Examining a Mathematician's Goals and Beliefs about Course Handouts

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In this qualitative narrative study, we employed Schoenfeld's theory of Resources, Orientations and Goals (ROGs) to analyze a mathematician's beliefs and goals in creating handouts for his students. Some of the instructor's primary goals in creating the handouts were: (1) to help students gain an intuition about Algebraic Topology, (2) to provide a resource for students to revisit the difficult material outside of class, and (3) to prompt students to complete exercises so that they could monitor their own mastery of the course content. As part of this study, one of the students in class took daily journals. These journal entries revealed that he appreciated the time and careful preparation that was necessary to create the handouts, particularly the pictures that the instructor drew in the margins to help students gain an intuition. However, one obstacle that the student faced was struggling to appreciate the instructor's goal of expecting students to monitor their mastery of content outside of class time through completion of ungraded exercises in the handouts.

Keywords: Algebraic Topology, handouts, beliefs, goals

Theoretical background

Giving out handouts is a common practice in many mathematics classrooms. In his book, *Mathematics Teaching Practice: A Guide for University and College Lecturers*, Mason (2002) shares a variety of reasons why mathematics instructors give handouts to students (see Table 1). In his view, "people have a mixture of aims, and so use different approaches at different times" (p. 64). Mason differentiated among handing out complete notes, writing everything on the board, and giving no notes during the lectures. In the case of providing complete notes, students may not see the need to attend lectures; in the case of writing everything on the board, students will turn into transcribers; in the case of providing no notes, students have nothing to fall back on to make sure they are gleaning the important points from the lecture. Teaching is a complex activity and clearly designing handouts and successfully implementing them in lectures requires careful thought.

Table 1. Pedagogical insights in using handouts in lectures (Mason, 2002, p. 64).

Aims	Expected Actions by Students	Styles of Handouts
Cover (explain, teach, transmit, or convey) the definitions, theorems, proofs, and techniques	Study (not just read) notes mathematically; work mathematically on set exercises	Complete notes as if in a book (available in advance or after the lecture) Definitions, theorems, and sample worked examples
Inspire	Appreciate overall flavour; pick up details from carefully working on notes and perhaps texts, not just working through exercises	Notes with headings but details left as spaces for students to fill in as you work through the exposition

		Explicit references to standard texts Extra suggestions not mentioned in the lecture
Work at understanding, making connections between topics or theorems	Re-construct topics for themselves from lecture notes and text, and, increasingly, independently from text alone	A succinct mathematical summary, perhaps with worked examples or challenging questions to explore
Teach how to carry out required techniques and solve sample problems	Work on 'worked' examples; justifications and theory found in text	Elaborated worked examples displaying choices, wrinkles, and use of theorems

The current qualitative narrative study is a part of a larger study with the main goal of understanding the mind of a working mathematician as he made pedagogical decisions (Stewart, Thompson, & Brady, 2017). In this paper, we describe what motivated a Geometer to design and employ 35 handouts in an Algebraic Topology course. In our holistic approach to investigate this instructor's teaching, we examined his handouts, his weekly teaching journals, and discussions that occurred during weekly meetings with a team of researchers. Additionally, we examined one student's daily journals to get the student's perspective on the instructor's handouts. We do not aim to prove or disprove whether handouts are ideal educational resources, rather, our ultimate goal is to understand the mind of the working mathematician by investigating what motivated him to create detailed handouts. To analyze the Geometer's motivations, we employed Schoenfeld's (2010) Resources, Orientations, and Goals (ROGs) theoretical framework. This theory helped us identify the knowledge and materials at the instructor's disposal, his values and beliefs, and what he wanted to achieve with the handouts. Schoenfeld claims that "if you know enough about a teacher's knowledge, goals, and beliefs, you can explain every decision that he or she makes, in the midst of teaching" (2015, p. 229). Resources, or knowledge, include "the information that he or she has potentially available to bring to bear in order to solve problems, achieve goals, or perform other such tasks" (Schoenfeld, 2010, p. 25). Orientations are "dispositions, beliefs, values, tastes, and preferences" (Schoenfeld, 2010, p. 29). Goals are what the individual wants to achieve. Although, the theory was originally applied to middle and high school teaching, (Aguirre & Speer, 2000; Thomas & Yoon, 2011; Törner, Rolke, Rösken, & Sririman, 2010), it has more recently been applied to university teaching (e.g. Hannah, Stewart, & Thomas, 2011; Paterson, Thomas, & Taylor, 2011).

Our current research questions are: What were the instructor's ROGs in making the handouts? Was the student aware of the instructor's goals for creating the handouts, and what were his reactions toward the handouts?

Method

In this qualitative narrative study (Creswell, 2013), our research team analyzed a Geometer's thought processes and pedagogical decisions while he taught a course in Algebraic Topology. The research team consisted of four members: a mathematics education researcher; a Geometer (the course instructor); a cognitive psychologist; and a mathematics postdoc. The Algebraic Topology course was the first in a two-semester sequence of courses; eight students were enrolled. During class meetings, the instructor (Noel Brady) passed out handouts to help students follow along with the topic of the day. Students actively solved problems together in groups, or individual students were called to the board to complete problems.

