Gestures as Evidence of Assimilation When Learning Optimization

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Teachers and students often produce gestures during communication about mathematical concepts and processes. Our goal in this study was to determine whether students would produce gestures similar to those used by the teacher. Each of five students in a first semester calculus course was asked to solve two optimization problems based on a video lesson in which the teacher used primarily pointing, primarily depictive gestures, or no gestures at all. Though our data do not show the students' gestures directly imitating the teacher's, they provide support for the claim that frequent gesture use during communication may indicate assimilation of new concepts and that assimilation improves student performance on optimization tasks.

Keywords: gesture, calculus, optimization, assimilation, accommodation

Background

Optimization problems are frequently difficult for students in first semester calculus. These problems often require the drawing of figures, definition of several variables, coordination of multiple equations, algebraic substitutions, application of derivatives, and ultimately interpretation of the final results. LaRue and Engelke Infante (2015) studied student responses to optimization problems and determined that students have the most difficulty during the early, "set up" parts of the problem. This part of the problem solving process is referred to as the *orienting phase* (Carlson & Bloom, 2005). During this early phase of problem solving, the student "deciphers the problem and assembles the tools he or she thinks may be required" (LaRue & Engelke Infante, 2015, p. 2).

Deciphering the problem may evoke for the student certain *concept images*. The notion of *concept image*, as defined by Tall and Vinner (1981), is "the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes. It is built up over the years through experiences of all kinds, changing as the individual meets new stimuli and matures." For instance, one's concept image of the derivative might include things like a prototypical example curve, tangent lines, slope, rate of change, "prime" notation, and processes like the power rule, product rule, and chain rule. Note that the concept image is *dynamic*: it changes in response to new experiences. Prior to learning about optimization, students in calculus will almost certainly have learned about derivatives, maxima and minima (in relation to curve sketching), and the second derivative test for concavity. When optimization is introduced, however, it may become a part of the student's concept image for any or all of these concepts.

A student's concept image may incorporate gesture. Gestures are a naturally occurring part of communication; as such, teachers frequently gesture when teaching. For example, it is not uncommon for teachers to trace the shape of a parabola in the air, or to point to an equation written on the board for reference. Students may internalize these gestures as part of their concept image, and in turn, they may produce these or similar gestures during communication. It has been shown that thinking about an object or an event activates the same regions of the brain that become activated during the actual physical perception of those objects or events, and thus regions of the brain responsible for reacting to these stimuli are also activated (Hostetter & Alibali, 2008). Hostetter and Alibali's (2008) *Gesture as Simulated Action* framework posits that the activation of these regions of the brain in response to simulated (mental) actions will sometimes result in the realization of an overt movement: a gesture.

Studies suggest that students are more likely to produce gestures when communicating difficult information (McNeill, 1992; Radford, 2009; Roth, 2000). Roth (2000) specifically noted, “This and other research documents a high incidence of gestures when individuals deal with unfamiliar situations” (p. 1711). In light of the results of LaRue and Engelke Infante (2015), we expect that when solving an optimization problem, students might produce more gestures during the orienting phase of solving the problem. This study aimed to answer the following question: Do students mimic the teacher’s gestures when solving problems similar to what the teacher presented? While we did not see evidence of this, we did observe evidence that students were more likely to produce gestures if they are *assimilating* new information, rather than *accommodating* it.

Theoretical Perspective

We frame our research using Piaget’s (1985) notions of *assimilation* and *accommodation*. *Assimilation* is “the cognitive *process* by which the person integrates new perceptual matter or stimulus events into existing schemata or patterns of behavior” (Wadsworth, 1975, p. 15). During the learning process, an individual is said to have assimilated new knowledge when they have made cognitive connections between the new information and their pre-existing knowledge. However, assimilation may not be possible: the individual may not possess an existing schema into which the new information fits. Under this circumstance, accommodation may take place. *Accommodation* is “the creation of new schemata or the modification of old schemata” (Wadsworth, 1975, p. 16). Piaget posits that cognitive systems exist in a state of dynamic equilibrium involving both processes of assimilation and accommodation (Piaget, 1985).

