The Next Time Around: Shifts in Argumentation in Initial and Subsequent Implementations of Inquiry-Oriented Instructional Materials

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Considerable learning is entailed in adopting an inquiry-oriented approach to teaching a class. In this analysis, we examine classroom video data of three instructors' initial implementation of an inquiry-oriented instructional unit and their implementation of the same unit one year later. We document consistent increases in instances of eliciting and building on student contributions across tasks and instructors, and use Toulmin's argumentation scheme to offer an illustration of how classroom discussions became more mathematically robust and student-centered from initial to subsequent implementations. Implications for instructor learning are discussed.

Key words: inquiry-oriented instruction, instructor learning, instructional practice

Enrollments in science, technology, engineering, and mathematics (STEM) programs in the United States must grow to meet projected workforce demands in coming years (PCAST, 2012). Following a growing body of research documenting the positive outcomes related to student-centered approaches to instruction in undergraduate STEM (e.g. Freeman et al., 2014), there is increased institutional and financial support for initiatives that promote this kind of teaching. Student-centered approaches range widely, from approaches that provide opportunities for students to practice things demonstrated by their instructor with groups of peers during class time, to inquiry-oriented approaches that aim to provide students with opportunities to participate in the reinvention of important mathematical ideas by working with peers to solve non-standard problems with many possible solution paths. Inquiry-oriented approaches are instructionally complex in that as students are inquiring into the mathematics, instructors inquire into students' mathematical thinking so it can be leveraged as a resource for moving forward the development of the class's mathematics (Kwon & Rasmussen, 2007).

The difficulties experienced by instructors attempting to implement research-based, inquiry-oriented instructional materials developed by others have been documented to include struggles in making sense of and building on student reasoning (Johnson & Larsen, 2012; Speer & Wagner, 2009; Wagner, Speer, & Rossa, 2007). While these findings suggest it is challenging to teach in an inquiry-oriented way for the first time, there is little work at the undergraduate level that examines what one learns as a result of teaching in this way. In this analysis, we draw on video data of three instructors' initial implementation of an inquiry-oriented instructional unit in linear algebra and their implementation of that same unit one year later. The research question is: How does instructors' facilitation of whole-class discussions shift from initial to subsequent implementations of inquiry-oriented instructional materials?

Literature Review & Theoretical Framework

To conceptualize the kind of knowledge needed for teaching mathematics, Hill, Ball, and Schilling (2008) developed a model of mathematical knowledge for teaching (MKT) which is split into two major domains: subject matter knowledge (SMK) and pedagogical content knowledge (PCK). This distinction builds on Shulman's (1986) argument that there is a distinction between knowledge of mathematics and the specific knowledge about mathematics that is needed to teach it effectively. In this work, we are particularly interested in how PCK might develop as a result of implementing inquiry-oriented instructional materials so we focus on that part of Hill and colleagues' framework. Hill et al. (2008) divide PCK into three subdomains. First, Knowledge of Content and Students (KCS) refers to the knowledge a teacher has about their students' prior knowledge of specific content and how student learn that content. Second, Knowledge of Content and Teaching (KCT) has to do with instructional decisions that require "coordination between the mathematics at stake and the instructional options and purposes at play" (Ball, Thames, & Phelps, 2008, p. 401). Third, Knowledge of Curriculum (KOC) refers to what teachers know about how ideas build in the context of a particular set of curricular materials.

At the elementary level, Remillard (2000) found that instructors can learn from curricular materials when materials focus on mathematical problem solving and analysis of student reasoning. Sherin (2002) conducted an analysis of two high school algebra teachers' work with a reform-oriented unit on linear functions, noting three kinds of learning: their subject matter knowledge, their views of curricular materials, and their ideas about student reasoning. However, little work at the undergraduate level has explored what teachers learn by engaging in inquiry-oriented approaches to instruction. Inquiry-oriented approaches require careful attention to student reasoning as well as skill at facilitating classroom discussion and argumentation, so we seek to document teacher learning by examining shifts in instructional practice as evidenced by shifts in classroom mathematical argumentation.

