A Preservice Mathematics Teacher's Covariational Reasoning as Mediator for Understanding of Global Warming

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I examine one preservice mathematics teacher's (PST's) covariational reasoning in relation to two functions involved in modeling global warming. I also discuss how her covariational reasoning mediates her understanding of important concepts related to global warming. Jodi, the PST, completed a mathematical task I created for the study during an individual, task-based interview. The analysis of Jodi's responses revealed that: (a) the level of covariational reasoning and conceptions regarding quantities can constrain/facilitate the understanding of concepts related to global warming, (b) overreliance on discrete variation can led to conflicting notions regarding global warming, and (c) reasoning about rate of change is necessary to make sense of mathematical models for global warming based on energy balance.

Keywords: Covariational Reasoning, Global Warming, Preservice Teachers, Modeling

Introduction

In recent years, there have been several calls to include global warming in school and college instruction (McKeown & Hopkins, 2010; UNESCO, 2012). Global warming is a contemporary and pressing issue affecting different people around the globe (Intergovernmental Panel on Climate Change [IPCC], 2013). Moreover, global warming provides a motivating scientific context to study important scientific and mathematical concepts. Mathematics teachers, however, are likely not prepared to incorporate global warming into their instruction. Researchers have demonstrated that the public have many problematic conceptions about important concepts related to global warming (Leiserowitz, Smith, & Marlon, 2010; Pruneau, Khattabi, & Demers, 2010). Also, teachers and students without sufficient scientific and mathematical literacy can have difficulties understanding concepts related to global warming (Barwell, 2013; Lambert & Bleicher, 2013). Therefore, there are both societal and cognitive needs for studies regarding global warming and mathematical reasoning.

In my research, I investigated how preservice mathematics teachers (PSTs) make sense of introductory mathematical models for global warming. By introductory models, I mean those for which the mathematics can be accessible to high-school students. The models require PSTs to think about a dynamic situation in terms co-variation between quantities. Existing research in mathematics education has demonstrated that students and future mathematics teachers can have persistent difficulties comprehending and mathematically expressing co-variation between quantities (Carlson, Jacobs, Coe, Larsen, & Hsu, 2002; Johnson, 2012; Oehrtman, Carlson, Thompson, 2008; Thompson, 2011). In this paper, I focus on one PST's covariational reasoning in relation to two functions: *the planetary energy imbalance function*, N(t), and *the planet's mean surface temperature function*, T(t). I also discuss how her covariational reasoning mediates her understanding of important concepts related to global warming.

Background Information

Earth's climate system is powered by the sun and there is a continuous flow of energy between the sun, the planet's surface, and the atmosphere. This continuous flow of energy is known as the *Earth's energy budget* (Figure 1). The sun warms the planet's surface (S). As the surface warms up, it radiates (infrared) energy to the atmosphere (R), the majority of which is absorbed by *greenhouse gases* (GHG) such as water vapor (H_2O), carbon dioxide (CO_2), and methane (CH_4) (B). The atmosphere re-radiates the absorbed energy in both directions toward space and toward the surface (A). This continuous energy exchange between the surface and the atmosphere is known as the *greenhouse effect* and influences the planet's mean surface temperature. The energy flows S, R, B, L, and A (Figure 1) are all magnitudes of energy flux density, while the abundance of GHG is a magnitude of concentration. *Energy flux density* is a flow of energy per unit of area per unit of time incident to a surface, usually measured in Joules per square meter per second ($J/m^2/s$). *Concentration* is the volume of a gas relative to the total volume of the mixture in which the gas is contained, usually measured in the same units of volume (e.g., m^3/m^3) or in parts per million by volume (ppmv). The parameter $0 < g < 1$ (Figure 1) is related to the greenhouse effect. Quantifying changes in the energy flows due to changes in the abundance of GHG is central to accurately model global warming. My study focuses on how variation in the atmospheric concentration of CO_2 produces variation in the energy flows over time, and how that variation affects the planet's mean surface temperature.

