Theorizing Silence

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Different norms govern the use of silence in mathematical collaboration and in every-day Anglo-conversation. Research is therefore needed into the ways students are enculturated into the distinctive uses of silences employed in mathematics collaboration. This project will require a new theoretical perspective that facilitates the study of silence. Drawing off studies of silence and embodiment from multiple disciplines, this paper advances a view of silence and the body, and so lays the groundwork for a rigorous study of silence in mathematics education.

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Two recent conference papers (Petersen, in press; Lim, 2017) have raised the prospect that the mathematics education community may benefit by attending to silence. First, Petersen argued that while engaged in intense collaborative mathematical activity, mathematicians remain silent for lengthy periods of time; a practice at odds with the every-day Anglo conversational norm that exactly one person speak at a time (Erickson, 2004; Liddicoat, 2011). This disconnect between the norms of every-day Anglo conversation and mathematical practice makes mathematicians' collaborative silences pedagogically interesting. Second, Lim, argued that though the reform movement has done a good job giving students voice in the classroom, introverted students who value silence and careful thought, may have a difficult time in reform classrooms.

As I have thought about these issues over the last year, two anecdotes have helped convince me that silence is in fact an important, though understudied aspect of mathematical activity and of learning to be a mathematician. First, while discussing silence with a mathematician, he told me that he had the distinct impression of being apprenticed into silence while working on both his Master's degree and Ph.D. The second episode occurred while tutoring students in a 300-level proof class. On one occasion, as I attempted to answer student questions, I realized that the solution I had worked out in advance was incorrect and, with the mathematics I then knew, I was unable to address the students' questions—though I could tell that my error was small, and relatively easy to fix. My natural strategy was not to speak, but to perform mathematical activity by attending to the problem closely and carefully, in silence. The students, however, responded to my silence as a lapse in conversation, and repeatedly attempted to engage me in further conversation. To my surprise, I quickly realized that though, on my own, I would be able to fix the bug in my proof quickly and easily, the interactional requirements of the tutoring situation, and the norms governing the conversation, made me incapable of performing the mathematical activity necessary to adequately address my students' questions.

Though these are just anecdotes, both stories provide corroborating evidence that in learning to collaborate as mathematicians, students need to learn to employ silence in ways that violate the norms of every-day Anglo-conversation. The second, in particular, points to potential difficulties students may face as they attempt to collaborate on mathematics. If research is to be conducted into silence, strong new theoretical and methodological papers are needed. This paper attempts to make a beginning in providing a theoretical foundation for the study of students and mathematicians uses of silence.

Communities of Practice

Several important strands of research in mathematics education attend to the way students learn mathematical ideas and concepts in communities of practice. Several are particularly relevant to research on silence.

First, research in the emergent perspective (Cobb and Yackel, 1996; Yackel and Cobb, 1996; Voigt, 1985, 1989, 1995) attends to sociomathematical norms, that is, ways students and teachers negotiate what sorts of answers are normatively treated as expressing mathematical concepts and practices (such as justification) (Cobb and Yackel, 1996; Yackel and Cobb, 1996; Voigt, 1985, 1989, 1995). If, as previous research suggests (Petersen, in press), there are distinct norms governing silence in face-to-face mathematics collaboration that differ from the norms governing silence in every-day conversation, there are peculiar sociomathematical norms governing silence, and, as students become mathematicians, they are enculturated to those norms. On the other hand, Yackel and Cobb attempt to "account for how students develop specific mathematical *beliefs and values*" (p. 458, emphasis mine), whereas though silence may be used peculiarly in mathematical collaboration, it does not itself signify any mathematical reality.

This focus on overtly mathematical aspects of the classroom, however, does not preclude attention to what Voigt (1985) calls "patterns of interaction", e.g. questions of who is authorized to speak when, or how much wait-time teachers give their students, behavioral patterns that are not overtly mathematical in nature, but which give the classroom a particular order in which explicitly mathematical practices can be learned. These patterns of interaction which undergird mathematical activity, however, are not strictly mathematical, and so the norms governing them are not sociomathematical norms; whereas, if the norms governing silence are discipline specific, because they are an aspect of mathematical activity, they are as sociomathematical norms.

On the other hand, a very different line of research in mathematics education attends not to mathematical concepts and beliefs, but to mathematical *activity* (Rasmussen, Zandieh, King, & Teppo, 2005). In learning mathematics, students and teachers engage in activities endemic to the mathematics profession, like justification, algorighmatization, and defining.