Piaget’s concepts of assimilation and accommodation describe two ways in which learners attempt to reconcile new information with their pre-existing knowledge. This includes the incorporation of sensorimotor input like gestures (Piaget, 1985). As evidence of the assimilation of perceived gestures into existing schemata, we observe the repetition of these or similar gestures during communication. Here, we adopt Sfard’s (2001) communicational approach to cognition, which views thinking as a special case of communication, “as one’s communication with oneself” (p. 26). With this perspective, gestures that are realized during interpersonal communication, as well as those performed during individualized thought, are taken as evidence of assimilation.

Methods

The goal of our study was to determine how student understanding is affected by the instructor’s gesture use in the classroom. We prepared a lesson on optimization for a first semester calculus course, and three scripts were prepared: one in which the instructor used only *pointing* gestures, one in which the instructor used only *depictive* gestures, and one in which the instructor made no gestures. Using the definitions in Alibali et al. (2014), a *pointing gesture* is one which “indicate[s] objects or locations in the physical world,” and a *depictive gesture* is a simulated action or a conceptual action grounded in a physical action, such as simulating the action of collecting objects as a metaphor for the conceptual action of adding numbers. Apart from the differences in gestures, these three scripts were identical. One member of the research team was filmed presenting each script, and three videos were prepared. It should be noted that this lesson used the second derivative to confirm that the answer that was obtained was a maximum/minimum instead of the first derivative.

Interview subjects were assigned one of the above videos to watch based upon the order in which they arrived for interviews. Students were permitted to take notes while watching their video. Immediately after watching the video, students were asked to solve two optimization problems:

Problem 1: If the perimeter of a rectangle must be 84 inches, what are the dimensions of the rectangle that has the largest possible area?

Problem 2: A company wishes to manufacture a rectangular box with an open top whose base length is twice as long as its base width. If the box must contain a volume of 32 ft^3, what are the dimensions of the box that will minimize its surface area?

Students were encouraged to speak aloud as they worked so as to ascertain why they took the steps they did to solve the problem. Interviewers prompted the students when they were quiet for long periods of time and after they had completed certain steps in their solutions. Interviews were filmed to capture students' thoughts and gestures during this process. Students were compensated for their time with a $10 gift card.

There were a total of five interview subjects who were assigned pseudonyms: Ben, Andrew, Eric, Lisa, and Mary. Ben and Lisa watched the "Pointing" video, Andrew and Mary watched the "Depictive" video, and Eric watched the "No Gesture" video. All five students were enrolled in first semester calculus at the time of their interviews. Students Ben and Mary reported having taken a first semester calculus course in the past, while Andrew and Lisa reported that they had not. Ben and Eric self-reported that they were international students.

We employed a thematic approach to the data set (Braun & Clarke, 2006). Each video was watched several times by each member of the research team who made notes about the students' problem solving activity, paying particular attention to the gestures being made. From these notes, it became evident that two of the participants were actively seeking to make connections between the new information that had been presented to them and their existing knowledge of functions and calculus. Hence, complete transcripts (all speech and gesture production) for Lisa and Mary were made and further analyzed to examine how they were assimilating the new ideas.

Data

Ben, Andrew, and Eric all displayed superficial understandings of the lesson presented in the videos. Evidence of accommodation was present in the form of utterances referring to the instructor's words in the videos, but little evidence of assimilation was demonstrated by any of these three subjects. Most of the actions performed by these subjects during their solution attempts were simply appealing to memorized rules they had learned either from the video or from some other source; little evidence of true understanding manifested. For these reasons, we focus on the results of interviewing Lisa and Mary, which we present here as case studies.

