A Model of Task-Based Learning for Research on Instructor Professional Development

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We present a theoretical framework that synthesizes and increases the descriptive power of existing models of task-based learning. Grounded in social constructivism and activity theory, the framework supports collegiate mathematics education researchers in identifying, investigating, and reporting on task-based learning in instructor professional development contexts. Relevant definitions and connections to the larger realm of inquiry-based, problem-based, and other general inquiry-oriented instruction are addressed. We conclude with a discussion and illustration of how the framework may be used in design, materials development, and evaluation research related to instructor professional learning.

Key Words: Activity, Task-based learning, Professional development, Teaching for robust understanding

Experts in the social and behavioral sciences, such as mathematics education, often deal with the challenge of specialized language. Technical terms can have shades of meaning that differ significantly from everyday language-in-use counterparts. For example, Cook, Murphy, and Fukawa-Connelly (2016) point out that the absence of a concise and consistently applied definition of *inquiry-based learning* (IBL) in science, technology, engineering, and mathematics education has meant researchers use the term to mean fundamentally different things. This divergence has created confusion. Kirschner, Sweller and Clark (2006) claimed that inquiry-based learning does not work, while Hmelo-Silver, Duncan and Chinn (2007) responded that the claim was based on a fundamental flaw: the authors had oversimplified and treated IBL as if it were unguided discovery (which, for Hmelo-Silver and colleagues was a very different thing).

The importance of concise and shared definitions is amplified in research on instructor professional development. Hayward, Kogan and Laursen (2016) note that presenting IBL as a broad and inclusive set of pedagogical practices appeared to be critical in the willingness of college mathematics faculty to adopt it. Instructors viewed questions (inquiry) and learning as existing aspects of their own practice. Faculty saw this new "inquiry-based learning" as an extension of something they already knew, as professionally relevant and useful. As faculty learned more, read more, spoke more about IBL, they practiced using a specialized language, an IBL lexicon, for describing and re-defining their goals, resources, and orientations about teaching, about what learning was, and about what constituted evidence of it.

Here we operationalize a theory of *task-based learning* (TBL). Our focus is in the context of faculty professional development. The goal is to create a sufficiently detailed framework that has descriptive power and is useful for evaluating professional learning and for doing design-based research. In particular, there is a need for a model of TBL for research and development work on professional growth among mathematics faculty new to teaching future school teachers (Masingila, Olanoff, & Kwaka, 2012).

Some might argue with the feasibility of singular definitions in mathematics education. At the same time, the attempt to negotiate a definition, to create a useful model of meaning, can have valuable descriptive power (Schoenfeld, 2000). It is this aspect of research and design in professional development, and the knowledge that there are linguistic and cultural norms related to particular views of teaching and learning, that influences our framework effort. Consider the case of college mathematics faculty in the U.S., most of whom are fluent in one or more natural languages (e.g., English and Chinese) and one or more dialects of research mathematics. These are people who also know the Western academic cultural norms of the transmission and product models for college instruction (Davis, Hauk, & Latiolais, 2009). Place a person with these multiple fluencies and areas of expertise in a room with 20 undergraduates whose life goal is to become a primary school teacher and tell the instructor: Teach them math. Three words: Teach. Them. Math. Each word has a cacophony of meaning. The layers of meaning are large in number and the likelihood of shared definitions for "teach," "them," and "math" are small. What does it mean to teach? What distinguishes "them" from "me" or "us" (if anything)? And *which* math does "math" mean? Indeed, many American teachers perceive mathematics as a static body of knowledge where knowing mathematics is equivalent to efficiently manipulating symbols without necessarily understanding what they represent (Thompson, 1992).

Mathematical Knowledge for Teaching (MKT) for Grades K-8

Several decades of research rooted in Shulman's (1986) work have indicated that there are particular understandings and skills associated with effective instruction, a sociological synergy of mathematics and mathematics education called *mathematical knowledge for teaching* (MKT; Ball, Thames, & Phelps, 2008). MKT for elementary grades as modeled by Ball and colleagues is made up of six kinds of knowledge. Three are types of subject matter knowledge: *horizon content knowledge,* about how topics are related across the span of curriculum; *specialized content knowledge* which is specialized in the sense that it is specific to the task of *teaching*, and is complementary to *common content knowledge*. In particular, specialized content knowledge includes ways to represent mathematical ideas, provide mathematical explanations for rules and procedures, and examine and understand innovative solution strategies from the student's perspective. This specialized knowledge for teaching K-8 is sparse or absent for many with advanced mathematics expertise but little teaching experience (e.g., mathematics professors; Bass, 2005). As an example, consider fraction division. Most novice instructors can readily use the invert-and-multiply algorithm to divide fractions. Thus, this piece of knowledge is *common content*. Yet, few can explain to someone *why* the algorithm is justified in some problem situations and not in others, thereby making knowing the "whys" *specialized*.