Like that from Cobb and Yackel's emergent perspective, research from this perspective focuses on overtly mathematical aspects of learning. This line of research, however, opens up the possibility that actions that are not overtly mathematical nevertheless play an important role in allowing people to perform mathematical actions, and so are an important aspect of mathematical activity and an important line of research in mathematics education. Though not from the same perspective, Savic (2015), can be read as an existence proof for this sort activity. He found that when mathematicians reach a proving impasse, they will sometimes resolve the difficulty by stepping away from the problem and doing something else, e.g. taking a walk, going to lunch with their family. Savic's research does not address the potential for aspects of mathematical activity that are *socially* interesting, and that require students to learn new sociomathematical norms, but together with Cobb and Yackel's research into sociomathematical norms, it raises the prospect that, as part of their collaborative mathematical activity, mathematicians follow norms contrary to the norms used in every-day interaction. If the results in Petersen (in press) hold up, silence falls into this category. This claim, however, needs unpacking unpacking.

Philips' (1792; 1983) ethnography of education on the Warm Springs Indian Reservation in Oregon provides helpful information regarding the ways different norms for silence can structure classroom interaction. According to Philips, the Native community she studied placed a high value on the difficult skill of effective, brief, speech; and therefore, lengthy pauses often preceded responses. On the other hand, in Anglo-conversation, pauses longer than a second carry meaning, often indicating a dispreferred response (Liddicoat, 2011). This disconnect between the norms governing silence for Anglo teachers and their native children meant that student silences, directed at both teacher and peer as signs of respect, were read by the teacher as signs of incompetence. Furthermore, the Anglo teachers would often cut native student's silences short, thus depriving them of the chance to speak. It goes without saying that, in the university mathematics classroom, the power relations are very different than in the elementary classrooms Philips studied, however, this example illustrates the possibility for deep miscommunication caused by different norms for silence. Furthermore, if those norms are specific to mathematical activity, they are sociomathematical norms.

Silence

Silence seems difficult to study scientifically for two reasons: First, silence seems to be the lack of speech or of sound and not a phenomenon in its own right. This issue has theoretical and methodological aspects: What is silence, and how can we attend to it? Second, silence does not regularly signify anything mathematical, and if it does, it only does so accidentally. It seems therefore, silence should be addressed when it happens to come up, but should not be a topic of research in its own right. This section will address the theoretical aspects of the first question, whereas the subsequent section will address the second question. A separate paper will be required to address the methodological aspect of the first question, though the final section of this paper provides a sketch of a methodology.

Silence is not a mere absence or a lack (Acheson, 2008; Ephratt, 2011), but a phenomenon, actively heard with our ears, that both frames sounds and words, and is in turn framed by sounds and words (Acheson, 2008; Chrétien, 2004). So, for instance, as Dauenhauer (1980) notes, a performance of music is only heard as a unity because of the silences that bracket it. On the other hand, Handel often underscores dramatic moments in his music with lengthy silences (Harris, 2005), which are only heard as dramatic parts of the music because they are surrounded by sound (cf. Kim, 2013). Nor is silence not one-dimensional: The sorts of sounds that bracket a particular silence, and the posture and gestures employed during a silence, give a particular color and meaning to silences (Margulis, 2007a, 2007b; Acheson, 2008). Finally, silence is not a default state, but is actively produced. Thus, for instance, silence can be a design feature of buildings (Kanngieser, 2011; Ergin, 2015; Meyer, 2015; Bonde & Maines, 2015); and we are all familiar with how difficult holding our tongue can be.

Because silences are actively produced and heard, they can bear particular meaning (Acheson, 2008; Ephratt, 2011). They are therefore perhaps best understood as a particular sort of gesture; a gesture which we can perform in concert with others, or alone while others are speaking (Acheson, 2008). For instance, Quaker worship is structured by lengthy collaborative silences (Lippard, 1988), and the bond of a nursing mother with her infant can be strengthened through mutual eye-contact and silence (Maitland, 2008); while, on the other hand, the children Philips (1983) studied on the Warm Springs Indian Reservation communicated that they were actively listening, not through eye-contact or back-channeling (e.g. "mhm"), but through silence.

Embodiment

Two recent papers (Abrahamson & Sánchez-García, 2016; Abrahamson, Sánchez-García, & Trninic, 2016) have called for attention to the ways students develop their bodily capacities and so, open up new avenues for action in the world. In this call, they have opened up a new avenue for research into the body in mathematics education, the ways skilled uses of the body are a prerequisite for mathematical learning, and how, by training our bodies to be capable of new actions, new affordances for action are opened. Following their lead, mathematics education research can attend to the various ways we train our bodies to perform otherwise difficult actions which can subsequently give rise to mathematical meaning. Since acting according to new norms is difficult, this perspective is a helpful starting point for theorizing silence.

