How may Fostering Creativity Impact Student Self-efficacy for Proving?

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Mathematical creativity has been emphasized by mathematicians as an essential piece of doing mathematics, yet little research has been done to study the effects of fostering creativity in the undergraduate classroom. In this paper, we seek to understand creativity in the classroom using Sriraman's (2005) five principles for fostering mathematical creativity by studying how these principles impact student self-efficacy for proving. Using online student surveys, interviews, and classroom observation, we demonstrate how Sriraman's five principles and Bandura's four sources of self-efficacy may be used to explore the impact of fostering mathematical creativity on student's self-efficacy for proving. Then from the interviews, we highlight how the use of the free market and scholarly principles may influence student self-efficacy via vicarious role-modeling, and explain why the use of these two principles may be of particular importance in fostering student self-efficacy for proving.

Keywords: mathematical creativity, discrete math, proving, self-efficacy

A large body of research has underscored the importance of *mathematical creativity* (1) in learning and doing mathematics (Mann, 2006), (2) in retaining students in STEM fields (Atkinson & Mayo, 2017), (3) in advancing the field of mathematics (Sriraman, 2005), (4) as a platform for creativity in general (Nadjafikhah et al., 2012), and (5) as one of the aims of mathematics education (Levenson, 2013). But how and to what extent should creativity be incorporated in an introductory proofs course? How does creativity in the classroom impact students' long-term mathematical development? In this paper, we endeavor to better understand these questions by studying the research question: how does instructor use of creative "principles" in the classroom impact students' development of self-efficacy for proving?

Literature Review

Importance of self-efficacy in mathematical learning and performance

Perceived self-efficacy (often referred to simply as *self-efficacy*) is one's belief in their own ability to accomplish something, and is highly predictive of general academic performance (Bouffard-Bouchard, Parent, and Larivée, 1991). Self-efficacy for mathematical problem solving is a better predictor of mathematical performance than mathematical ability or prior experience with mathematics (Siegel, Galassi, & Ware, 1985; Pajares & Miller, 1994). Why is this?

Bandura (1997) explains that "academic performances are the products of cognitive capabilities implemented through motivational and other self-regulatory skills." (p. 216). A wide range of studies in cognitive psychology have confirmed that beliefs of self-efficacy mediate the skills that determine how consistently and effectively students apply what they know (Pajares & Kranzler, 1995; Randhawa, Beamer, & Lundberg, 1993). Students with high self-efficacy show increased motivation and use of strategic thinking (Boffard-Bouchard, 1990), are more successful in solving conceptual problems, manage their time better, are more persistent, and are less likely to reject correct solutions (Bouffard-Bouchard et al., 1991). Thus, we suggest that self-efficacy for mathematical problem solving and proving mediates learning in a way that results in long-term benefit of educational principles and practices correlated with increased self-efficacy.

How can self-efficacy be influenced or changed? Social cognitive theory identifies four primary sources of self-efficacy: enactive attainments, vicarious influence, verbal persuasion, and physiological reactions (Bandura, 1997). *Enactive attainments* are experiences of mastery in a given task and are often the most reliable indicators of future capability. *Vicarious influence,* through observation of someone else's competencies in comparison with one's own, provide the next most reliable indicator of ability. *Verbal persuasion*, or direct verbal appraisal of one's ability by someone else, is a somewhat less reliable source of self-efficacy depending on the credibility of the persuader. Lastly, *physiological reactions,* are feelings such as strength, stamina, stress, fatigue, or pain that can provide indicators of ability or inability. Here we note that these sources of information themselves do not influence self-efficacy, but through cognitive processing of this information and reflective thought, they are selected, weighted, and integrated into self-efficacy judgements: "a host of personal, social, and situational factors affect and direct how socially mediated experiences are cognitively interpreted" (Bandura, 1997, p. 79).

Definition and Principles of Mathematical Creativity

There is considerable variation when seeking definitions of mathematical creativity (Mann, 2006). In this project, while considering mathematical creativity relative to the student, we chose a view influenced by the perspectives of Liljedahl and Sriraman (2006): "a process of offering new solutions of insights that are unexpected for the student, with respect to his/her mathematical background or the problems s/he has seen before" (Savic et al., in press, p, 2). This is based on relative (Beghetto & Kaufman, 2007), process-oriented (Pelczer & Rodriguez, 2011), domain-specific (Baer, 1998) mathematical creativity.