We draw on Toulmin's (2003) model of argumentation that was originally used to examine how individuals supported claims in front of an audience. Krummheuer (1995) extended the model to examine mathematical argumentation in classroom discussions that involved input from multiple individuals. Others have used this framework as a tool of analysis for both individual mathematical argumentation (e.g. Wawro, 2015), and collective mathematical discussion (Ramussen, Wawro, & Zandhieh, 2014; Conner, Singletary, Smith, Wagner, & Francisco, 2014). In this study, we draw on four core components of Toulmin's argumentation model: claims, data, warrants, and backings. Wawro describes a claim as "the conclusion that is being justified" whereas data is the evidence that supports the claim (Wawro, 2014, p. 320). Warrants are "statements that connect data with claims" (Conner et al., 2014, p. 404) or alternatively "clarification [statements] that connects the data to the claim" (Rasmussen et al., 2015, p. 263). On occasions when a backing is given, it can be defined as the support that gives a warrant authority (Rasmussen et al., 2015). Claims and data can also be a pair; once a claim has been established in a discussion, it can be used as data for a subsequent argument (Conner et al., 2014).

Study Design (Study Context, Participants, Data Sources, Methods of Analysis)

This work is part of a broader NSF-funded project aimed at developing shareable, research-based resources for instructors interested in using teaching inquiry-oriented linear algebra. The primary data used in this analysis consists of video-recordings of three instructors who implemented a 4-6 day instructional unit on span and linear independence two years in a row. The instructional approach is detailed in Wawro, Rasmussen, Zandieh, Sweeney, & Larson (2012). The instructional sequence consists of four tasks, starting in a context where students are given two "modes" of transportation (a magic carpet and a hoverboard that can travel in particular directions that are represented as vectors) and start at home (the origin). Students work to figure out if they can reach a particular location (to introduce ideas related to linear combinations of vectors), determine if there is anywhere they can't reach (to introduce span), and explore when they can take "non-trivial" journeys that start and end at home (to introduce linear in/dependence). In the final task, which we examine in greater detail, students work to generate examples of linearly dependent and independent sets of specified numbers of vectors in R^2 and R^3, as well as generalizations that emerged from their efforts to generate these examples.

The instructors participating in this study were 'best case' implementers in many ways: they all expressed interest in implementing the materials and all were situated in departments that were supportive of these efforts. The instructors represent a variety of institutional contexts including two small, private teaching-focused institutions (one religious, one not) and one large research institution. One instructor was a mathematician interested in RUME research, one was a RUME researcher, and one was teaching faculty in a mathematics department at a large research-focused institution. Class sizes ranged from 8-35 students.

Our analysis had four phases: content logging, development of codes for whole class discussions, coding of whole class discussions, and a comparative case study. We began distilling the data by generating a content log for each day of classroom instruction that was video recorded as part of the instructional unit for each instructor for both the first and second year of implementation. The content log was organized in four columns: time stamp and discourse structure (whole group, small group), key events, new language or notation introduced, and other notes. A new row was created when there was a change of discourse structure or shift in topic.

In our second phase of analysis, we drew on data from the analysis of the first year's data to identify four levels of eliciting and building on student ideas: 1. Getting students to talk, 2. Getting students to explain, 3. Using student ideas to explain or formalize, and 4. Using student ideas as the basis for a new mathematical question or task. In our third phase of analysis, two coders separately coded each whole class discussion according to the highest level of eliciting and building on student ideas that was observed. There was difficulty coming to agreement about when one whole class discussion ended and another began based on the criteria of "topical shifts," but by aggregating all scores to the "maximum" score observed in all whole class discussions that took place after students worked on each task in the instructional sequence, agreement was reached.