There are, however, two aspects of their theoretical perspective that make it inadequate for theorizing research on silence. First, they do not attend to the ways bodies are used in social interaction. But silence is an interactional accomplishment and challenge. It is relatively easy, however, to modify their perspective to incorporate social interaction. As McDermott (1978) notes, in interaction, we are the environment in which our peers act. Thus, developing new ways of acting in the world means developing new ways of acting on our peers, and of being acted on by our peers. Abrahamson Sánchez-García's (2016) perspective can be modified to say that as we acquire new skilled uses of the body, new affordances are opened up not only for learning mathematics, but for orienting ourselves and our peers collaboratively toward mathematics.

Second, their focus is still on actions that signify mathematical realities—the actions just do not yet have mathematical significance when learned. But there is another way actions, in our bodies both natural and social, can be connected to doing mathematics: They can order the parts of the body in a way that gives the capacity to do mathematics. Morgan and Abrahamson (2016) take something like this tack in their preliminary investigation of the ways meditative practices like tai chi and yoga could be utilized to enable students to engage with difficult mathematics, and Savic (2015) showed that not doing mathematics is an aspect of doing high-level mathematics. But otherwise, I have not encountered research that examines ways the body is used to give agents the capacity to do mathematics; and none that examine interaction. However, ordering the parts of our bodies, natural and social, in a way that facilitates the doing of mathematics is a necessary condition for doing mathematics, and so is a valid topic for mathematics education research. Furthermore, as noted above, if in doing mathematics, the body social is ordered in a novel way that relies on social norms different from those used in every-day interaction, this order, and the way it is learned, is educationally *relevant*.

While little mathematics education research that attends to the ways bodies are utilized to give an agent, or a group of agents, the capacity to perform mathematics, this perspective on the body is akin to some perspectives employed in anthropology. In particular, Marcel Mauss' (1935/1968) concept of a *habitus*, a pre-reflective, bodily know-how, that gives a subject the capacity to engage in an activity, has proven fruitful in examining a number of different phenomena, e.g. the transmission of oral literature (Saussy, 2016), and to the mosque movement (Asad, 2003; Mahmood, 2005). Mauss's concept also has a deep resonance with the theorization of the body Targoff (2001) employed in her investigation of poetry and prayer in early modern England (Mahmood, 2005). Finally, though not related to Mauss, Esaki (2016) argues that Japanese-American gardeners employ silence to give them the capacity to tell what sorts of cuts they should perform on their trees.

Interaction

The issues surrounding silence, however, are not individual, but arise in interaction. Petersen (in press) argues that, while engaged in intense mathematics, mathematicians collaboratively engage in lengthy silences, in violation every-day conversational norms. How do students learn these norms? And how does conflicting interpretation of silences, and conflicting norms governing its use, influence students capacities to engage in mathematical activities? In order to address these questions, we need to theorize not only the body, but interaction. In this section I will argue that the claim that interacting participants form a complex dynamic system, or what some researchers call a synergy (e.g. Chemero, 2016), is a plausible hypothesis.

A pair of recurring question in behavioral sciences concern the mechanisms involved in the bodily coordination presupposed by the pursuit of a behavioral goal, either by an individual or by a group of individuals (Takei, Confais, Tomatsu, Oha, & Seki, 2017; Ashraf et al., August 24, 2017). Though the addition of multiple agents makes the second question more complex, there are reasons to believe that similar dynamics underlie both. As Marsh (2015) claims "in both cases, some kind of information…leads to entrainment; each involves the creation of a coordinative structure or synergy" (p. 321).

Researchers studying the material aspects of interaction have found that participants mutually entrain multiple aspects of each other's movements, including posture, limb-movement, speaking rate, vocal intensity, and, critically, length of silences (Marsh, Richardson, & Schmidt, 2008; Shockley, Santana, & Fowler, 2003; Sebanz, Bekkering, & Knoblich, 2006; Shockley, Richardson, & Dale, 2009; Noy, Dekel, & Alon, 2011; Fowler, Richardson, Marsh, & Shockley, 2008; Schmidt & Richardson, 2008; Capella & Planlap, 1981). What functional goal has this entrainment evolved to serve? The answer seems to be that it allows people to join together in a common activity, in pursuit of a common good (Richardson, Dale, & Marsh, 2014). This hypothesis is, partially, confirmed, by a recent paper on professional string quartet performance (Chang, Livingstone, Bosnyak, & Trainor, 2017). They demonstrated the body-sway of the musicians is auditorily and visually coupled, is a tool musicians employ to shape performance, and more coupling is correlated to the musicians' perception of successful performance.