In seeking to study students' experience of creativity in the classroom, we searched for observable characteristics of creativity that instructors and students engaged in the classroom. According to Sriraman's (2005) extensive review of the literature on mathematical creativity, there are five principles that "maximize potential for mathematical creativity" (p. 26):

- the *Gestalt principle* conveys the importance of students engaging in "suitably challenging problems over a protracted time-period, thereby creating the opportunities for discovery and to experience the euphoria of the 'Aha!' moment of illumination" (p. 27);
- the *aesthetic principle* is concerned with appreciating the beauty in discovering and connecting new ideas;
- the *free market principle* encourages risk-taking and atypical thinking;
- the *scholarly principle* encourages students debating and challenging the validity of teachers' and peers' approaches to problems; and
- the *uncertainty principle* embraces ambiguity in mathematics, emphasizing the importance of giving open-ended problems and providing "affective support to students who experience frustration over being unable to solve a difficult problem" (p. 28).

How does instruction seen through the lens of these principles directly or indirectly impact student development of self-efficacy for proving?

Methods

Pilot data collection was conducted in an 8-week summer session of a discrete mathematics course serving as an introduction to proofs. Five classes were videotaped, one of which was transcribed, then coded by both authors for evidence of the five principles. Differences in codes were discussed until arriving at an agreement for each coded action; otherwise, both would continue for the remainder of the class period. An online survey was given to the students to measure student *experience*[1] of the five principles consisting of ten questions, two per principle. The survey was given twice, once for student experience in *prior* math classes, and once at the end of the semester for experience in *this* class. Then, in both instances, students rated their confidence in their ability to do five tasks related to proving with respect to three specific problems classified as moderately routine, moderately non-routine, and very non-routine (Selden & Selden, 2013). Selection of task questions was based off the EP-spectrum (Hsieh, Horng, & Shy, 2012) deconstruction of the proving process and followed Bandura's guide for constructing self-efficacy (2006, p. 307-337).

At the end of the course, two randomly selected students (Abe and Ben) and the instructor (Dr. One) were interviewed. Each interview was transcribed and coded once by both authors (again compared for inter-rater reliability); once for explicit and implicit examples of instructor use of each of Sriraman's (2005) five principles in the classroom, and again for examples of the student's experience of each of Bandura's (1997) four sources of self-efficacy.

Results

For the purposes validating the test questions (Bandura, 2006), the surveys were given two weeks apart. The mean scores of the self-efficacy surveys 79 compared to 80 only two weeks later, showing that both surveys were of similar and appropriate difficulty for distinguishing levels of efficacy. Also, the correlation between the two questions for each of the five principles ranged from 0.45 to 0.92, showing convergent validity.

Only nine students participated in both surveys, so with this data we cannot yet say which of the five principles are quantitatively correlated with changes in student self-efficacy. Also, from classroom observation, we were not able to observe enough instances of instructor actions demonstrating the use of the five principles of creativity. Thus, the remainder of this paper focuses on evidence from individual student surveys and interviews.

In the interviews, both students reported teacher actions for the free market and scholarly principles. Abe explained two actions coded as free market and three coded as scholarly. For example, his comment that "quite a few people went up to the board," and that "if we saw, or said what was on our mind," were both coded as an implicit use of the free market principle, since it showed that students were comfortable enough to take risks: going up to the board and saying what was on "our mind." Abe saying "[The instructor] would break questions down to the point where everyone could possibly have an input on why, or on the steps building up to the proof," was coded as scholarly principle, as well as part of the following:

Interviewer: Is there any way that the classroom environment did help you learn some of those skills?

Abe: It helped me learn a lot faster because if I didn't know the solution right away in most cases someone else did, and once someone else was called up to the board and started writing their proof**, I would follow along and then at a certain point I'd be able to figure out this is where they were going**. I'd finish out the proof and try to continue on. But having people around me that were like-minded and also enjoyed doing these types of proofs... It helped my learning because I wasn't just having to rely on what I gained from the instructor. I could rely on what other people brought to the class as well.

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[1] In future research, we realize it will be more directly answer our research question to measure to student observation of *instructor use* of the five principles.

Ben described that the instructor would “engage the class like earlier on in the semester and I felt comfortable about like speaking up and answering occasionally.” This was coded as implicit use of the free market principle since it showed Ben was able to take the risk of speaking up at that point in the semester. He also stated, “a lot of times [the instructor] would introduce a new problem and tell us to work on it... it'd be like a completely brand-new problem, which I guess is good to try to be able to think of how you'd approach like a brand-new concept.” This was coded as uncertainty principle because the instructor did not answer the problem immediately, thus perhaps implicitly allowing the students to tolerate ambiguity.

In coding for sources of self-efficacy, instances of enactive experiences and vicarious role-modeling were identified in both interviews. Abe described seven instances coded as enactive experiences, for which we provide one.

> *Abe*: I'm very confident now. I feel like these eight weeks have really given me enough time to work on the formats for everything, to be able to look at a problem rather than just as a solution and more as a problem within a problem and I think I'm pretty confident with solving it now.”

However, he did not indicate whether they were due to any actions from the instructor. Ben cited four instances coded as enactive experiences, all on his own, outside of class. He cited *not* having the confidence to engage in class without the “right answer.”