The other three categories in MKT are types of *pedagogical content knowledge* (PCK) and are neither purely pedagogical nor exclusively mathematical. *Knowledge of curriculum* includes awareness of the content and connections across standards and texts (i.e., of the intended curriculum; Herbel-Eisenmann, 2007). *Knowledge of content and students* (KCS) is "content knowledge intertwined with knowledge of how students think about, know, or learn this particular content" (Hill, Ball, & Schilling, 2008, p. 375). *Knowledge of content and teaching* (KCT) is about teaching actions or moves (i.e., productive ways to respond in-the-moment to students to support learning). So, in our fraction example, teachers who are aware that students often invert the dividend instead of the divisor are demonstrating KCS and might use fraction diagrams to scaffold understanding if they have the appropriate KCT. Both KCS and KCT are associated with improved student learning (Hill et al., 2008; Hill, Rowan, & Ball, 2005).

A related idea at the college level is *mathematical knowledge for teaching future teachers* (MKT-FT) held by college instructors who teach pre-service teachers (Hauk, Jackson, & Tsay, 2017). A rich and textured MKT-FT is especially vital in the inquiry-oriented or activity-based approaches to teaching shown to improve student learning, increase persistence, and reduce inequities (Bressoud, Mesa & Rasmussen, 2015; Freeman et al., 2014; Holdren & Lander, 2012; Laursen, Hassi, Kogan, & Weston, 2014). College instructors acquire MKT-FT in many ways: grading, examining their own learning, observing and interacting with students or colleagues, reflecting on and discussing practice (Kung, 2010; Speer & Hald, 2009; Speer & Wagner, 2009).

Defining and Illustrating Tasks

With a focus on MKT and MKT-FT in mind, we examined *task-based learning* (TBL) and the task and activity framework of Christiansen and Walther (1986). Growing from social constructivist roots, their view of TBL is as adaptation, human behavior in response to the conditions between the individual and the social, physical, and cognitive environments perceived by the individual. In other words, human behavior is a result of goal-directed seeking for a regulation of mutual relationships between the individual and environment(s). Within this framework, the terms *task, activity, action,* and *plan* each play distinctive roles. Christiansen and Walther explicitly characterize *task* as *interplay* among teacher, students, curriculum and objectified mathematics (see Figure 2). They implicitly express activity as inherent in the relations among the various components indicated by the unlabeled arrows in Figure 2.

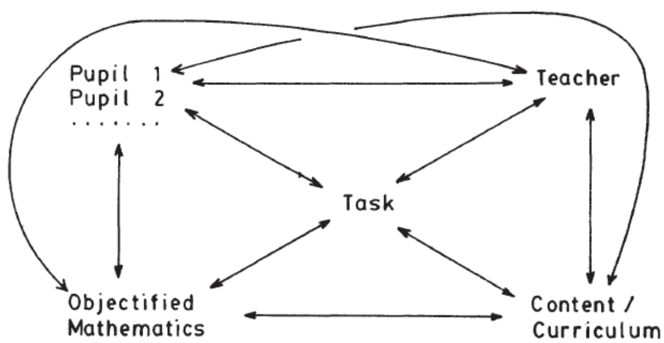


Figure 2. The relational character of task and activity (Christiansen & Walther, 1986)

A *task* is the "goal of an action, with the goal being framed by distinct conditions" (p. 256). Specifically, a task is the assignment set by the teacher, which is the object for students' activity. A mathematical task generally includes one or more problems whose solving is expected (by the task designer) to involve mathematics. The task also includes a set of instructions, directives, and/or extensions to which learners are expected to respond. Two caveats here: (1) how explicitly the goals and conditions of the task are communicated varies widely, and (2) replacing "mathematics" with "MKT" or "MKT-FT" in the paragraph above provides parallel definitions for tasks in the context of college instructor professional development for teaching.

Activity is a process that includes reactions and adaptations by the student that are in response to the changes in task conditions that arise during the students' work on the task (these are theorized to be based upon student-specific needs and motives). Activity is realized through a collection of *actions*, goal-directed processes arising from the students' motives:

> Activity exists only in actions, but *activity* and *actions* are different entities. Thus, a specific action may serve to realize different activities, and the same activity may give rise to different goals and accordingly initiate different actions. (p. 255).