We noted that in the second year implementation, whole class discussions felt "smoother" in that they seemed to have a clearer mathematical direction, but this distinction was difficult to operationalize. In order to better understand shifts in discussion from the first to the second implementation, we decided to closely examine the mathematics that emerged in whole class discussions following the final task in the sequence in the first and second year. We selected mathematically analogous ten-minute segments from the same instructor, transcribed them, and used Toulmin's (2003) argumentation model to examine the mathematical arguments that emerged, who contributed what to these arguments, and the role of the instructor in the construction of these arguments.

For our final phase of analysis, we transcribed the selected segments, noted important gestures, such as writing on the board or pointing to work, and identified the role group of the speaker (teacher or student). We analyzed the transcripts to identify core claims being argued and from there identified the data and warrants that supported these claims. When they occurred, backings were also noted. Argument components were numbered according to the order in which they occurred in the transcript. Once we had identified and numbered the components of argumentation, we constructed diagrams of the mathematical discussion. We adapted Conner et al.'s (2014) convention of using solid and dotted line in these diagrams to distinguish the primary contributor of the statement. The Toulmin mappings allowed us to count the types of contributions by role to quantify shifts in argumentation relative to who made contributions.

Findings

To show how instructors' facilitation of whole class discussions shifted from their initial implementation of inquiry-oriented instructional materials, we leverage our coding scheme for eliciting and building on student reasoning to show consistent, quantifiable growth. Table 1 shows the maximum score each instructor received in all whole class discussion after students had worked in small groups on each of task 1 through 4 in year 1 and year 2. Every instructor elicited and built on student reasoning in whole class discussion as much or more in the second year's implementation. However, this does not capture the way in which year 2 discussions seemed to generally hold a clearer sense of direction and seem more mathematically rich while also building on student ideas. To explore this, we examine two mathematically similar ten-minute segments of discussion for instructor A in year 1 and year 2 through a Toulmin analysis.

Table 1: Instructors' Eliciting and Building on Student Reasoning

	Year 1					Year 2					Year 1 to 2
Task:	**1**	**2**	**3**	**4**	**mean**	**1**	**2**	**3**	**4**	**mean**	**Mean change**
Instr A	3	4	2	3	3	3	4	3	4	3.5	+0.5
Instr B	3	3	3	2	2.75	3	3	4	4	3.5	+0.75
Instr C	2	2	3	2	2.25	4	3	3	3	3.25	+1.0
Mean	**2.67**	**3**	**2.67**	**2.33**	**2.67**	**3.33**	**3.33**	**3.33**	**3.67**	**3.42**	**+0.75**

Toulmin Analysis of Selected Exchange: Year 1

Figure 1 shows a Toulmin mapping of a whole group discussion in instructor A's class in year 1 related to the claim that any three vectors in R^2 must form a linearly dependent set. The class had already concluded (in previous discussion) that two vectors in R^2 are linearly dependent when they are scalar multiples of one another or point in the same direction (note the zero vector case had not been teased out). The instructor then pointed out one group's example of a linearly dependent set of three vectors in R^2 (marked as Claim 1). Upon the teacher's request, students offered somewhat vague data (2) that alluded to scaling vectors to get a non-trivial solution (without specifying *what* equation would have such a non-trivial solution), supported by the warrant (3) that this was possible because the three vectors were not parallel.

The instructor built on students' ideas by writing the equation $a\begin{bmatrix}1\\2\end{bmatrix}+b\begin{bmatrix}3\\4\end{bmatrix}=\begin{bmatrix}5\\6\end{bmatrix}$ on the board and asking "how are you sure of that without finding the coefficients?" (Data 4). When the students seemed unsure of how to answer, the instructor asked, "What if we just had these two? [*pointing to two of the three vectors in the original set of vectors*] Is this an independent set?" Students agreed with a choral response (Warrant 5). The instructor continued, "they're not multiples of each other, easy to check with two vectors, throw the third in, now I might be able to write a linear combination of two that gets me the third, that's our triangle… um pretend this is your magic carpet and your hover board cause I claim this is no different from that situation. They look different directions, they're not in the same line, they're an independent set. Where can you get on your magic carpet and hover board?" Students chorally responded, "Everywhere"

(Data/Claim 6 and Warrant 5). The instructor then rearranged the equation in Data (4) to point out that the homogeneous vector equation corresponding to the set of vectors proposed by the students has a non-trivial solution, so that the set is linearly dependent by definition (Warrant 7 and Backing 8).

