The Authority of Numbers: Fostering Opportunities for Rational Dependence in a Mathematics Classroom

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This cross-case analysis of quantitative literacy instruction at the undergraduate level compares three different settings where activities were introduced that required students to seek out and make use of information outside of the classroom. These activities provided students with opportunities to engage with quantitative claims made by experts and by comparing these cases I was able to identify several axes of variability that affect the extent to which the problems supported the practice of rational dependence, or the reasoned dependence on the knowledge of others. These variables include the extent to which students are held accountable for their choices of information sources, the way in which the teacher frames what it means to critically appraise a quantitative claim, and the role that mathematics plays in the activity.
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Rational dependence (Erickson, 2016) is the reasoned dependence on the expertise of others. If mathematics instruction is to help prepare students for the quantitative claims that they may expect to encounter in their everyday lives, then they need to be given the opportunity to develop rational dependence. In order to explore what happens when such opportunities are created, I collaborated with several teachers of quantitative literacy-focused undergraduate mathematics courses in order to introduce information-based problems (Walraven, Brand-Gruwel, & Boshuizen, 2008), or those problems that require students to seek out and evaluate information sources outside of the classroom. I developed case studies for each location which informed cross-case observations about how these teachers used information-based problems. The resulting multiple case analysis allowed me to answer questions about whether and how opportunities for rational dependence arose in the context of the activities.

My analysis of classroom work suggests that opportunities for rational dependence were associated with the way that teachers used information-based problems and, in particular, the structure of the academic tasks through which the problems were implemented. All of the teachers prioritized their students' development of a critical stance towards quantitative claims over direct assessment of the credibility of sources. I found that the classroom tasks that contributed to opportunities for rational dependence included (a) how students were held accountable for the sources that they found, (b) how the teachers operationalized their students' development of a critical stance towards quantitative claims, and (c) the role that mathematics played in the tasks.

Review of the Literature

Quantitative Literacy

Mathematics instruction has long had an instrumental role with respect to training in the STEM fields, but there has been a more recent push targeting the development of the general mathematical skills and attitudes that might best serve students in their daily lives. This aggregate of skills and dispositions is sometimes referred to as quantitative literacy (Steen, 2004), mathematical literacy, quantitative reasoning or statistical literacy (Cullinane & Treisman, 2010; Watson, 2013). Accordingly, the development of quantitative literacy has become an important goal of courses offered by many colleges for non-STEM majors who need to fulfill a mathematics requirement as part of their liberal arts education. Examples of quantitative literacy are usually directed at quantitative claims made by journalists, politicians, or advertisers rather than experts, but the question of how students should relate to experts does occasionally arise. For example, Gal (2002) in his analysis of statistical literacy, suggests that critical skills, i.e., knowing which questions to ask about sources of information and their biases, is a component of statistical literacy, and that this must also be accompanied by a disposition to maintain a critical stance towards statistical claims. However, one of the most thorough attempts to understand how students can come to interact productively with expert information can be found in the field of scientific literacy.

Learning to Live with the Expertise of Others

Norris (1995) breaks down what science education might look like if it served to prepare students to attain an intellectual independence tempered by their dependence on expert communities. He stressed three components of this program: a) learning science in the sense outlined by Moje's (2007) "usable disciplinary knowledge", b) learning about the history and philosophy of science, and c) "learning to live with science" (Norris, 1995, p.214).

This last component is elaborated by Norris, "the only access to scientific truth for most of us is through the efforts of scientific experts [...] therefore, students need to acquire the disposition to question, and to seek other opinions on scientific issues that matter in their lives and in their community" (Norris, 1995, p.215). But this questioning disposition should not, per Norris, be indiscriminate,

> A skeptical disposition is not sufficient if one does not know how to exercise wisely that skepticism. […] [Students] should be taught how to use criteria for judging experts: the role and weight of consensus; the role and weight of prestige in the scientific community; the role and weight of publication and successful competition for research grants; and so on. As part of learning to live with science, students need practice in judging the credibility of scientific experts. This practice should be based on real-world problems that currently affect their lives. (Norris, 1995, p. 216)

Gaon and Norris (2001) go on to argue that there are *content-transcendent* modes of inquiry into claims made by experts and that a non-expert can, and should, ask questions about scientific claims:

> Does this scientific belief embody or support any particular social hierarchies such as those based on race, on gender, or on class? If so, what normative assumptions have been made? Have these norms been thematised and justified scientifically, or are they simply assumed? Have alternate accounts of the same phenomenon been developed? By whom? What were the grounds for choosing one account over another? Are these grounds themselves free of normative assumptions; are they as certain as they appear? Who decided? (Gaon & Norris, 2001, p.200)

These questions apply equally well to quantitative claims and serve as a road map for thinking about what it looks like to live with expertise in any disciplinary area. This gives rise to a further question: How could activities be created that would provide students with the opportunity to develop these skills?