These results in mutual entrainment allows the tentative conclusion that the body functions, in part, to knit people together into a body social with a common end, either through the mediation of shared representations, or immediately by allowing them to engage in joint activity (Marsh, Richardson, & Schmidt, 2008). Because research in silence is not attending to mental constructions, the second option seems to better fit for research on silence: In interaction, we utilize our bodies not only to signify the world, but to order each other and ourselves toward a common good, e.g. discovering and proving a new theorem, symbolizing and defining a mathematical object, etc. (cf. Rasmussen, Zandieh, King, & Teppo, 2005), and so to give a body social, and its individual members, the capacity to pursue that good.

This theoretical perspective on interaction requires that the activity of the body social—the linked dynamic system (Richardson, Dale, & Marsh, 2014), synergy (Chemero, 2016), or teleodynamic system (Walton, Richardson, & Chemero, 2014)—be the unit of analysis, not the isolated actions of the particular persons in the interaction. However, because the individual mathematicians are material parts of the body social, the analysis cannot be carried out in abstraction from the bodily actions of the individual mathematicians. Rather, the unit of analysis is the body social precisely because the actions of the individual mathematicians are treated as constraining, and constrained by, the activity of their peers. As certain activities—say, in piano playing—are not difficult for each hand individually, but in the coordination between hands; so some activities that are not difficult for individuals, when working alone, may be difficult to achieve *in common* (Marsh, Richardson, & Schmidt, 2009). On the other hand, because of the mutual entrainment, social order belongs to the body social—that is, to the dynamic system—and cannot be understood merely as the work of the individual participants, considered in isolation. Two points are key here: First, if peers engaged in joint interaction act according to different interactional norms, joint action may be particularly difficult. Second, when participants in face-to-face collaborations engage in high-level activity, and act according to the same norms, the fact that each member of the body social perceives the others are engaged in the same activity should serve to strengthen their own engagement (Walton, et al. 2014).

This theoretical position is heavily influenced by, and very similar to, the position McDermott (1978; McDermott, Gospodinoff, & Aron, 1978) employed in his ethnographic descriptions of a classroom, and to Erickson's (1996, 2004) microethnography. McDermott (1978), attended ways two teacher and student reading groups established an order through the postural positions of each member of the groups and the way the orders facilitated, or did not facilitate, learning to read. Similarly, employing the concept of the *habitus* mentioned above, Erickson (2004) argues that when there is a disconnect between the *habitus* actors attempt to employ in joint activity, "seemingly automatic workings of the players' *habitus* are no longer effective for engagement in the collective activity…If the player is to be able to stay in the new game, that player's *habitus* must change" (p. 12).

On the other hand, it shares similarities with several influential perspectives in mathematics education, while differing from them in key respects, namely Cobb and Yackel's (Cobb & Yackel, 1996; Yackel & Cobb, 1996) emergent perspective on classroom activity, and a Realistic Mathematics Education (RME) (Rasmussen et al., 2005) discussion of mathematical activity.

My research shares with Cobb and Yackel an emphasis on the ways participants in concerted activity co-create the activity, mutually conditioning the activity of all the others, and so forming the group into a single "dynamic system" (Yackel and Cobb, 1996, p. 460), and with a concern with the sociomathematical norms that govern this activity. It differs from them in two key, interconnected, respects. First, though Cobb and Yackel are concerned with one aspect of the way students and teachers mutually position each other around mathematics; the bodily aspects of that activity are not relevant to their investigations. But research into silence attends to one aspect of the way individual students and mathematicians engaged in mathematical activity hold themselves, physically, and so mutually orient themselves and peers toward doing mathematics. Second, their fundamental goal is to determine how mathematical *beliefs* are learned in learned in concert; whereas my focus is more like Rasmussen's research, in that it is focused on joint mathematical *activity*. This second difference shapes a methodological divergence: Whereas they envisage zooming in to a psychological investigation of student beliefs and understandings; I envisage zooming in from an investigation of the materiality, including the silences, of interaction, to an investigation of the bodily activity and gestures, including silence, of each individual mathematician or student.