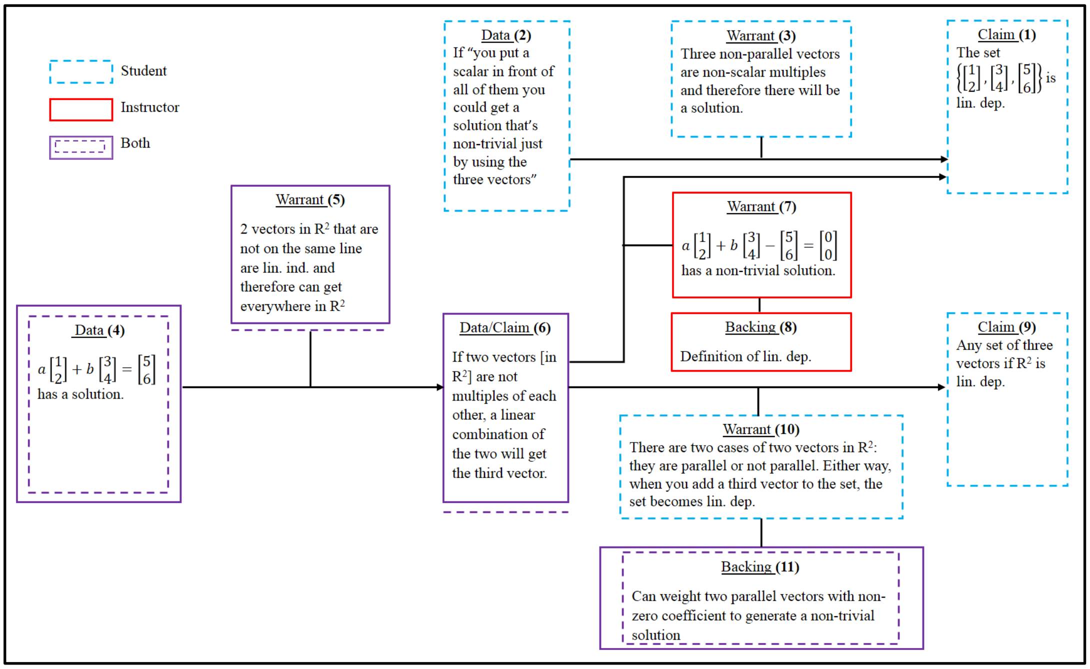

Figure 1: Year 1 Toulmin Mapping of Selected Episode for Instructor A

The instructor then asked, "Say I didn't take (1,2) and (3,4) say I take any two vectors that didn't lie on the same line, I throw a vector into that set. What's true about it?" A student suggested, "You can get back to the start… because you can get to that point using those two non-parallel vectors." After the instructor rephrased this idea in terms of span, a student observed, "We just showed that if we have the two [vectors] that are not parallel and then a third one, we can get anywhere. So if we have two that are parallel, then we can just go out one and come back the other. So either they're parallel or not parallel; there's no third case. So in every case they're going to be dependent" (Claim 9 and Warrant 10). The instructor then acknowledged that she set students up to make this observation, and a student asked, "If we have two parallel vectors and the third one, are we allowed to throw a zero on that third one?" The instructor confirmed that that is still considered a non-trivial solution and offered an example (backing 11) linking this case to the definition.