Information-Based Problems

The seeking out of information on the internet can become a dilemma for the instructor once they allow this activity to take place in their classroom. At this point, the question becomes not so much about classroom management (e.g., “What should the smartphone policy be?”, “How do I keep students from surreptitiously texting?”) but rather about managing the classroom’s *didactical contract*. I borrow this last term from Guy Brousseau (1997) who refers to the division of labor and system of accountability that specifies how a classroom activity provides evidence that the envisioned learning has in fact occurred. In particular, this type of activity amounts to a modification of the traditional terms of the mathematical *tasks* (Herbst, 2006; Doyle & Carter, 1984), or the actions associated with a given problem along with an established set of resources. This may help explain why, even though the information-seeking behavior of academics differs across disciplines (Palmer & Cragin, 2008), there are few attempts to educate students in the discipline about how that discipline-specific information-seeking is carried out (Grafstein, 2002). This would not necessarily be a problem if students were able to learn how to seek out information through generic instruction that could then be used to support quantitative reasoning, but research has shown that content knowledge is deeply tied to successful information-seeking (Walraven et al., 2008).

Theoretical Framework

Rational Dependence as an Educational Goal

Successful engagement with real-life quantitative claims is predicated on our *epistemic dependence* (Hardwig, 1985) on others, or the fact that much of what we know is dependent on our trust in the expertise of others. This observation must be tempered by the fact that an individual can rely on others in a more or less rational way (Siegel, 1988). What does all of this mean for mathematics instruction? Although the importance of information-based problems for disciplinary literacy is easy to justify as long as one accepts that information-seeking is an important part of practice in the disciplines, it requires a little more unpacking to explain why this type of instruction might have a place in mathematics instruction. One way to begin such an explanation is to imagine an applied mathematics problem -- say students are given an editorial in which the author argues that federal guidelines on fuel efficiency will end up costing the country more money than it will save (Diefenderfer, 2009). Students are asked to read the editorial and then provided with several guiding questions that encourage the students to analyze the numerical argument contained in the article while noting some of the additional information that might be required prior to coming to a final verdict on the validity of the editorial’s argument. If a reader were to actually want to determine whether a quantitative claim was true or not, they would want to locate the relevant *epistemic community* (Haas, 1992), i.e. that community that possesses the expertise to tentatively rule on the truth of the claim. In other words, they would need to engage in the practice of rational dependence by finding experts on whom the students have good reason to rely. An *information-based problem* (Walraven et al.,

2008) provides such an opportunity by requiring students to seek out and evaluate sources outside the classroom. In order to come to a better understanding of an information-based problem, the student must "identify information needs, locate corresponding information sources, extract and organize relevant information from each source, and synthesize information" (Walraven et al., 2008, p.2) in a process called *information-problem solving*. My inquiry can be framed, then, as a question about how mathematics teachers and their students cope with the introduction of information-based problems, and whether and how these problems afford opportunities for rational dependence in the classroom.

Accordingly, this study seeks to answer the following research questions:

1. How can opportunities for the practice of rational dependence be introduced to a quantitative-literacy focused mathematics class?
2. What aspects of information-based problems in a quantitative-literacy focused mathematics classroom most influence students' opportunities to practice rational dependence?

Research Methodology

I investigated the research questions outlined above through a multi-case analysis (Stake, 2013) of collaborations with three teachers of terminal undergraduate mathematics classes targeting non-STEM majors. We worked together to design activities in which information-based problems would be introduced to their students. Table 1 provides more information about the sites where this research took place. At Phi University students were assigned to argue one side in a classroom debate. To prepare, they were required to research their topic and provide some statistical evidence supporting their side of the issue. At Rho University, we developed a two-part activity where students were asked to look for articles in which a conjecture about causation was being studied (e.g., vaccines and autism). They were asked to locate the quantitative evidence used to claim that the two variables were or were not correlated, and then engaged in a small-group discussion with their peers about the topic. Their groups tried to come to a consensus on the issue at stake and then shared their verdict with the rest of the class. The students at Delta University also worked in small groups, but here they were asked to create a presentation in which they would analyze the way that statistics were used in a research article for the rest of the class. The focus of this analysis would be on the sampling methodology, but they were free to talk about other facets of the article if they chose to do so.

