# Examining Students' Problem Posing Through a Creativity Framework

Steven Silber
Virginia Tech

*Understanding how students pose problems can inform the development of posing activities to further enhance students' understanding of mathematics. Analyzing students' problem posing through the lens of mathematical creativity provides insight into the creative process of posing problems; namely, the cognitive tools students use to formulate questions. Three undergraduate students, enrolled in a developmental mathematics course, participated in a problem-posing intervention to examine the cognitive resources students used as the foundation for their mathematical problem posing. Session transcripts were analyzed using an analytical framework derived from an investment perspective on creativity, and identified resources were organized into two categories: mathematical knowledge and skills, and social interactions and experiences. Preliminary findings from the fifth session suggest that students associated the mathematical content of a posing task with previously encountered problems, as well as appealed to their familiarity with the situational context of the posing task.*

*Keywords:* Problem posing, Undergraduate Students, Developmental Math, Math Creativity

Problem posing is considered as the creation of mathematical problems, often from a given set of information or from previously presented problems (Christou, Mousoulides, Pittalis, Pitta-Pantazi, & Sriraman, 2005; English, 1997; Silver, 1994). Problem posing is a naturally-occurring activity in which individuals engage during their daily interactions. Kilpatrick (1987) notes that people encounter and recognize problems frequently, proceeding to solve those problems as they arise. Problem posing further occurs as part of the problem-solving process, acting as the foundation for developing a solution strategy (Brown & Walter, 2005) or as a form of reflection on and verification of solution strategies (Carlson & Bloom, 2005). In this capacity, individuals engage in problem posing to gain an improved understanding of the problem scenario in front of them. Understanding how individuals pose mathematical problems can inform the development of problem-posing activities to further enhance students' understanding of mathematics.

One way to examine individuals' problem posing is through the lens of mathematical creativity. Silver (1997) describes mathematical creativity as "closely related to deep flexible knowledge in content domains" (p. 75), viewing the connection between problem posing and problem solving as venue for mathematical creativity. Silver notes, "It is in the interplay of formulating, attempting to solve, reformulating, and eventually solving a problem that one sees creative activity" (p. 76). As students closely examine a mathematical situation, they can begin to generate hypotheses about the situation, develop flexibility in the ways in which they think about the situations, and begin to develop new ideas that expand upon their understanding of mathematics. In other words, students have an opportunity to use and develop the cognitive tools they have cultivated for doing mathematics. Viewing problem posing as an act of mathematical creativity, the descriptors of the creative process can be used to illustrate the creation of mathematical problems.

In this proposal, a framework based on the investment theory of creativity (Sternberg & Lubart, 1996) is used to describe undergraduate developmental mathematics students' problem posing. Under investment theory, individuals use a confluence of cognitive resources, such as content knowledge, thinking styles, and environmental influences, to "invest" in their ideas and develop them over time. These resources are tools individuals use as the foundation of their creative process. The guiding question to this inquiry is, "*How do students use cognitive resources to pose mathematical problems?*" Undergraduate students in developmental mathematics courses are an interesting population to observe; knowledge of mathematics is a mediating factor while posing problems (Kontorovich, Koichu, Leikin, & Berman, 2012; E. A. Silver, Mamona-Downs, Leung, & Kenney, 1996), and these students have been identified as underprepared for the expectations of college mathematics courses. As knowledge is a resource for creativity, one wonders to what extent these students use their knowledge as a resource for posing problems.

## Methods

### Participants & Study Design

Three undergraduate students enrolled in a developmental mathematics course at a mid-Atlantic public university participated in a five-week problem-posing intervention during the spring semester of 2016. The purpose of the intervention was to examine how students' problem posing evolved after learning about two problem-posing strategies described by Polya (2009): accepting the given, and "what-if-not". When accepting the given, students posed problems using only the numerical information and situational context provided in a posing task. When using "what-if-not", students were asked to pose problems by either changing the information they were given or adding new information to the scenario. The intent for the instruction on the two posing strategies was to encourage students to reflect on the given information in each task, using their understanding of the scenario as a resource for creating math problems. This proposal will focus on students' resource use during the final session of the intervention, to illustrate the variety of resources the students' used.

The final session of the intervention consisted of one posing task called "Payment Plan", shown in Figure 1. In this task, students were presented two payment options: one option where the payment increased by $1,000 each day, and a second option where the payment doubled each day. The posing scenario presents an opportunity for students to examine the two rates of growth and make comparisons between the two options. Students worked together to pose ten problems for the task as a group and were not required to use any specific posing strategy when creating their problems. After posing the ten problems, the primary investigator asked the students to describe their thinking behind the problems they posed. Students were not asked to solve the problems they posed. At the end of the session, students were asked to reflect on their experience and discuss their thoughts about posing problems. Recordings of conversations with the students were transcribed, and students written work was collected.

**Payment Plan**

You are given the choice to be paid in one of the following two ways:

1. You will be paid $1,000 the first day, $2,000 the second day, $3,000 the third day, $4,000 the fourth day, and so on for one month.
2. You will be paid $0.01 the first day, $0.02 the second day, $0.04 the third day, $0.08 the fourth day, and so on for one month.

(1) Work with your partner(s) to write ten mathematical word problems.