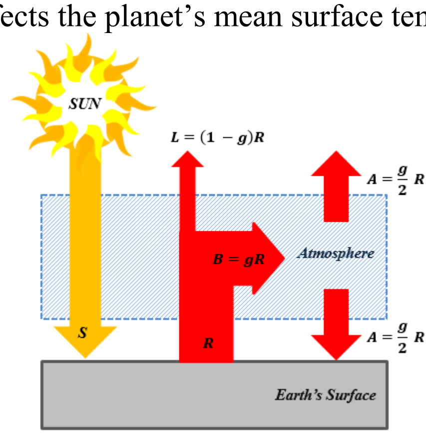

Figure 1: The Earth's energy budget, assuming a one-layered atmosphere

The *planetary energy imbalance function* N(t) is a measure of the energy imbalance in the Earth's energy budget over time. In particular, N(t) can be defined as a difference between the downward radiation and the upward radiation at the planet's surface, or mathematically $N(t) = (S + A(t)) – R(t)$. The Earth's energy budget is said to be in radiative equilibrium when $N(t) = 0$ (downward radiation equals upward radiation), which implies that the *planet's mean surface temperature function* T(t) remains constant. However, there are factors or *forcing agents* that can push the energy budget out of equilibrium, producing $N(t) \neq 0$. The present study focuses on how N(t) and T(t) vary over time after a *positive forcing by* CO_2 occurs at $t = 0$. An instantaneous increase in the concentration of CO_2 results in an atmosphere with more capacity to absorb surface radiation R(t). This translates into a value for A(0) such that $N(0) = (S + A(0)) – R(0) > 0$, which means that the downward radiation exceeds the upward radiation. As a result, the planet's surface starts warming up (i.e., an increasing T(t)); a hotter surface produces more radiation (i.e., an increasing R(t)). The atmosphere absorbs even more radiation, increasing its own radiation back to the surface (i.e., an increasing A(t)), further warming the surface. The expression $N(t) = (S + A(t)) – R(t) = S – \beta R(t)$, where S is the solar constant and $\beta = 1 – g/2$, indicates that R(t) continues to increase until the upward radiation equals the downward radiation since $N(t) \to 0$ as $t \to \infty$. This in turn indicates that T(t) increases at a decreasing rate as it approaches to a new equilibrium temperature. In fact, mathematical models for global warming commonly known as Energy Balance Models (EBMs) rest on the idea that $\frac{dT}{dt} = \alpha N(t)$ for a constant $\alpha > 0$ (Widiasih, 2013).

Conceptual Framework

Carlson et al. (2002) defined covariational reasoning as "the cognitive activities involved in coordinating two varying quantities while attending to the ways in which they change in relation to each other" (p. 354). Based on this definition, Carlson and colleagues developed the Covariation Framework as a theoretical instrument to examine and assess a student's covariational reasoning abilities relative to a mathematical task showing two co-varying quantities. Their framework describes five mental actions involve in reasoning about quantities that vary together. *Mental Action 1 (MA1)* involves coordinating the value of one variable with changes in the other (e.g., labeling the axes with verbal indications of coordinating the two variables such as "y changes with changes in x"). *Mental Action 2 (MA2)* involves coordinating the direction of change of one variable with changes in the other variable (e.g., constructing an increasing straight line or verbalizing an awareness of the direction of change of output while considering changes in the input). *Mental Action 3 (MA3)* involves coordinating the amounts of change in one variable with changes in the other (e.g., plotting points, constructing secant lines, or verbalizing an awareness of the amount of change of the output while considering changes in the input). *Mental Action 4 (MA4)* involves coordinating the average rate of change of the function with uniform increments in the input variable (e.g., constructing contiguous secant lines or verbalizing an awareness of the rate of change of the output while considering uniform increments of the input). *Mental Action 5 (MA5)* involves coordinating the instantaneous rate of change of the function with continuous changes in the independent variable for the entire domain of the function (e.g., constructing smooth curve with clear indications of concavity changes, verbalizing an awareness of the instantaneous changes in the rate of change for the entire domain of the function, or correctly interpreting concavities and inflexion points). The collection of mental actions inferred from the student's responses is examined to determine the student's overall level of covariation reasoning relative to the task. There are five levels of development, each more sophisticated than and built upon the previous one: *dependency of change* (L1: y changes when x changes), *direction of change* (L2: y increases as x increases), *amounts of change* (L3: a change Δy in y correspond to a change of Δx in x), *average rate of change* (L4: y increases more rapidly for successive changes Δx in x), and *instantaneous rate of change* (L5: y increases more rapidly as x continuously increases). If a student's covariational reasoning is classified at a particular level, then it is implied that the student's covariational reasoning supports the mental action associated with that level *and* the mental actions associated to all previous levels.