The *quintain* (Stake, 2013), or the phenomenon of interest for this cross-case analysis, is the introduction of information-based problems to an undergraduate mathematics course. The data for this study includes pre- and post-interviews with the instructors at each of the three sites, supplementary interviews with teaching assistants and students, field notes taken while observing instruction prior to the introduction of the information-based problems, video and audio-recordings of the in-class component of the activities, and copies of the work that the students submitted. These data sources informed the writing of individual case reports which were, in turn, used to develop the cross-case analysis. Following Stake (2013), I developed themes based on my research questions that I then used as an analytical lens for the development of case reports for each of the three sites. After writing up the case reports, I cross-referenced case-specific with the themes of the larger study. This allowed me to warrant theme-based assertions and used those to inform the final cross-case assertions (see Figure 1) about the introduction of information-based problems to undergraduate mathematics classrooms.

University Name[a]	Course Name[a]	Students	Topics	Structure
Phi University (Research)	Topics in Mathematics	22 entering Freshman, Liberal Arts Majors	Gun Control, Marijuana Legalization, Single-Sex Education, Death Penalty	Debate Format
Rho University (Regional)	Quantitative Reasoning	14 Juniors and Seniors, many are prospective Nursing students	Autism and Vaccination, The Mozart Effect, Gun Control, Health Care Reform	Small-group Discussions
Delta University (Doctoral)	Mathematics in Today's World	24 Juniors and Seniors, many are prospective Nursing and Education students	Autism and Vaccination, Gun Control, Murder Rate, Vehicular Accidents, Employee Prospects	Small-group Presentation

[a]These are pseudonyms

Table 1. Description of research sites and student population

Results

The three cases provided instructive examples of how and why the actions of a teacher can open up or limit opportunities for rational dependence. I have broken down these observed axes of change into three categories. First, the degree to which students are made accountable for their choice of sources to draw from; second, the way in which a critical stance to quantitative claims is framed by the instructor; third, the role of mathematics in the information-based problem.

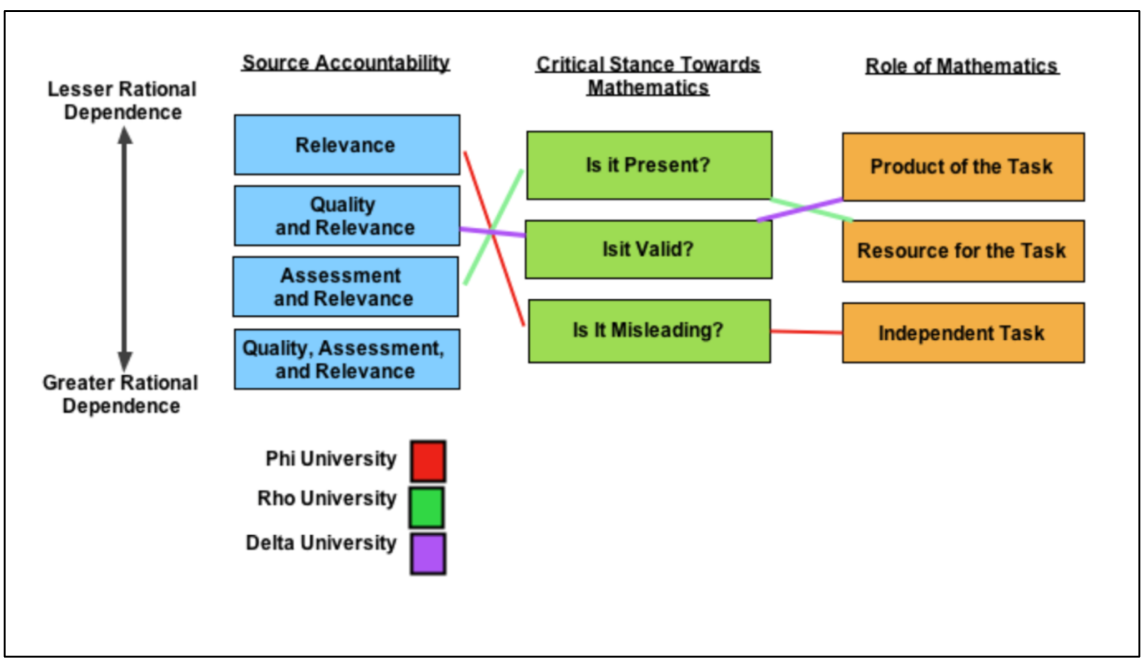

Figure 1. Opportunities for rational dependence in each of the three cases.

