Guiding *Whose* Reinventions?
A Gendered Analysis of Discussions in Inquiry-Oriented Mathematics

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The under-representation of women in STEM fields is well-documented and undisputed. Evidence suggests that students' experiences in undergraduate mathematics courses contributes to this disparity, and that student-centered approaches to instruction may be more equitable than lecture-based approaches. However, the generalizability of this finding has not been established. In this study, we explore how female students are positioned in whole class discussions in two inquiry-oriented mathematics classes selected to reflect differences in how female students reported experiencing whole class discussions.

Key words: inquiry, equity, linear algebra, argumentation

Gender-based disparities in various forms of mathematics participation are well documented. For instance, women make up 50% of the workforce but only 25% of the STEM workforce (Beede, Julian, Langdon, McKittrick, Khan, and Doms, 2011). Two possible explanations for this disparity are that biological differences give rise to different abilities or preferences, or that these differences are socially constructed. In a critical analysis of literature related to gender and mathematics learning, Leyva (2017) argues that studies of both achievement and participation in mathematics suggest these differences are socially constructed. The rates at which women choose to discontinue study in math-intensive fields following first semester college calculus suggest that inequities in the way students experience collegiate mathematical learning environments likely contribute to these gender disparities (Ellis, Fosdick, and Rasmussen, 2016).

Recent research has suggested that student-centered approaches to instruction in undergraduate mathematics are related to improved and more equitable outcomes for students, particularly when considering gender differences (Laursen, Hassi, Kogan, & Weston, 2014). However, the mechanisms by which such instructional approaches relate to more equitable outcomes for women are not well understood. Some have raised questions about whether Laursen and colleagues' (2014) findings in the context of Inquiry Based Learning (IBL) classes apply to their own efforts to teach in student-centered ways (e.g. Hagman, 2017). This begs the question: Are emerging research-based, student-centered approaches to instruction contributing to or disrupting the pattern of underrepresentation of women in mathematics? More broadly, when and under what conditions are student-centered approaches to instruction more equitable? These broader questions are beyond the scope of this preliminary report, but we aim to move toward answering them by taking on the following, more modest set of questions:

- How do instructors distribute opportunities to contribute to whole class discussions in inquiry-oriented mathematics classes, and how does this relate to the gender composition of the class? (Who is invited to / does contribute and how are contributions framed?)
- How do whole class mathematics discussions vary in relation to how male and female students experience them?

Literature & Theoretical Framing

We broadly adopt a socio-political perspective, taking the view that knowledge is constructed through social discourses, and that power and identity play important roles in the construction of that knowledge (Adiredja & Andrews-Larson, 2017). Inquiry-oriented instructional approaches aim to support students' reinvention of important mathematical ideas through sequences of carefully designed tasks; they are instructionally complex in that instructors inquire into students' thinking as students are inquiring into mathematics (Kwon & Rasmussen, 2007). Such approaches reposition students to take mathematical authority in a way that may reorganize traditional norms of knowledge construction associated with lecture-based classes. We aim to relate this instructional approach to literature on gender equity in mathematics, as well as settings of cooperative learning and decision making.

Some literature suggests that learning environments that require students to develop their own problem-solving strategies may favor male students in that development of invented approaches aligns with traits traditionally valued as masculine (e.g. independence and confidence), whereas use of standard algorithms aligns with traits like compliance, which are traditionally valued as feminine (Fennema, Carpenter, Jacobs, Franke, & Levi's, 1998; Hyde & Jaffe, 1998). Other literature suggests that female students acclimate better than their male peers to learning environments that emphasize collaboration, work on open-ended problems, and conceptual understanding (Boaler 1997; 2002). Laursen et al.'s (2014) work suggests that reform-based approaches that involve collaborative problem solving may 'even the playing field' for male and female students.

Research from other fields on group decision-making suggests female students are likely to experience marginalization in instructional settings that involve collaborative group work. Karpowitz, Mendelberg, and Shaker (2012) found that when a group was charged with arriving at a decision, women spoke significantly less and were interrupted more frequently (undermining their ability to influence the group's decisions) when they were in the minority. When a group was required to come to consensus, women did not experience this. In mathematical classrooms where discussions are facilitated, students from non-dominant groups (including women) are often marginalized (Becker, 1981; Black, 2004; Walshaw & Anthony, 2008). If emergent research-based instructional approaches are to broaden participation in mathematics by constructing more equitable learning environments, it is important that we consider the gendered and racialized experiences of students in these courses. In this preliminary report, we contribute to this goal by examining gender dynamics in two inquiry-oriented classrooms.