Methods

This paper is part of a larger study that investigated how PSTs make sense of introductory mathematical models for global warming. That larger study consisted of two parts: (1) exploring PSTs' conceptions of intensive quantities commonly used to model global warming, and (2) examining PSTs' covariational reasoning relative functions commonly used to model global warming. To address these goals, I created an original sequence of six mathematical tasks involving intensive quantities, functions, and concepts related to global warming.

Three secondary PSTs enrolled in a mathematics education program at a large Southeastern university participated in the larger study. The PSTs have completed three mathematics content courses (calculus I, calculus II, and introduction to higher mathematics) and were completing a mathematics education content course (connections in secondary mathematics). In this paper, I focus on the case of Jodi, one of the three PSTs who participated in the larger study. Specifically, I focus on Jodi's responses to the sixth mathematical task in my sequence. Her case is interesting for two reasons. First, Jodi's responses were markedly different from her peers, which represent a unique case for discussion. Second, her case shows clear examples of how covariational reasoning can mediate the understanding of scientific concepts related to global warming.

I started by showing Jodi a 7-minute long video introducing the *Earth's energy budget*, *radiative equilibrium*, and *greenhouse effect*. The video was retrieve from the NASA YouTube channel *NASAEarthObservatory*. Then, I answered any questions she may have had concerning the concepts discussed in the video. Next, I presented her with a diagram of the energy budget (Figure 2a) and the following task:

> *An increase in the atmospheric concentration of CO_2 results in an energy imbalance in the Earth's energy budget. This initial imbalance is known as forcing by CO_2. We want to examine how the planetary energy imbalance N(t) and the planet's mean surface temperature T(t) vary over time after the forcing. Use what you learned about the Earth's energy budget, the greenhouse effect, and the definition $N(t) = (S(t) + A(t)) – R(t)$ to determine: (a) how N(t) varies over time and sketch its graph and (b) how T(t) varies over time and sketch its graph.*

Jodi completed the mathematical task during a 60-minute, semi-structured, task-based interview (Goldin, 2000). The interview was video recorded and transcribed for analysis. All of Jodi's work on paper was collected as well.

Videos and transcripts were analyzed through Framework Analysis (FA) method; this method five inter-related stages of data analysis: familiarization with data, developing an analytic framework, indexing and pilot charting, summarizing data in analytic framework, and synthesizing data by mapping and interpreting (Ward, Furber, Tierney, & Swallow, 2013). Through these stages, the researcher creates and refines framework analysis' distinctive feature: the *matrix output*, a table arrangement into which the researcher systematically reduces, summarizes, and analyzes the data. I utilized the mental actions in the Covariation Framework (Carlson et al., 2002) as themes for coding interview transcripts. Then, I re-read all transcript texts categorized under a particular mental action. I selected and summarized those transcript texts that were more representative of that particular mental action. I repeated this process until I selected representative texts for each mental action. Then, I organized the selected texts into a matrix output containing five columns (one for each mental action) and two rows: one for N(t) and another for T(t). The matrix output allowed me to develop an idea of Jodi's: (a) overall level of covariational reasoning, (b) understanding of N(t) and T(t), and (c) conceptions of the energy budget and radiative equilibrium.