Each action in activity serves to attain a goal of the task: the collection of actions is goal directed and together forms a *plan*. For Christiansen and Walther, the teacher is the central agent of authority. We argue that in contexts where students are adults, the locus of control may well lie with the learner (e.g., future elementary teachers, faculty who are learning about teaching). And, social, mathematical, and socio-mathematical mediation occur among students and between students and instructor. That is, how an activity induces action depends on the agents and their relationships. Moreover, moving between actions and from actions to related plan (and back again) involves many decisions. Figure 3a summarizes our interpretation of the framework, overall, and Figure 3b illustrates one possible decision process across actions and planning.

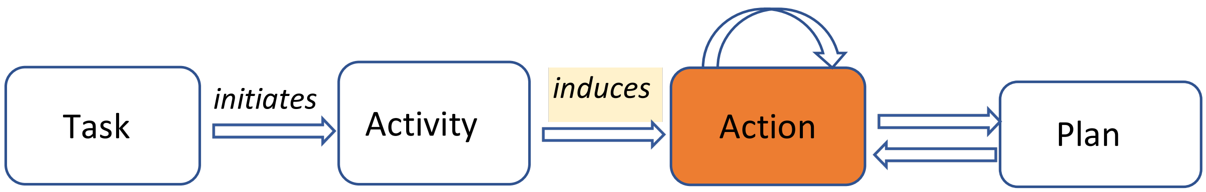


Figure 3a. Detailed task and activity framework.

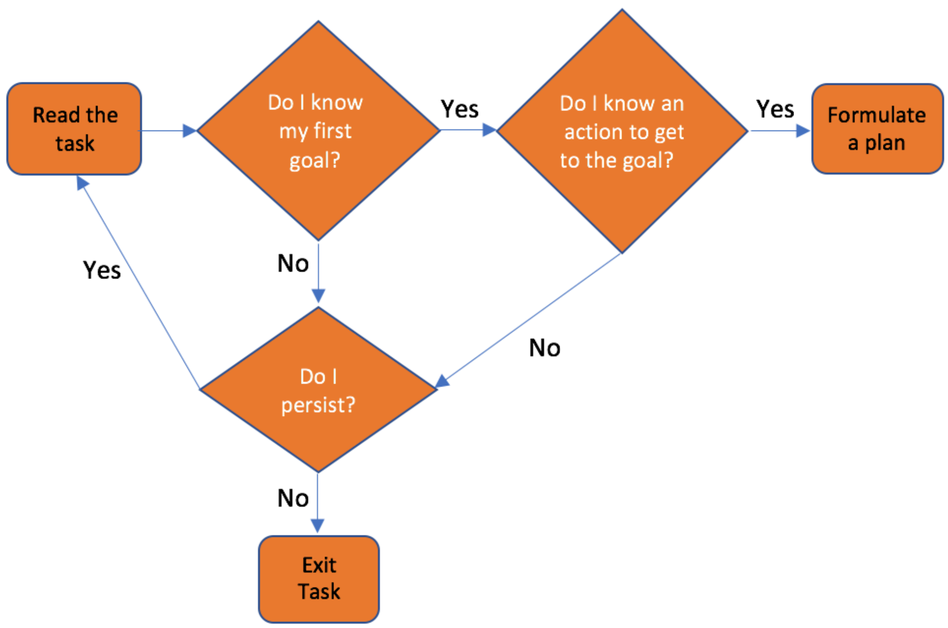


Figure 3b. Example Action – Plan feedback tree.

Christiansen and Walther (1986) offer different non-exhaustive types for each element in the framework. For instance, they distinguish different tasks by the type of mathematical activity in which students will engage: exploratory, constructive, or problem-solving. Smith and Stein (2011) offer a further delineation of problem-solving tasks as those that (a) call on memorization, (b) use procedural knowledge but require limited connections to other knowledge, (c) require procedural and connected knowledge, and (d) engage students in actually doing novel (to the learner) mathematics by calling for conjecturing, reasoning, and justification.