Abe describe three instances of vicarious role-modeling. Two were from the instructor: “[the instructor] was very excited when he was talking about proofs and I feel like I feed off the energy of my professors… I think that really helps me learn,” and “as he broke it down, he would slowly work it out with us, as we were talking to him.” Abe also experienced vicarious role-modeling from other students in the class: “I would follow along and then at a certain point I'd be able to figure out this is where they were going” (quoted above).

Ben described four instances of vicarious role-modeling: two as positive sources of self-efficacy from the instructor: “seeing a teacher like do the proofs repetitively” and “seeing it how you're supposed to approach a proof… seeing like where to start” (in response to the question “what made you confident?”), and two as negative sources from the students:

> *Ben:* In this class setting I felt like there were people in this class that already knew, like there's like two people in particular that would always answer all the questions and they seemed… I just kind of like deferred, if it… like the questions to them, so if they didn't… like if the teacher posed a question to the class and they didn't answer it, then I felt it like well, I definitely can't answer it if they can't.

> *Ben:* There are other times when he would engage the class like earlier on in the semester and I felt comfortable about like speaking up and answering occasionally, but [later on] I didn't feel comfortable and like around my peers to like answer questions, because I didn't have the confidence.

Lastly, we found that although Ben expressed some confidence in his proving ability in his interview, his self-efficacy score (from one survey) was 8 points lower than average. Abe’s self-efficacy score was 6 points (average of two surveys taken) above average.

Discussion

In Abe’s interview, we might infer a potential association between the free market and scholarly principles and increased self-efficacy via vicarious role-modeling. In the same quote, he believed that students were comfortable enough to say, “what was on [their] mind,” (free market) then the instructor “would slowly work it out with us, as we were talking to him” (vicarious role-modeling). But, without more data, it is difficult to understand the degree which the free market principle promotes, or may be necessary, for the positive influence of vicarious role-modeling in the classroom. Abe's experience also highlights the importance of the scholarly principle in gaining self-efficacy via vicarious influences. Engaging and considering other's solutions appears to have improved his self-efficacy because he "wasn't just having to rely on what I gained from the instructor." According to Bandura (2007), these kinds of influences (i.e. from peers), provides a stronger source of self-efficacy (than that from the instructor), since the attainments of those who are more like oneself gives better indication of one's own ability.

Although Ben's interview contained evidence of instructor use of scholarly and uncertainty principles, the use of the scholarly principle may not have promoted self-efficacy in this student's case. Firstly, his self-efficacy scores were lower than average. Secondly, when asked, "did you become more confident by the end [of the course]?" he responded, "I still don't think I'd be confident… like if the teacher posed a question to the class and [the two confident students] didn't answer it, then I felt it like well, I definitely can't answer it if they can't." This is an example of vicarious role-modeling giving a negative source of self-efficacy information for the reason described by Bandura: "observing others perceived to be similarly competent fail lowers observers' judgment of their own capabilities and undermines their effort" (1997, p. 87). Interestingly, Ben did not cite any evidence of the free market principle.

After comparing both interviews, we believe that the scholarly with the free market principles *together* may better promote positive student self-efficacy: creating an environment where students can take risks (where mistakes are ok) levels the playing field, helping students see others' experience of both success and failure as part of the proving process. Thus, the way an instructor handles the free market principle may mitigate potential negative effects on self-efficacy associated with the threat of being wrong (inherent with the scholarly principle).

Conclusion

Although not directly identified thus far in our research, there are some potential connections between Sriraman's principles and self-efficacy suggested by other research. For example, the Gestalt principle may foster self-efficacy via enactive attainment since students' experience of creating proofs though sustained time and effort gives evidence of their future proving ability. The aesthetic principle may serve as verbally influencing student self-efficacy, by convincing students of the joy and beauty inherent in mathematics.

Additionally, as we found above, other principles may have a stronger combined influence on self-efficacy, which may be seen via other mediatory mechanisms such as intrinsic motivation. For example, giving students opportunities to state and defend their solutions (free market) while promoting the understanding of problem design (scholarly) may give students greater ownership, intrinsic motivation, and promote the development of evaluation. As a result, when doing tasks with high intrinsic motivation (i.e. proving), task feedback can have a stronger effect on self-efficacy (Arnold, 1976).

Furthermore, student self-efficacy may influence how students engage or experience these principles. Students with high self-efficacy tend to pursue higher challenges (Pintrick & DeGroot, 1990), suggesting that students with high self-efficacy are more likely to engage the free market principle in and out of class. Also, student experience of the gestalt principle can be influenced by their persistence, which may be supported by self-efficacy beliefs (Selden & Selden, 2010). More research is needed to understand the connections between mathematical creativity and self-efficacy for proving and problem solving. How might this study be modified or applied in a larger context? How might the results inform our use of creative principles in the classroom? What other connections might one find with these theoretical framings?

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