Overall, the two core claims in this exchange (1 and 9) came from students, though the instructor offered significant support that build toward the formulation of claim 9. Specifically, an incomplete data and warrant (2 and 3) were initially offered by students. The instructor added information to these by formulating them as an equation (data 4). She then built those into a new claim (6) and warrant (5), with students contributing to these in the form of choral responses. The instructor then used claim 6 as data to support claim 1, also providing the warrant (7) and backing (8). As such, we argue that the instructor provided the majority of the justification for claim 1 that built on student ideas while making clear efforts to involve students in the development of that justification. On the other hand, claim 9 and warrant 10 were fully articulated by a student after an initial question from the instructor that the instructor later explicitly acknowledged was intended to lead students in this direction.

Toulmin Analysis of Selected Exchange: Year 2

In year 2, the conclusion that three vectors in R^2 cannot form a linearly independent set was arrived at rather differently (see Figure 2). The class had previously established, as suggested by a student, that two vectors in R^2 are linearly dependent when "they are scalar multiples of one another." The discussion began with one group's (incorrect) claim (1) that the set of vectors $\left\{\begin{bmatrix}1\\2\end{bmatrix},\begin{bmatrix}7\\2\end{bmatrix},\begin{bmatrix}8\\5\end{bmatrix}\right\}$ is linearly independent[1]. The instructor did not correct the students but asked them how they checked they were independent. One student from the group explained, "you can't add vector one and vector two to create…vector three" (marked as Data 2). The instructor then offered what we refer to as an "empathetic" warrant (3): "Because you can't generate uh, by sort of observation, a dependence relation between those, they're independent." Another student then (correctly) noted that any pair of vectors from this set "can reach anywhere in the plane…" (Claim 4) because they are linearly independent (Data 5). The instructor provided the warrant (6) that any two of these vectors are not multiples of each other and "point in different directions". The student continued, noting that you "can reach eight, five" (Warrant 7), so the set is linearly dependent (Claim 8).