Results

Jodi's responses to the first part of the task suggest covariational reasoning abilities at the direction of change level (L2) when her object of reasoning was the situation (i.e., how the energy budget evolves after a positive forcing). When her object of reasoning was the graph of N(t), she demonstrated abilities at the amounts of change level (L3). To start the task, I told Jodi to imagine that an instantaneous increase in the atmospheric concentration of CO_2 produces an imbalance of energy equal to $N(0) = 5$ J/m^2/s (positive forcing by CO_2). Jodi is then given a diagram of the Earth's energy budget showing the initial values: $S = 240$ J/m^2/s, $R(0) = 390$

J/m^2/s, B(0) = 310 J/m^2/s, L(0) = 80 J/m^2/s, and A(0) = 155 J/m^2/s (Figure 2a). Notice that N(0) = (S + A(0)) – R(0) = (240 + 155) – 390 = 5. Jodi was expected to visualize how N(t) varies as time t increases. Jodi imagined energy moving from R to B, then to A, and finally back to R, what she labeled as *cycles*. Using these cycles, Jodi determined the following values for the energy flows R, B, and A: R(C_1) = 395 J/m^2/s; B(C_1) = 313 J/m^2/s, and A(C_1) = 157 J/m^2/s, and R(C_2) = 397 J/m^2/s; B(C_2) = 315 J/m^2/s, and A(C_2) = 158 J/m^2/s (Figure 2a), where C_i represents *cycle i* after the positive forcing. When I asked Jodi whether N(t) was increasing or decreasing, she stated "I guess it would increase? [*Pauses*] but, I don't see an argument for why it wouldn't stay the same." I then asked her to determine the values of N(t) for each one of her cycles. Jodi determined the values N(C_0) = 5 J/m^2/s, N(C_1) = 2 J/m^2/s, and N(C_2) = 1 J/m^2/s, where C_i represents *cycle i* after the positive forcing. Jodi stated that she was not expecting N(t) to decrease over time ("I though N would be larger"). When I asked her to interpret this decreasing N(t), Jodi replied "[it means] that we are going back to an equilibrium, or we are not as far from equilibrium as we were." When Jodi was able to establish the direction of change of N(t), she began to conceive N(t) as a measure of the energy imbalance. Also, the direction of change helped her develop the idea that the energy budget moves towards (radiative) equilibrium after a positive forcing. These represent foundational concepts to understand introductory mathematical models for global warming.

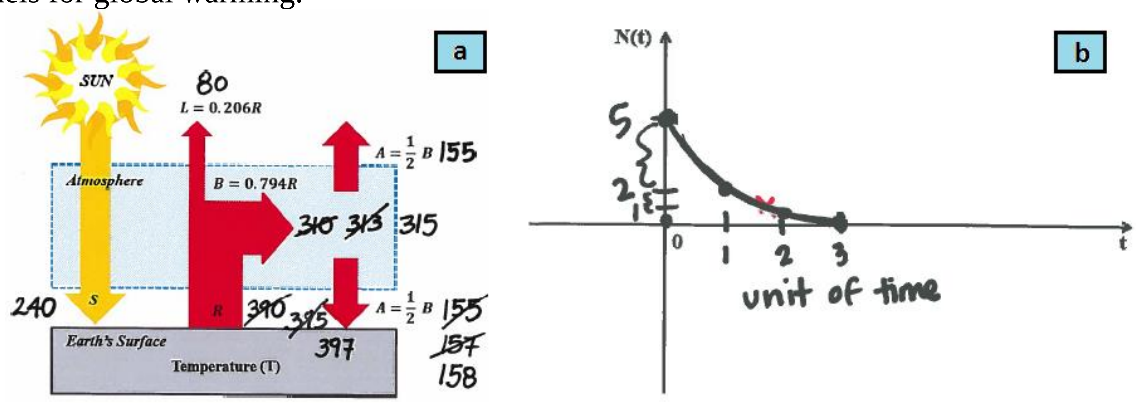

Figure 2: (a) Jodi's work on the diagram of the energy budget. (b) Jodi's final graph of N(t).