Lisa

Lisa began the explanation of her solution to Problem 1 with several pointing gestures referring to her written perimeter and area formulas. When asked to explain how she knew she had the correct formulas for area and perimeter, she initially stated that "Teachers have beat those into my brain," and "That's just what I've always been told." However, when asked if these formulas have meaning for her, Lisa immediately explained that the perimeter is the sum of the lengths of the sides of the rectangle she had drawn, and that the area was the product of the side lengths, pointing to the relevant sides of her figure as she spoke about them. She elaborated that she thinks of the area as "tiny squares everywhere, so how many squares on this side [points to one side of rectangle] times how many squares on this side [points to a perpendicular side] will give you how many squares in total [mimes shading in the figure]."

To determine the length needed to obtain the maximum area, Lisa found the derivative of her area function and set it equal to zero. When asked why she chose to do that, Lisa explained that, on the graph of the area function, this would be where the graph changed from increasing to decreasing. While explaining this, Lisa traced a "concave down" shape in the air. While continuing her explanation, Lisa also drew a rough sketch of the curve she pictured in her mind, then also quickly sketched the graph of the derivative of this curve to explain that a maximum would occur when the values of the derivative changed from positive to negative.

Before beginning Problem 2, Lisa indicated that she did not know the formula for the surface area of the shape in question. However, she began to think aloud as she reasoned through what the formula should be. She vocalized that the figure had five faces, and initially suggested "5 times length times width." At the interviewer's prompt, Lisa drew and labeled a picture of the figure. She stated that surface area is "kinda the area-perimeter of everything on the outside… all the material on the outside." When asked to elaborate on what she meant by the term *area-perimeter*, she explained, "It's kinda both in a way. 'Cause it's all around [moves her hand in a circle around an imaginary object] the object, but it's the area of each face as well [mimes touching the five faces of the box by holding her hands in parallel then rotating them 90 degrees to indicate the next pair of parallel sides]." She was then able to determine the area formula for each face, pointing to the appropriate faces on her figure as she did so, and add them together to obtain the surface area formula for her figure.

Continuing, Lisa used the volume formula and the constraint that the base length is twice the base width to rewrite her surface area function in terms of one variable. After staring quietly at her formula for about 30 seconds, Lisa said, "Well, I'm thinking about taking the derivative of this, but I don't want to." She explained that she didn't like the quotient rule, but then she acknowledged that she could avoid using it by rewriting her formula using negative exponents, and she then differentiated her function:

Interviewer: And why do we take the derivative?
Lisa: So that we can find the critical point, which will be our, hopefully our minimum. I mean, it'll likely be concave up, but…
Interviewer: OK, and so, you're hoping that it's concave up. Why are you hoping that it's concave up?
Lisa: If it's concave up, then it will be like a U [puts her thumbs together and extends her index fingers into a U-shape], and then it'll have a minimum value [puts a fist at the bottom of the U-shape and points at it with her other hand] at the critical point where the slope is zero.

She then set her derivative equal to zero and found the critical number to be the cube root of 24. Initially, she wrote "$\pm\sqrt[3]{24}$," but she decided that her answer must be positive because it defines a width. After another moment, however, she concluded that her solution must be positive, as 24 is positive, and "negative times negative times negative would be a negative number."

When asked how she knew that the dimensions she obtained would minimize the surface area, Lisa answered, "'Cause I'm gonna take the second derivative, and if it's a positive number then I'll know it's concave up." She talked briefly about testing the function for points of inflection, but then decided against it. She then spoke about substituting a value into the second derivative, but she was unsure what value to use. She concluded that she could choose any number in the domain, so she chose to substitute $w=1$ into the second derivative. Lisa seemed unconvinced by her result, so she then used the first derivative test to confirm that the graph of her surface area function was concave up at the critical number. With this information in mind, she then returned to the second derivative and substituted $w=3$ to again confirm that the graph was concave up, as she expected it to be.