For Christiansen and Walther, *educational activity* is what leads to work in response to a task-driven behavioral goal (e.g., produce a graph), while *learning activity* is activity that results in someone achieving the intended learning outcomes. When engaged in an activity, learner actions may be *preparatory, observational/reflective, control-focused, safeguarding,* or *corrective*. Preparatory actions are those that establish conditions for success or which facilitate another action (e.g., *formulating a plan* is a preparatory action). Observational and reflective actions develop or identify information needed to complete or plan other actions. Safeguarding actions ensure that information and results obtained along the way in the task are readily available to the learner later in the task. Control actions are calibrations: learners compare the intended goals/actions with those that were actually achieved/performed. Corrective actions refer to acts by learners to anticipate or remove possible errors.

As an illustrative example of the framework, suppose an instructor of pre-service elementary teachers gives students a collection of questions similar to the one in Figure 4.

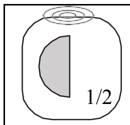
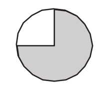
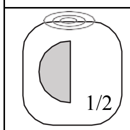
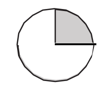
	Suppose a whole serving is 1/2 of a cookie. How many servings (whole or fractional) can I make from 3/4 of a cookie?	
	Suppose a whole serving is 1/2 of a cookie. How many servings (whole or fractional) can I make from 1/4 of a cookie?	

Figure 4. Fraction word problems (adapted from Gregg and Gregg, 2007).

Notice that based upon definitions given here, a collection of a dozen such word problems, in and of itself, does **not** constitute a task because there are no instructions or extensions asking for response/resolution. However, the collection might be transformed into a task with an MKT development goal by inducing two different activities:

1. Suppose you are a 6th grader who is completing this activity for the first time. You have never been exposed to an algorithm for fraction division, and so you do not have that knowledge going into the activity. Do the activity accordingly.
2. What algorithm for fraction division does the activity suggest would be appropriate? HINT: To answer this, think about HOW you got your answers to each of the questions.

Now students are being asked to do more than to solve problems. They are required to engage in several activities. One activity introduces planning and actions for imagining (and then thinking like) a 6th grader, and as an extension, deducing the common denominator algorithm from the task by thinking about another (imaginary) person's solution process. In the course of completing the task, common learner actions tend to include reflection on how what they have done in solving each of a parallel set of problems about serving size as it relates to an algorithm, safeguarding as they search for patterns to determine the algorithm, and control actions as they begin to think about what a typical 6th grader might know.

Characteristics of Task-based Learning (TBL)

Having now established definitions and a framework relating task and activity, we are in a position to elucidate the defining characteristics of task-based learning:

- Learners work on a task collaboratively (usually in groups of 2 to 4 members). Often tasks will include activity with manipulatives, video, and/or other technology.
- As learners work to complete the task, they consistently engage in activity that is mathematical and/or pedagogical in nature. The task is designed to elicit actions such as sense-making, conjecturing, reasoning, justifying, problem posing, questioning, challenging, role playing, reflecting, and anticipating.
- The task makes explicit queries about the nature of learners' thinking, reasons for steps they take, and what they produce as they work to complete the task. Teacher utterances include challenges to student productions, questions that extend activity or call for re-planning, and brokering guidance for struggling students.

Note that the first element is collaboration – working together towards a group goal or outcome. This is different from cooperation – working together for mutual benefit towards individual goals/outcomes. Both can be powerful supports for building community (Banilower et al., 2013).

Juxtaposition with Other *-Based Learning

Task-based learning certainly shares characteristics with most uses of "inquiry-based learning" we have encountered. It requires inquiry-oriented instruction (Rasmussen and Kwon, 2007) in that teacher and student play important roles in the process. Within the science education community, inquiry-based learning is often categorized as *structured*, *guided*, or *open* (Biggers and Forbes, 2012; Chinn and Malhorta, 2002; Kuhn, Black, Kesselman, & Kaplan, 2000). In structured inquiry, the instructor provides the materials and procedures necessary to complete the task, with the expectation that students will discover the intended learning outcomes in the process. In guided inquiry, the instructor poses a problem and provides necessary materials, leaving students to devise their own solution methods. In open inquiry, students pose their own problems and seek their own solutions. By design, TBL is either guided or structured, depending on how the task is presented to the learner. This is in contrast to *problem-based learning* which is an open model starting with something problematic for the learner rather than *problems,* which are the starting point for TBL. Likewise, TBL is different from *project-based learning* because tasks as defined here are not generally projects that require synthesizing significant amounts of information over time.