Source Accountability

The information-based problems, as specified by these teachers, differed with respect to the way in which the students were held accountable for the sources that they found. This feature appears as the leftmost column in Figure 1 and contains four categories that I have ordered based on the degree to which the feature presents opportunities for rational dependence. The least conducive to rational dependence are those problems for which students are held accountable for finding sources that are *relevant* and nothing more. For example, students participating in the debates at Phi University did not need to say anything about the quality of their sources nor did they have to account for the process through which they decided on those sources. At Delta University, students were required to assess the quality of their sources through an analysis of the researchers' sampling methodology – this also provided students with an opportunity to bring to bear the statistics that they had been learning. The greatest opportunity for engaging in rational dependence with respect to sources of information, however, occurred at Rho University where students were accountable for the process that they used to find sources and were asked to compare sources to one another. Thus, students engaged in the type of work described by Gaon and Norris (2001) by comparing different accounts of the same phenomenon and the grounds by which one account might be prioritized over another. For example, students prioritized research published in scholarly journals by relevant experts in the field over reports by journalists who had a record of supporting one side of the debate over the other.

Critical Stance Towards Mathematics

In the second column of Figure 1, I refer to the manner in which the teacher intends their students to take a critical stance towards mathematical claims. As these were mathematics classes, the students were expected to be critical of mathematical content specifically but, perhaps surprisingly, this expectation took a different form in each of the three cases: the students at Phi University were encouraged to watch for biased mathematical content, at Delta University they were asked to assess the validity of the mathematical argument supporting the claims, and at Rho University they were simply told to check for the presence of mathematical backing in the form of a correlation coefficient or a confidence interval. This last approach is positioned as least conducive to rational dependence in Figure 1 because the presence or absence of mathematical content does not say anything consequential about either the validity of the arguments being made by a source or whether a source's claims are supported by a broader epistemic community. Assessing the validity of the mathematical argument used by a source is more conducive to rational dependence because answering that question gives a better sense of whether a sources' claims are supported. However, that approach fails to account for the possibility that the author of an article might present a mathematically invalid argument due to their lack of mathematical knowledge even if the claim is held to be true by those with expertise in the area. Indeed, it is commonly held that popular science reporting falls prey to this exact problem (Bubela et al., 2009). Thus, the approach most consistent with the development of rational dependence is to determine whether a source has a bias and to stay conscious of how the relevant research is being framed by the source in question (Bubela et al., 2009).

Role of Mathematics

Finally, in the third column of Figure 1, I address the role of mathematics in these information-based problems. Ideally, this role would be consistent with the manner in which rational dependence arises in out-of-school contexts where the goal is to educate oneself more broadly about a specific claim rather than a mathematical concept in isolation. Information-based problems where the mathematical task is independent are the most consistent with rational dependence since this approach best reflects the reality that mathematics is one tool among many for approaching such problems. This is how the debates at Phi University treated the use of statistical charts, it was a component of the debate, but not treated as the determining factor for deciding on the credibility of a source. It is less consistent with rational dependence to give the identification of mathematical content a central role in credibility assessment, as occurred at Rho University. For example, by directing students to focus on the presence or absence of a correlation coefficient, the teacher led students to use this one element of a source as a marker of credibility over and above other important elements such as its institutional affiliation or the presence of corroborating sources. The activity at Delta University positioned the role of mathematics in a way that was least consistent with the practice of rational dependence as the product of the task prioritizes mathematics as the sole determinant of credibility.

Discussion and Conclusion

Although this cross-case analysis is only a first look at the introduction of information-based problems to quantitative literacy-focused instruction, the present findings provide some guidance for future iterations of these activities. First, they suggest the importance of holding students accountable for both how and why they choose sources to address the problems that they are assigned. This was deftly achieved at Rho University by the teacher when she facilitated small-group discussions in which students had to defend their choices to one another and come to a consensus about which sources were the most credible. Second, pushing students to reflect on the larger context in which their sources are providing information will provide them with an opportunity to identify the existence of competing explanations for the phenomenon under investigation as well as possible sources of bias. Finally, even though mathematical concepts must necessarily remain the focus of a mathematics class, it may be productive to provide students with a perspective on their problem-solving work that foregrounds the limited role of mathematics in establishing the credibility of sources. This does not have to mean that the mathematics itself is given short shrift, after all, the students need to be provided with the opportunity to develop the quantitative literacy skills that will help them engage in information problem-solving in their everyday lives – however, it does mean that students should be reminded that there are other facets to the information problem-solving process and that quantitative reasoning is only one, albeit important, element of their repertoire.

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