Data Sources, Case Selection, and Methods of Analysis

We draw on data taken from a broader project interested in the teaching and learning of undergraduate mathematics through inquiry-oriented pedagogy. Instructors in this project received three forms of instructional support: access to research-based instructional sequences with implementation notes, a 16 hour summer workshop, and facilitated weekly online workgroups in the semester when they implemented the instructional materials. In this analysis, we use student surveys from seven classrooms at different institutions to select cases in which there was evidence of gender-based differences in how students were experiencing whole class discussions. We then analyzed video recordings of the selected cases to examine the relationship between mathematical discussions and gendered interactions in these classess.

Students' views of their experience in the course were captured using the Student Assessment of their Learning Gains in Mathematics (SALG-M) instrument (Seymour, Wiese, Hunter, & Daffinrud, 2000; accessible at http://salgsite.org/); surveys were conducted at the end of the semester via Qualtrics, an online survey platform. Classroom videos were recorded from two different instructional units, each of which included 2-3 days of classroom instruction. We selected video from the second instructional unit to analyze, as it took place later in the semester when instructors were more likely to be familiar with the instructional approach and classroom norms were more likely to be well-established. The instructional unit is described in Zandieh, Wawro, and Rasmussen (2017).

To select cases, we first eliminated classess with survey response rates lower than 40%. We then disaggregated students' survey responses by their self-reported gender for each class, and identified classes in which female students reported learning more from whole class discussions than male students and vice versa. We selected two classes, one from each of these categories, with similar class size and gender composition (15-20 students, approximately 25% female). On the survey, no students in either of the classes selected for analysis identified as a gender other than male or female. In video analysis we relied on visual and audio cues (e.g. hair length and style, clothing, vocal pitch, names and pronouns used) to make inferences about the gender of participants when analyzing video data. As such, all claims about participants are based on the researchers' interpretation of gender expression.

Following the selection of two cases, we created summaries of the video recordings of the two classrooms, where we first attempted to characterize instruction in each class broadly. We paid particular attention to the framing of student contributions by the instructors, how students and the instructor attributed mathematical authority, and distribution of opportunities for students' participation to the mathematical discourse in the classroom – with an eye toward gender throughout. We then transcribed the whole class discussions to analyze the mathematical argumentation, paying particular attention to how opportunities and expectations for female students to participate were framed. To this end, we selected mathematically similar focal episodes of similar length (~9 minute long whole class discussion) in the two classrooms. Both discussions addressed the image of $\begin{bmatrix}-2\\2\end{bmatrix}$ under a linear transformation from $R^2 \rightarrow R^2$ that fixed points along the line $y = x$ and stretched points in the direction $y = -3x$ by a factor of 2.

Initial Findings

The number and nature of contributions made during whole class discussions by students of each gender in each class appear in Table 1. We note that female students contributed 50% of the student ideas in instructor B's class, which is a greater portion than instructor A's class. Table 2 reorganizes student contributions according the ways in which those contributions were solicited (also sorted by the gender of student who made the contribution). Similar portions of female students offered unsolicited contributions in both classes, but instructor A called only on male students by name, and instructor B called on only female students by name. In instructor A's class, female students volunteered to speak at slightly lower rates than in instructor B's class.

Table 1: Number of contributions by nature of student contribution

	Word or phrase		Question		Idea	
Instructor	Female	Male	Female	Male	Female	Male
A	1	5	0	2	1	7
B	6	17	0	2	3	3

Taken together, this suggests instructor B made deliberate efforts to include female students in whole class discussion. Based on this information, the reader might think that female students reported getting more out of whole class discussions in instructor B's class than instructor A's class. Interestingly, this is not the case. To better understand the nature of differences in whole class discussions, we describe for each instructor, the task set-up, group formation and composition, and mathematical content of whole class discussion of the focal episode.

Table 2: Number of contributions by nature of instructor solicitation

	Unsolicited		Called by name		Volunteer requested	
Instructor	Female	Male	Female	Male	Female	Male
A	1	4	0	3	1	7
B	1	4	3	0	4	19

Development of Mathematics: Instructor A

Task setup and grouping: Before students began working on the task in groups, the instructor framed the task as difficult to understand, noting that she did the problem incorrectly in her first attempt. The instructor asked students to read the task to themselves, then to explain their understanding of what is happening to make sure they are all interpreting the task the same before the students began working on the task in small groups. Students were in groups before the camera was turned on. In interviews, the instructor had indicated an explicit effort to avoid isolating female students in predominantly male groups.

Whole class discussion: After students had worked in the groups for some time, the instructor stops students, telling them she has asked one particular student to share his idea. The instructor noted that this student didn't have it all figured out but that his group's ideas might be helpful for everyone to consider.