Throughout her interview, Lisa utilized a variety of gestures to express various notions relating to work she had done on her paper. When explaining written parts of her work, she used pointing to refer to the relevant portions of her work; when describing more general concepts like slope or maximum and minimum, she often used depictive gestures, as exemplified in the following excerpt:

> *Lisa*: The derivative will be zero when L is 21.
> *Interviewer*: OK, so why, why do we care about that?
> *Lisa*: Um, because that is, uh, that'll be the critical point. So that'll be [traces a concave down arc in the air] when it's changing directions from, um, from increasing to decreasing on the graph, and we care about that because we want the maximum area.

In addition, she frequently referenced the notes she had taken while watching the video any time she felt unsure as to how to proceed.

Mary

When Mary began working through Problem 1, she generated the correct formulas for area and perimeter. When asked how she knew that her formulas were correct, she replied in similar fashion to Lisa, initially citing memorization but elaborating a clear conceptual understanding via similar gestures to those used by Lisa.

Throughout solving Problem 1, Mary referred to her notes on the video to confirm her procedure as she worked. Mary expressed frankly that she was unsure what the significance of the second derivative was in solving these problems. Despite this, Mary worked quickly through most of Problem 1, and used her notes on the second derivative to confirm that her solution would yield the maximum area for the rectangle. Like Lisa, Mary utilized a combination of pointing and depictive gestures.

Mary began Problem 2 by sketching a box with an open top and labeling its dimensions with l, w, and h. Similar to the other interview subjects, Mary said "I don't even know the surface area of a cube to be perfectly honest." However, she knew that the surface area represents "the area of, like, all of the outside… let's say rectangles, added together [uses her hands to depict the parallel pairs of faces of a box]." After a brief conversation about this idea, Mary was able to determine the correct formula for the surface area of her open-topped box. She then proceeded to solve Problem 2 using the same method she employed to solve Problem 1. She continued to express doubt about her use of the second derivative, but she followed the rules stated in the video.

Prior to stating that "If it is a max or a min, then the derivative has to be zero," Mary said that if the derivative is equal to zero, "it either has to be a max or a min… well – no, not necessarily." Following the exchange in the previous paragraph, the interviewer returned to this comment to ask Mary how she convinced herself that this original statement was false. She answered, "I was just, um… Because, like, in cubic functions, [sketches a graph similar to that of $y=x^3$] um, you can have a point where the derivative right here [draws a point at the point of inflection] would equal zero, but it's not necessarily an absolute max or a min."

At the conclusion of the interview, Mary asked if it was necessary to test the endpoints in addition to the critical number by substituting them into the original surface area equation, apparently thinking about the test for absolute extrema on a closed interval (the domains for both Problems 1 and 2 are open intervals). However, she correctly identified that, if she were to do this, the input value which yielded the smallest surface area would give the location of the function's minimum.

Discussion and Conclusions

In response to our research question, we did not observe that students consistently mimicked the instructor's gestures when solving similar problems. However, we argue that assimilation of new information increased the frequency of gesture production and increased subjects' degrees of success in solving the problems in this study. In the data, it is clear that Lisa and Mary performed a significant number of gestures, while Ben, Andrew, and Eric did not. We now discuss evidence of Lisa and Mary's assimilation and its correlation to their success on Problems 1 and 2.

The new information presented in the videos in this study is twofold: the context of the problems (optimization), and the use of the second derivative in the determination of extrema. Data collected from interviews with Ben, Andrew, and Eric are minimally discussed here, as we observed little evidence of assimilation of this new information. These subjects showed some evidence of accommodation. For Ben, Andrew, and Eric, the use of the second derivative in this way appeared to be detached from their prior knowledge of calculus. Rather than assimilating this knowledge, they appear to have simply accommodated it by adding it to a collection of disconnected procedures. For example, Eric initially claimed that both the first and second derivative tests were necessary to confirm the location of the maximum in Problem 1. However, he concluded that the second derivative test alone was sufficient, as the instructor in the video had said this, and because "The second derivative is negative two; negative number is concave down… uh, concave down. The first derivative is equals to a positive number, so that's why we got, like, a maximum value." Eric's response is typical of these three students. While often incorrect, each of these students used what they believed to be appropriate rules in an algorithmic manner with no evidence of attempting to make connections between concepts. These three subjects were largely unsuccessful in their solutions of both Problems 1 and 2. Correspondingly, Ben, Andrew, and Eric gestured minimally when discussing their solutions.