Jodi constructed the graph of N(t) by plotting the points (C_i , N(C_i)), and then joining them by a concave-up, decreasing curve (Figure 2b). Jodi looked at the curve and stated that "we are decreasing at a decreasing rate." When asked to elaborate, Jodi said

> Each time we are increasing t, we are decreasing N by smaller amounts. Like here, we decrease 3 [curly brackets on Figure 2b], and then we decrease 1 … I am trying to make sure I know what the graph looks like. OK, when you have a graph and you do like this [*draws a concave-up, decreasing curve*], this is one and this is two [*makes two equally-spaced marks on the horizontal axis*]; you would be decreasing by smaller amounts each time. The same thing what we are doing here [*draws the curly brackets on Figure 2b*], so I want to say that the graph looks like this: decreasing at a decreasing rate

Jodi's responses regarding the rate of change of N(t) and how N(t) decreases by smaller and smaller amounts were a result of reasoning about the graph of N(t). Jodi did draw a concave-up curve, but the concavity was the result of joining all points by a curve. Notice that she need not reason beyond L2 to accomplish that. Jodi did not notice that R, B, and A were also increasing at a decreasing rate. This suggests that she was not attending to the situation when thinking about amounts of change. It was by using the graph as her object of reasoning that Jodi attended to the variation in amounts of change in N with respect to changes in time. This appears as a version of L3 covariational reasoning, a version that makes use of the graph as an object of reasoning. It did not seem that this version of L3 helped Jodi understand the energy budget since the latter was not the object of reasoning. Also, Jodi's verbalization regarding the rate of change of N(t) must be taken with caution. Jodi's responses suggest that she was reasoning in terms of amounts of change rather than rate of change. It is, therefore, unlikely Jodi's covariational reasoning was at the rate of change levels L4 or L5.

Jodi provided an interesting interpretation of N(t) in relation to the variation in energy (or heat) in the surface. Jodi stated that the surface was losing heat because N(t) was decreasing. When asked to elaborate, Jodi stated that "we would need to be losing energy so that we can go back to equilibrium." For Jodi, a decreasing N(t) represented an energy budget moving towards equilibrium, but in the sense that *thing were going back to their original state* (i.e., a budget before the positive forcing). Jodi conceive N(t) as a measure of energy imbalance as in measuring *how far* the budget was from its *original radiative equilibrium*. Jodi's conception of energy imbalance did not involve N(t) as a difference between downward radiation and upward radiation. Jodi's conception of N(t) shaped her understanding of radiative equilibrium.

Jodi's responses to the second part of the task suggest covariational reasoning abilities at a discrete version of the amounts of change level (L3) when her object of reasoning was the situation (i.e., how the energy budget evolves after a positive forcing). For this task, Jodi attended to the way R and A were changing between cycles as shown in Figure 2a. Specifically, Jodi attended to the amounts of change in R and A with respect to changes in time.

> It increased by two (A changes from 155 $J/m^2/s$ to 157 $J/m^2/s$), and then it decreased by two (R changes from 395 $J/m^2/s$ to 397 $J/m^2/s$) [*pauses*]. So, it is almost as if there was no change in temperature because I associate energy as kind of having a relationship with temperature. So, if the energy increases, then the temperature increases. But, in this scenario an equal change in energy was an equal change in output [*simultaneously points at A and R*]

Jodi saw that any increase in A, or radiation from the atmosphere towards the surface, was match by the same increase in R, or radiation from the surface towards the atmosphere. She interpreted it in the following way: "the Earth would heat up because it got more energy [*points at A*], but then it would release it within the same cycle [*points at R*]." This suggests that the discrete approach to estimate the values of R, B, and A was shaping Jodi's thinking about the situation. Jodi conceived time varying in discrete units, or *cycles*. For *cycle i*, A instantaneously increased by an amount $\Delta_i A$ (at the beginning of *cycle i*), while R instantaneously increased by an amount $\Delta_i R = \Delta_i A$ at the end of *cycle i*. Following this reasoning, Jodi concluded that the surface energy was oscillating over time, which led her to conclude that T(t) was also oscillating over time. She represented this oscillatory variation by two periodic curves (Figure 3). Jodi drew two different periodic curves (*arcs* curve and *dashes* curve) because she was not sure whether the energy, and consequently the temperature, was increasing and decreasing within each cycle (*arcs* curve) or increasing within a cycle and instantaneously decreasing at the end of it (*dashes* curve). Her responses showed evidence of L2 covariational reasoning since she described the direction in which T(t) was changing over time (i.e., as t increases, T(t) increases and decreases).