*Figure 1. Prompt for the Payment Plan task.*

## Data Analysis

The transcript of the session was partitioned into three episodes based on the session activity, and each episode was partitioned into several smaller events based upon what students were doing within the activity. Events were established around the topic of conversation, typically a student's explanation of a response or continued discussion around an idea. As a result, events were varied in length so that a more complete picture of each event could be achieved (Schoenfeld, 1985). Across the three episodes, there was a total of thirty-one events in the transcript. Using the categories of resources outlined by Sternberg and Lubart (1992) as a basis, students' actions, concepts mentioned by the students, and students' experiences were grouped into three types of resource types: task resources; mathematical knowledge and skills; and social interactions and experiences.

This proposal focuses on mathematical knowledge and skills, and social interactions and experiences. *Mathematical knowledge and skills* refers to students' mathematical thinking during the posing activities. This category relates to what students know about mathematics, such as students' understanding of concepts, association with previously encountered problems, use of mathematical terminology, as well as problem posing strategies. The design of the intervention focused on the use of the accepting the given and what-if-not posing strategies; therefore, it was expected that the students would use these strategies. *Social interactions and experiences* refer to students' personal experiences, non-mathematical knowledge, and interactions with other individuals. This resource category primarily relates to students' use of the situational context provided in a posing task, but also includes interactions between the students during the session, such as building from other students' thinking or seeking verification from other students.

# Students' Resources for Posing

Students exhibited use of both their mathematical knowledge and skills, and social interactions and experiences as resources for creating mathematical problems. To illustrate how students used these resources, an example of a resource type under each category will now be discussed. Under the mathematical knowledge and skills category, students associated the posing task with problems they previously encountered. Under social interactions and experiences, students related the situational context of the task to their personal experiences.

## Mathematical Knowledge – Problem Association

To engage with the mathematical content in the posing tasks, the students would relate the posing task to types problems they had previously encountered in the past. Brianna associated the Payment Plan task to comparison problems, posing the problem, "How long will you have to work for the second plan to equal the first plan?" She noted, "We've done problems like this before, where you have two rates of growth, and you compare them. So I was just curious at what point would they intersect?" Brianna recalled that with previous comparison problems, she would often be asked to identify the moment that two mathematical relationships would have the same value.

Students would also recall specific examples of problems they had encountered. Jason posed the problem, "Which would make you more money, a minimum wage job, or the second option?" (Jason later clarified that minimum wage stood for $7.50 an hour with an eight-hour work day.) When asked what motivated him to pose this problem, Jason recalled a previous experience with a teacher he had in high school:

> I looked at the problem, and I remembered that I had seen a similar math problem posed by one of my old teachers, as a…example to show what exponential growth was. Would you rather have a minimum wage job, or the one that starts off paying one cent and then doubles every day? I made it [the second option] because it's pretty obvious that the minimum wage is never going to beat the [first] job in terms of pay.

In recalling this past experience, Jason focused on the exponential relationship presented in the second option, explicitly naming the pattern as an example of exponential growth. It was a combination of his recognition of the exponential relationship and the similarity of the posing task to his prior experience that led him to associate the task with the previously encountered example.

**Social Experience – Familiarity with Context**

Appealing to the situational context of the posing task, the students framed their posing around their familiarity with the context. Brianna interpreted the two options as two jobs offering different pay. Brianna posed the problem, "If there was a 5% tax that was taken every day, how much would you have at the end of the month?", relating the scenario to a recent experience: "I was thinking of real life. Over spring break, I just worked, so that's what I was thinking about. A percentage is taken out every time. It's just a real example." Brianna took her recent employment experience as a resource for the problem she posed, introducing a type of income tax to the payment options.

Students could be familiar with situational context yet not have personal experience with that context. Kelsey had posed the problem, "If you take a sick day on the seventh day, what will your pay be at the end of the month for the first job?" Trying to describe how she created the problem, Kelsey questioned whether her response was realistic:

> *Kelsey:* Well, I just did a real-life problem… I just thought of another problem that could come from this. I don't know if the pattern would just continue and you would just lose that $7,000, or is it going to continue? I don't know. Do you get paid during the sick day?
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> *Interviewer*: Possibly. I guess it would depend upon the job. What do you think?
>
> *Brianna*: It depends upon the job.
>
> *Jason*: I think that for this problem, just the way it's worded, you're supposed to imply that you aren't paid for it.
>
> *Kelsey*: That's what I thought, but I didn't know if you would get paid or not, because I only volunteer. I don't work. When I was thinking about this [problem], I didn't think of the fact that you might not get paid.

Kelsey began to pose a problem by introducing the idea of sick days and how payment would be impacted by a sick day. Kelsey ultimately revealed that she had not had employment experience, which led her to be uncertain about how her problem would fit within the situational context. Although she did not have employment experience, she was familiar enough with the situational context to pose a problem with a context related to employment.

## Concluding Remarks

The preliminary findings suggest that students can use both their knowledge of mathematics and their social experiences as foundations for engaging in mathematical activity. Students could use this foundation to spring into further discussion around mathematical ideas. This could be especially valuable for students in developmental math courses, as it provides another venue for students to gain access to mathematical activities. A notable limitation of this study is that students were not asked to solve the problems they posed. Because students were only asked to pose problems, it is difficult to determine the extent to which students' resource use shaped their mathematical thinking while posing problems. Due to this limitation, feedback sought from the audience will focus on identifying the depth of students' engagement with the mathematical content while posing problems in the absence of students working towards a solution.

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