MS1: So, basically what we did is we started by sketching $y={}^{-}3x$ and $y=x$. We decided to draw parallel lines next to it so we could get a better visual understanding to see how to sketch. We understand from the above these parts are going to stay and these are going to stretch like this. So, we tried to fix points on corners of the box to see how it goes. What we understood is, the farther it gets from the $y=x$ axis, you could say, the points will stretch farther. So it'll have this sort of diagonal look, if that makes sense.

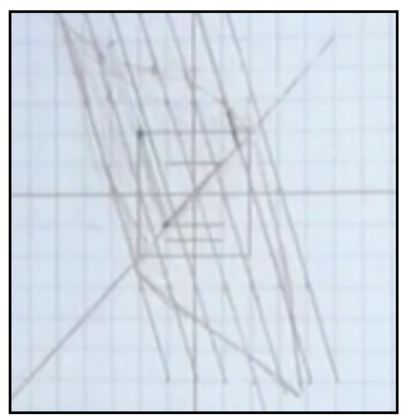

Figure 1. MS1's drawing

After the student had argued that points farther from the "$y=x$ axis" will stretch further than points close to that axis, the instructor asked students where points were stretching "from" – eliciting responses that revealed disgreement on this point. One student (incorrectly) suggested they stretched from the origin, so the instructor drew a line from the origin to the point and noted that wouldn't be in the direction of stretch stated in the problem. The instructor clarified that points stretch from the y=x axis, as the presenter had indicated, before extending this argument to geometrically show how $\begin{bmatrix}-2\\2\end{bmatrix}$ maps to $\begin{bmatrix}-3\\5\end{bmatrix}$ under this transformation by doubling its distance from the $y = x$ line in the direction of the $y = -3x$ line.

Development of Mathematics: Instructor B

Task setup and grouping: Students were expected to complete the first part of the task before class. Students were asked to "share with everybody in your table what you think this image looks like." Then, students spent about three minutes in their groups to talk about their drawings. Although students were in groups before the camera was turned on, the instructor changed that formation, saying, "I wanna form a did-the-work-[assigned at home] group over here. The rest of you can work for two minutes without the benefit of […] the people who did their work." In contrast with instructor A's class, there were no female majority groups.

Whole class discussion initiation: The instructor requested a group to volunteer to share their solutions for the second part of the task following their work in small groups. There were no volunteers, and the instructor called a female student by name to ask if her group would share. The discussion began:

FMS1: We use that matrix to transform the two vectors $\begin{bmatrix}2\\0\end{bmatrix}$ and $\begin{bmatrix}-2\\2\end{bmatrix}$ to see what their transformed values would be. So, we did matrix multiplication with matrix A times $\begin{bmatrix}2\\0\end{bmatrix}$ and then times $\begin{bmatrix}-2\\2\end{bmatrix}$ to find this. Any questions?

I: I got a question. How did you go about finding the matrix with those two you knew?

FMS2: Well, so, we say that T is x_1, x_2, and then x_3, x_4, and then you multiply that out again then using the rules of matrix multiplication. You get these four… equations and then you can use equations to solve for the four unknowns and hence you get that. [pointing to the matrix on the board.]

I: Did anybody do it a different way?

MS: I used linear combinations… first of all $\begin{bmatrix}2\\0\end{bmatrix}$… I got the linear combination of 1.5 times $\begin{bmatrix}1\\1\end{bmatrix}$ and 0.5 times $\begin{bmatrix}1\\-3\end{bmatrix}$. So, then when you multiply the $\begin{bmatrix}1\\-3\end{bmatrix}$ times 2 you got $\begin{bmatrix}2.5\\-1.5\end{bmatrix}$, which I was glad to see because that is a, that's what it would look like in my graph. And, then I used the same procedure to get a vector transformation of $\begin{bmatrix}-2\\2\end{bmatrix}$ to equal to $\begin{bmatrix}-3\\5\end{bmatrix}$.

The instructor then agreed that both methods were sensible and correct, referring to the first group's approach using matrix muliplication as "Method 1" and the second group's approach using the linear combinations as "Method 2." The instructor recapped the two methods and offered an explanation for how they related to one another.

Discussion

Our analysis suggests that identifying female students with correct solutions and asking them to share does not ensure a more equitable learning environment for female students. Taken together with the literature, our findings suggest that female students report getting more out of whole class discussions in the class where we observed explicit discussion of the ambiguity of mathematics and underlying meanings, intuition, and interpretation. It is both plausible and likely that factors beyond what we were able to observe in whole class discussion contributed to different student experiences, and we are eager for feedback to inform our ongoing analysis.

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