Lisa and Mary both demonstrated significant evidence of assimilation, and we observed significantly more gesture from these subjects, so we focus on the results of their interviews. First, we note that Lisa and Mary were the only two subjects who were able to articulate a conceptual understanding of the perimeter and the area formulas. From their oral descriptions, one has the sense that both of them have a clear concept image of perimeter and area, at least for rectangles. This knowledge assisted them in determining the formula for the surface area of a box. By using gesture to help them visualize the sides of the box in a manner similar to perimeter, both were able to construct an appropriate formula.

Furthermore, both subjects demonstrated a rich concept image of the first derivative as it relates to maxima and minima. Lisa's explanation in Problem 1 for setting the first derivative equal to zero rested on the idea that the function should be increasing to the left and decreasing to the right of this point, and her gesture of drawing a concave down arc in the air is further evidence of her understanding. Moreover, without prompting, she was able to quickly sketch an example curve and its first derivative to support her claim that the derivative should be equal to zero. Mary's concept image of the first derivative as it relates to maxima and minima contains counterexamples to erroneous claims. Solving Problem 2, when trying to explain why she set the first derivative equal to zero, she said that if the first derivative is equal to zero, then "it either has to be a max or a min…" but quickly corrected herself, as she appeared to have internally convinced herself that this statement was false. When probed about this later, Mary was able to provide the example $y=x^3$, a function which she explained contains a point where the first derivative is equal to zero but that point is not an extremum of the function. In these and other examples in the data, we see evidence of very detailed concept images of the first derivative.

The data suggests that Lisa readily assimilated the new information about the second derivative into her existing schema for finding maxima and minima. In solving Problem 2, Lisa expressed not only an intention to find the second derivative of her surface area function, but also her expectation that her calculation should yield a positive result: "If it's concave up, then it will be like a U [puts her thumbs together and extends her index fingers into a U-shape], and then it'll have a minimum value [puts a fist at the bottom of the U-shape and points at it with her other hand] at the critical point where the slope is zero."

Despite Mary's expressed lack of confidence in the use of the second derivative to solve these problems, she tried to use this method. When she did so, her comments reflected an internal struggle in which she sought to reconcile this new knowledge with her existing schema for extrema: "this is… where she took the second derivative, which I didn't really understand the purpose, but… [writing] So A double-prime of l is 2… which means that it's going to be a min… at 2. Um… [looking at her work] … we're trying to maximize the area, yeah, I don't know. This is where I get a little confused," and later, "Is it because of taking the second derivative and getting that max that you know that those are the dimensions that give you the largest possible area?" Though it doesn't appear that Mary had fully assimilated this use of the second derivative during her interview, there is evidence to suggest that she was making a concerted effort to do so.

Ben, Andrew, and Eric showed little evidence of assimilation; rather, we observe only the most basic accommodation. They appear to remember snippets from the videos they watched, but none of them appears to have a complete picture. Of these three students, only one of them obtained the correct solution to Problem 1 via a logically valid procedure, and none of these students obtained a correct solution for Problem 2. None of these students attempted to justify their solution without being prompted to do so, and none of them provided an accurate explanation for how to do so. Lisa and Mary both demonstrated evidence of at least an attempt at assimilation, if not success. Not only were these two the only subjects to obtain complete solutions to both Problems 1 and 2, but they were also the only subjects to attempt to justify their solutions without prompting. They were the only subjects to use logically sound reasoning about the second derivative in their justifications.

The results of this study point to frequent gesture use as a potential indicator of assimilation of knowledge. Future research might investigate: Does student gesture use facilitate assimilation, or might it simply indicate that assimilation has occurred? More research needs to be done to better understand the role gesture plays in assimilation of new concepts.

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