Interestingly, Jodi constructed a third graph for T(t) by attending to the variation in the amounts of change in the energy flows in the budget. She attended to the variation in the amounts of change in B with respect to changes in time (Figure 2a). Since B was increasing by smaller and smaller amounts, Jodi thought that T(t) was still oscillating, but its *amplitude* was decreasing between *cycles*. Jodi probably saw the decreasing increments in B as consistent with her idea of a budget returning to the original radiative equilibrium. She represented this quasi-periodic variation by drawing a quasi-periodic curve whose *arcs* were decreasing in size (Figure 3). Her response and graph suggest that Jodi, in a way, was reasoning about how T(t) was changing in relation to time. Since she attended to the variation in amounts of change, I consider Jodi's covariational reasoning a version of L3, which was shaped by a discrete conception of time variation. Notice that her L3 covariational reasoning led her to conclude that T(t) was decreasing over time (i.e., the planet's surface was cooling down). This may become an obstacle to understand the link between CO_2 pollution and global warming.

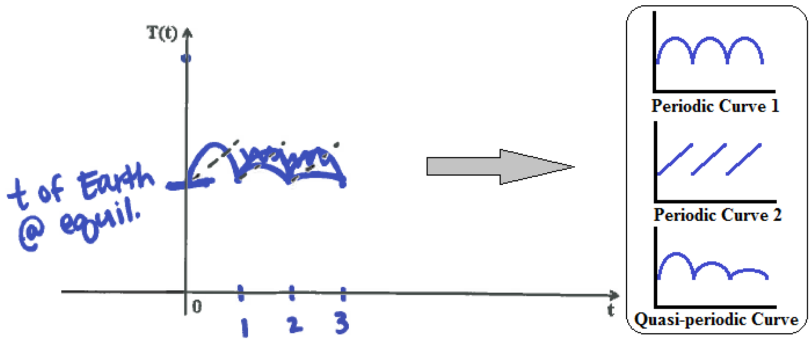

Figure 3. Jodi drew three different curves for T(t): two periodic curves and one quasi-periodic curve

Conclusions

The study's findings suggest that Jodi's covariational reasoning mediates her understanding of concepts related to global warming. Covariational reasoning at the direction of change level (L2) appears to facilitate the understanding of the budget moving towards radiative equilibrium after a positive forcing by CO_2. This is a foundational understanding for introductory mathematical models for global warming since it highlights the impact of CO_2 pollution over the planet's flow of heat. Jodi's case also shows the importance of developing covariational reasoning at the amounts of change level (L3) by using the situation as object of reasoning. Without this connection, L3 covariational reasoning can be of little use to understand global warming. Moreover, L3 covariational reasoning based on a discrete conception of variation can led to misunderstanding regarding the energy budget. In the case of Jodi, her discrete L3 led her to conclude that the planet was cooling down after a forcing, which contradicts the link between CO_2 pollution and global warming. Additionally, Jodi did not make use of N(t) to construct the graph of T(t). This suggests that Jodi did not see N(t) as a measure of the rate of change of T(t). This may be explained by Jodi's inability to reason about co-variation at the rate of change levels (L4 or L5). Another explanation involves Jodi's conception of N(t). She did not see N(t) as a difference between downward radiation and upward radiation. Without such understanding, it is unlikely to see the relationship between N(t) and T(t). Also, her conception of N(t) led her to think that the planet's surface was cooling down. This contradicts the long-term impact of CO_2 emissions on the planet's average surface temperature.

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