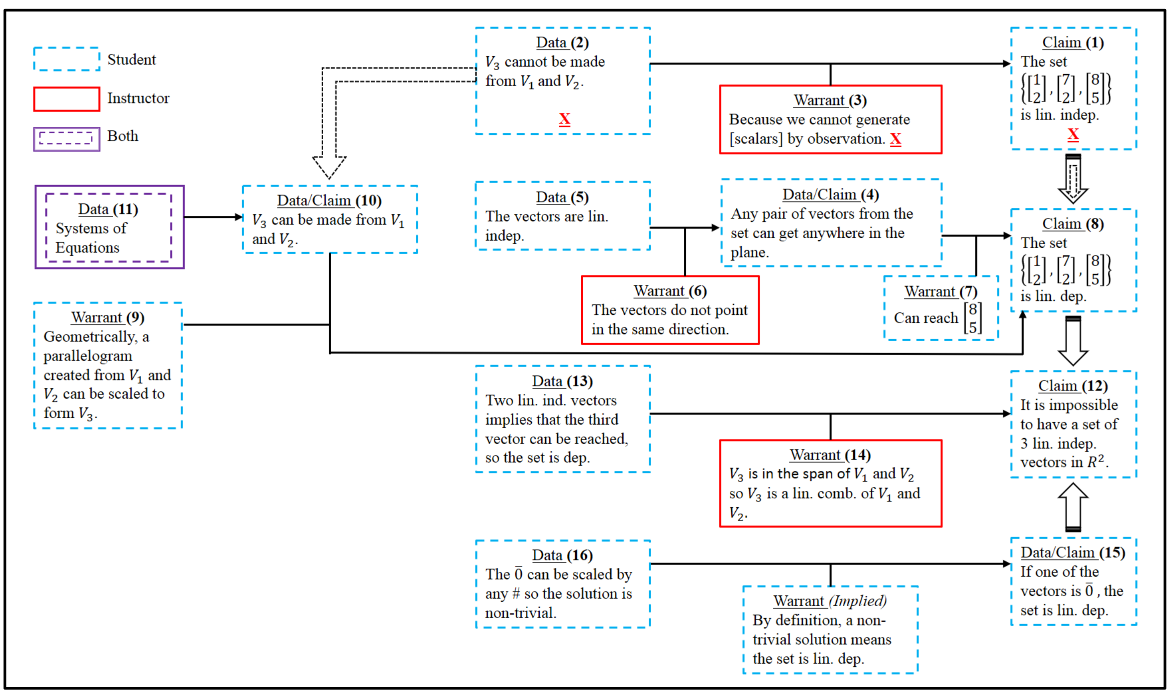

Figure 2: Year 2 Toulmin Mapping of Selected Episode for Instructor A

The instructor then revisited the question of how one can tell if the third vector $\begin{bmatrix}8\\5\end{bmatrix}$ can be made from a combination of the other two vectors. A student rectified the previous reasoning from Data 2, arguing that a parallelogram created from the other two vectors could be scaled to

[1] In the Toulmin mapping, statements that are not mathematically correct are marked with an X. Incorrect statements that resolve to correct statements are marked with dotted arrows.

make the third vector (Warrant 9 and Data/Claim 10). Another student suggested you could make a system of equations (Data 11), which the instructor elaborated somewhat extensively.

The instructor then said, "So this isn't an example of an independent set. Can we come up with three [vectors] that are? Who came up with a different example? Back corner, you guys have something written for that." A student claimed that it's "impossible" to create a set of three linearly independent vectors in R^2 (marked as Claim 12) because two linearly independent vectors in R^2 "can reach wherever the third is going which by definition makes it to become dependent" (marked as Data 13). The instructor elaborated, "Whatever third vector you pick it's already in the span of these. It's a linear combination of these. It's dependent on the previous two" (Warrant 14).

A student-to-student exchange about making the third vector the zero vector follows. A student suggested that if one vector is the zero vector, then the set is linearly dependent (Data/Claim 15), with another student noting "you could scale the zero vector by any number you want, then it's not trivial" (Data 16). The warrant remained implicit, as the instructor confirmed the second student's reasoning that "you can always put something non-trivial there" and ended the class period.

Comparing Argumentation in Year 1 to Year 2

In comparing these two exchanges, we noted a shift in the interconnectedness of mathematical ideas discussed, as well as a shift toward students taking authority and contributing more claims and data in year 2. Table 2 identifies how many claims, data, warrants, data/claims, and backings were articulated primarily by the teacher, and primarily by students.

Table 2: Count of Components of Arguments by Primary Contributor

	Claim		**Data**		**Warrant**		**Data/Claim**		**Backing**	
	T	S	T	S	T	S	T	S	T	S
Y1	0	2	0	1	2	2	1	0	2	0
Y2	0	4	1	4	3	3	0	2	0	0

We argue that in year 1, the teacher elicited correct student ideas, built on, and reshaped them so as to achieve her mathematical goals. In order to accomplish this, the teacher ended up formulating a larger portion of the mathematical argumentation as compared with the following year. In year 2, the instructor made space for students to explore their ideas in a supported way, even when those were incorrect. By engaging an incorrect response as a starting point, and allowing students to articulate and explore the ambiguity and their own uncertainty (supported by both the instructor's empathetic warrant and her invitation to discuss their uncertainty), the students were able to correct their reasoning and provide a larger portion of the mathematical argumentation.

Discussion

Our analysis offers insight into the shifts that take place between initial and subsequent implementations of inquiry-oriented instructional sequences among instructors with interest in implementing (and some access to conversations with curriculum developers). Our analysis suggests that teachers' knowledge of curriculum, content and students, and content and teaching all increased. Our study also functions to contribute a potential methodological approach of using Toulmin's argumentation model for investigating shifts in instructional practice, an important form of instructor learning.

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