Second, this perspective is closely related to a RME understanding of mathematics not merely as individual belief, but as particular sociocultural activity (Rasmussen et al., 2005). The key difference is that their work is focused on a different aspect of mathematical activity than research into silence is. Though they sometimes attend to gestures (e.g. Rasmussen, Stephan, & Allen, 2004), these gestures are relevant because of their ability to symbolize and communicate mathematical realities, whereas whether a student working to mathematize is, at that instant, seated or standing, motionless or pacing, etc. is irrelevant. Cooperative student mathematizing is, however, supported by shared bodily orientations that order group participants toward the mathematics at hand, norms regarding what bodily actions are appropriate, etc. It is to these norms that facilitate mathematical activity that research into silence should attend.

Sketch of a Methodology

The first methodological challenge a study of silence faces is that traditional transcripts make silences invisible, rather than highlighting them (Ochs, 1979). A new form of transcription is therefore required that highlights silences, both collective and individual, and the postural nuances that give silences distinct characteristics. Figure 1 contains a sample transcript from three calculus students' attempt to identify which of three functions represent the position, velocity, and acceleration of a car. Not all the transcription conventions are relevant, but the following are most salient: Individual students' verbal utterances are placed in columns on the left, and non-verbal gestures, on the right. Footnotes show when, relative to the speaking and silences, individual gestures occurred. Mutual silences are highlighted in dark grey, and their length notated in the center column. Silences that do not include all students are highlighted in light grey, and their duration indicated to the left of the column. In the gesture columns, "eg" abbreviates "eye-gaze". These conventions highlight silence and allow its investigation.

In this episode, how do the three students respond to the lengthy mutual silences at the beginning? Andy, responds to the silence by speaking, approaching the board and seeking eye-contact with his peers (bottom row; 2, 3, 5, 7, 8, 10). Jason and Katherine, however, remain mostly still, do not move in response to Andy, and avoid eye-contact with him. A much longer analysis of this episode is possible, but these facts suggest Andy responds to the silence as an awkward pause, and attempts to resume the lapsed conversation; whereas Jason and Katherine, treat the silence as a part of their mathematical activity, and seek to continue it. This leads to the tentative conclusion that Jason and Katherine hear Andy's talk as an interruption of their silent mathematical activity; whereas Andy hears Jason and Katherine's silence as silencing him.

Conclusions

Silence, though a peculiar aspect of mathematicians collaboration, does not fit well with existing theoretical perspectives in mathematics education research. Novel theorizations are therefore required for the study of silence. This paper provides the beginnings of a new theorization of silence, and the body, and provides a brief analysis of silence.

Verbal				Non-Verbal		
Andy	Jason	Katherine		Andy	Jason	Katherine
[1]	of [1]**b** //but] **c** is still °just the derivative of **a** [a]ri:ght°[z]	\\of **b**]		[1]steps toward graph	[a]turns head slightly to right (still at board) then motionless	[z]starts slowly rocking back with rigid body
35 Seconds		2 Sec	2s			
[1]		[z1]>right<		[1]raises paper		[z]pulls arm back (hand still pointing) nods
[1]	10 Seconds [a]	[z]	3s	[1]steps back ↓eg to paper	[a]tilts head to right ^squares toward board	[z]lowers hand, tilts head to left.
<so> [a](.) anyway[1] I'm just gonna go back to the basics here[2]-	[a]			[1]hands apart [2]steps forward pointing toward graph	[a]^squares hips toward board, crosses arms, tilts head slightly left.	
	Actually (.) ↑well,[z]	1 minute, 44 seconds [z]				[z]slight pulse of left hand (otherwise motionless)
So we do know we have a [1]velocity [2](.) [3]'cause this graph and we have a position[4] from that based[5] on all of our data from[6] all these[7] little tangles and everything based[8] on the position[a] well[9] we [b]can say that (.) and then[c] we[z] have >an acceleration< of **c**[d10] //so[11]	30 Seconds [a] [b] [c] [d]	[z]		[1]points to where they have written "velocity" [2]→eg peers, ↑back to board [3]traces along graph a [4]beats near word "position" then waves at graph [5]→eg to peers [6]↑eg to graph touches graph **a** with pen [7]draws lines down from **a** to **b** [8]traces hand along **b** [9]pulls hand back then gestures forward & turns hand over [10]traces hand along **c** [11]scratches nose	[a]head right [b]weight to right foot ↓eg toward floor [c]↑weight on both feet eg and pelvis toward board [d]turns head very slightly to right	[z]steps back, head straight (eg still board), crosses arms.

Figure 1: A transcript of three calculus students attempt to determine which of three sketched functions represent the position, velocity, and acceleration of a car.

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