatrix} b \\ a \end{bmatrix}$ so $F\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} a \\ b \end{bmatrix} + \frac{1}{2}\left(\begin{bmatrix} b \\ a \end{bmatrix} - \begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} \frac{a+b}{2} \\ \frac{a+b}{2} \end{bmatrix}$ so F is a linear transformation. All vectors $\begin{bmatrix} t \\ -t \end{bmatrix}$ (t a real number) will be mapped into the zero vector or $H = \left\{\begin{bmatrix} t \\ -t \end{bmatrix} \middle\vert\, t \in \mathbb{R}\right\}$

The sub-categories for standard 'usage' are auxiliary drawing, algebraic interpretation and justification. The scores based on students' responses ranged from zero (0) to three (3) as shown in Table 2. In each of the three problems presented in Table 1, we expected that the student would use the given visualization object as a tool to advance his introspective visualization (Phillips, Norris and Macnab, 2010, p. 10). We also expected the student to produce an auxiliary drawing. We consider that would indicate higher level of VLC of the students. Students had opportunities in class to see how linear transformations transform particular sets of points in the plane, but we could not measure to what extend they have developed their intuition and benefitted from those opportunities in their solutions to Problem 3. Some of the difficulties students had with problem 2 could be result of inexperience with the notion of the inverse of a composition of bijections, which they have encountered in their previous courses.

Table 2. Scores and explanations

Score	Explanation of expected response in the standard of usage
3- excellent	Adequate and effective auxiliary drawing Accurate and relevant algebraic interpretation Valid and relevant justification
2- satisfactory	Adequate but ineffective auxiliary drawing Accurate but irrelevant algebraic interpretation Inappropriate justification
1- fair	Inadequate or ineffective auxiliary drawing Inaccurate or irrelevant algebraic interpretation Invalid justification
0	No-response

Preliminary Findings

In Figure 1, we present the results of three test questions in the percentage distribution bar chart. (Y-axis represents the percentage of students achieving the measured category)

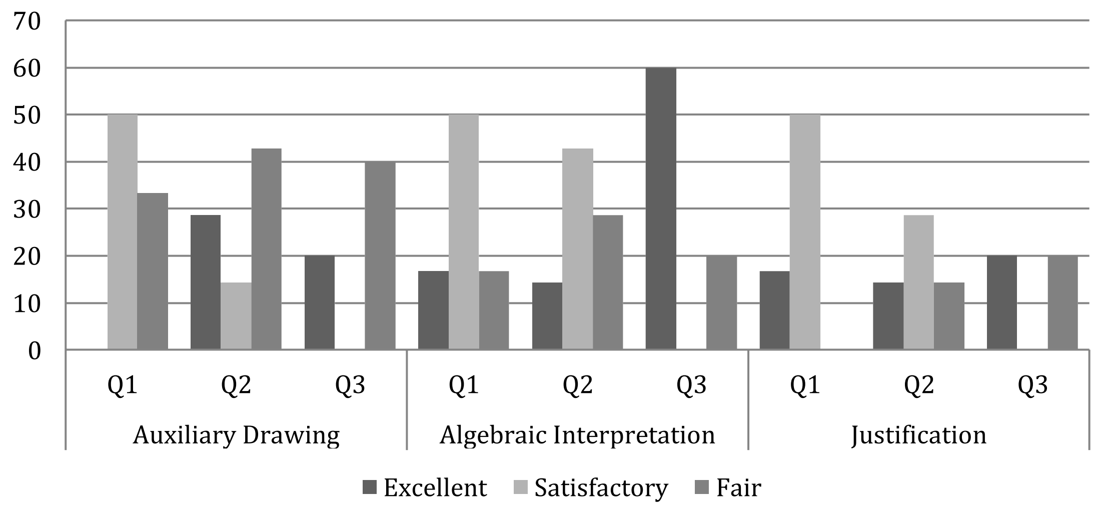


Figure 1. Using visualization objects

The bar chart (Figure 1), gives students' performance on these three questions and the three categories: effectiveness of their auxiliary drawing, the extent of their algebraic interpretation of a problem and appropriateness of their justifications. Notice that auxiliary drawing in all three problems is present, but it varies depending on the complexity of the question. One can also see a very mixed show of algebraic interpretation in all three questions. Students' ability to justify mathematical statements was weak, with good results only for question number one.

We will illustrate some of these conclusions with two examples of students' responses to question 3 and a question that was given as a part of the interview process.

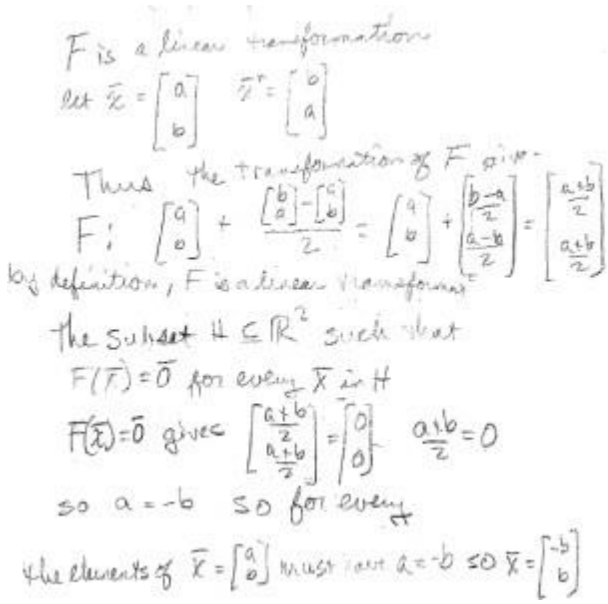


Figure 2a. An example response to the third visual linear algebra question

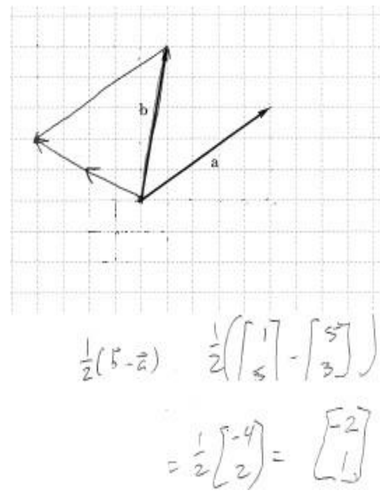


*Figure 2b. An example response to interview question: Sketch (**b-a**)/2*

The above two examples of student responses illustrate their primary learning strategy in solving the problems. Figure 2a illustrates a solution to problem 3 with correct response and conclusion to which student arrives in purely algebraic way, almost ignoring the image showing *F* as an orthogonal projection on the line *y=x.* The required sketch of *H* is obtained *after the* analytic solution to the problem was obtained. We observe a similar pattern in Figure 2b, where the student uses the unit grid as a coordinate system and assigns specific coordinates to given vectors. After computing the components of the vector (***a-b***)/2, the student sketched the answer. Again, the analytic reasoning precedes the visual way and illustrates the low 'usage' level of the student.

Conclusion

In this ongoing research, we proposed to use a new framework for assessing undergraduate mathematics students' visual literacy competency based on ACRL's (2011) standards and presented the findings of this usage standard. We found that students struggle to use given visualization objects in linear algebra. Students did not use auxiliary drawings very much, despite their usefulness, a phenomenon reported in Krajcevski and Keene (2017). We intend to continue developing the framework in our further research.

Intended Questions for the Audience

1. Are the components of the adjusted framework (perception, understanding, analysis and interpretation, usage, creation, and evaluation) sufficient to characterize visual literacy?
2. What sub-categories might be useful for assessing students' visual literacy?

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