Every task starts with a novel (to the learner) problem (i.e., not an exercise involving a single stream of well-rehearsed actions). The activity and actions of students required in TBL ensure that they are doing mathematics. Actions that occur during task activity form the basis for self-regulation, a critical component of metacognition which is crucial for effective and efficient problem solving. Self-regulation is a behavior that can be acquired over time as learners engage in authentic problem solving regularly (Schoenfeld, 1992). That is, repeated exposure to tasks that scaffold agency and self-regulation can support the taking up of agency and self-regulation. The teacher's role in TBL mirrors that in teaching problem solving: as a cultural broker of mathematically rigorous meaning and facilitator of self-aware use of mathematical language.

Enacting or assigning tasks does **not** guarantee learning. In much the same vein, presenting students with problems to solve does not constitute teaching of problem solving. Other criteria must be met by instructor and students. For example, having future teachers use base ten blocks to demonstrate operations does not mean they can explain common algorithms for the operations. Concretism does not always ensure that intended learning activity will follow. To achieve the desired activity and, ultimately the goal learning outcome, it takes focused effort by the expert (teacher, instructor, facilitator of professional development) during activity in the task to direct attention as needed. Thus, task-based learning for faculty, where the goal is to build MKT-FT must do more than tell participants to watch some mathematics classroom video and reflect on it (Seago, 2004). Specific prompts before video viewing might direct people to prepare themselves to notice and identify evidence of student thinking about the meaning of slope. Twice. That is, the task includes purposeful repetition of activity. The prompt for two viewings makes explicit the goal and sets expectations that participants will do a particular kind of intellectual work (notice, identify) about particular aspects of the video (student utterances and actions that can be considered evidence, slope). These prompts are intentional in preparing the participant for possible extensions like: Create at least two potential responses to the noticed thinking.

Why Promote TBL Among Mathematics Instructors and in Professional Learning?

First, TBL is a form of active learning and active learning has been shown to significantly improve undergraduates' performance in science, technology, engineering, and mathematics courses by half a letter grade (Freeman et al., 2014). Second, the recent *Standards for Preparing*

Teachers of Mathematics by the Association of Mathematics Teacher Educators (2017) calls for the use of task-based learning in courses for future teachers:

> In such settings, learners are typically provided challenging tasks that promote mathematical problem solving and … discuss their thinking in small and full-group discourse, thus promoting important mathematical practices (Webb, 2016) (p. 31).

As does the *Mathematical Education of Teachers II* (MET II, 2012):

> Courses should also use the flexible, interactive styles of teaching that will enable teachers to develop [mathematical] habits of mind in their students (p. 19).

Indeed, a task-based approach empowers the skilled teacher to meet many (if not all) of the criteria in the Teaching for Robust Understanding (TRU) framework for high quality instruction (Schoenfeld, 2014, 2017). Moreover, Connolly and Millar (2006) noted that faculty in teaching workshops wanted professional development that used the TBL methods being advocated in the workshop. In a current project by the authors, we are offering faculty a task-based approach to professional learning about task design and task use in their own classrooms.

Conclusion and Avenues for Further Investigation and Research

We end by giving some examples of tasks for faculty professional learning. Our focus is ways to teach mathematics courses for future K-8 teachers. Note that the main goal of these professional learning tasks is not to build mathematical knowledge, but to foster development of MKT-FT. A task requires a problem. Rich problems of instructional practice might center on pedagogical content or specialized content knowledge for teaching future teachers or building understanding of the MKT that future teachers need.

In this spirit, consider the cookies task discussed earlier. In a task for faculty, participants are asked to "put on your student hat" and do the task. Then the nature of the task and activity are discussed. Then comes a meta-aware extension to the task, "Imagine you are a pre-service teacher and have been given this task, what is challenging? Why?" Faculty work involves *knowledge of content and (pre-service teacher) students,* a component of MKT-FT parallel to knowledge of content and students in MKT. Faculty then read a transcript of pre-service teachers completing the original task. They identify things that the pre-service teachers struggled with and compare that with their anticipations. The task has two intended learning outcomes: (1) faculty build knowledge of (pre-service teacher) student thinking and (2) faculty unpack the demands and consequences of designing/revising tasks for achieving particular learning goals.

Areas for use of the TBL framework in research and development include addressing questions such as: How do designers and facilitators know that they are effectively implementing task-based learning in faculty professional development? What constitutes evidence of this? Also, what are indicators of success of a task-based professional learning experience? The productive use of tasks by participants in their own practice is one important factor, but are there others? Finally, the scant research literature on professional development for teaching in higher education has yet to delineate the conditions that promote (or hinder) faculty success. For example, what experiences and supports may be needed for faculty to use, as an instructor, a TBL model they have experienced as learners in professional development?

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