One source of data we analyzed was a series of teaching journals that contained the instructor's reflections on his preparations for class, what happened during class, as well as some descriptions of the events that took place during office hours. The research team read his daily journal entries and discussed them during weekly research meetings. During these meetings, the research team asked the instructor further clarification questions, and he often drew additional pictures as he described the course content. These meetings were audio recorded, and the meeting transcripts were also used as a source of data. Our team also analyzed data from a student in the instructor's class, who wrote daily journals. These student journals provided an additional perspective into the events that took place in class. The final source of data was 35 handouts that the instructor created for his students.

The data were analyzed thematically, meaning we mainly considered the key issues that emerged in this study. One of the main themes that emerged from this instructor's journals was "teaching". Forty-six percent of all instances from his journals were coded with the "teaching" code and 20% of those codes fell into the sub-category of "handouts/notes." More details about data coding and analysis is described in Stewart, Thompson and Brady (2017).

Results and Discussion

The instructor's resources included: (1) his knowledge of mathematics and the subject area, (2) many years of teaching experience, (3) course notes from when he was a student, (4) the textbook (Hatcher, 2001), and (5) many hand-drawn images. Analysis of the instructor's 35 handouts illuminated his motives. These handouts gave the research team a more authentic glimpse into the mind of the mathematician than his teaching journals. The instructor noted that he was self-aware when he wrote the journals, as he knew the research team would subsequently analyze and discuss them. On the other hand, he created the handouts solely for his students.

Apart from the usual dose of definitions, theorems, and proofs, the instructor's mostly handwritten handouts included headings such as, "intuition," "motivation," and "application," which are often lacking in textbooks. Table 2 summarizes the essence of the instructor's goals and beliefs as indicated in his teaching journal entries.

Table 2. The instructor's pedagogical goals and beliefs about handouts.

- Presented the information in a conversational tone
- Accompanied class activities
- Inspired by notes from his own graduate courses
- Contained relevant examples and exercises that referenced his published research
- The chosen textbook (Hatcher, 2001) gave a "muddled discussion" when the topic was "highly non-trivial and non-obvious" for the students
- Referenced alternative discussions of difficult topics that other mathematicians had posted on their websites
- Inspired by assigned homework
- The instructor wanted to present his students with "ultra-detailed" arguments that had taken him at least an hour to develop.
    - The handouts helped the instructor feel organized and not scattered.

o He was able to add onto existing handouts from semester-to-semester. o If he wanted to be particularly precise, he used LaTeX to type up the handout, but those handouts were time-intensive to create. • Some handouts synthesized information for the students. The instructor synthesized this material on his own when he was a student. • When the instructor was unable to cover all of the material that he wanted to in his lectures, he created handouts on the topic. o Handouts gave the instructor the license to go through the material more quickly because the students could revisit the information at their own pace outside of class time. • Helped students build intuition o The instructor drew images by hand that represented the complex mathematics. o The instructor noted that some students may be learning about the topics for the first time. • Handouts were helpful when the material was particularly complex. • The proof of the E-S axioms (for singular homology) was a component of the course. The textbook withholds the axioms until the end of coverage for singular homology, but the instructor decided to present them first. He believed that the axioms could serve as a framework, or table of contents, for the topic. • The instructor created a summary handout to give students a preview of what was to come in the second course on Algebraic Topology. He expected that the students would attempt to solve some of the problems when they were on break between the Fall and Spring semesters.

In this section, we will discuss some of the instructor's goals and beliefs in more detail and match them to the student's comments.

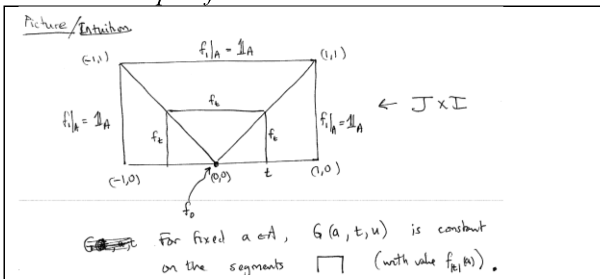
Helping Students to Build Intuition

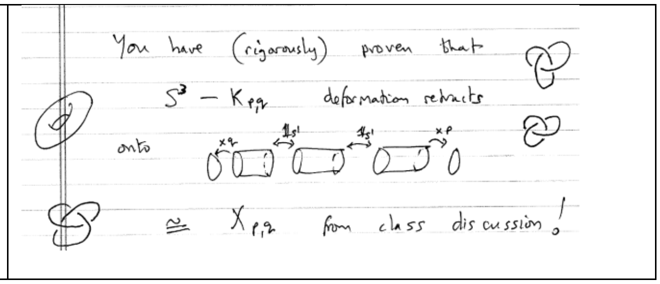
One of the instructor's main pedagogical goals was to help students build intuition. The instructor mentioned this goal often during the research meetings, which indicated his strong belief in and the importance he placed on helping his students build intuition. He drew images by hand (see Table 3) that represented the complex mathematics to help students who were learning about the topics for the first time. Research team members noticed that the instructor often used phrases such as "carefully and slowly" and "careful proof" in his teaching journals. These phrases indicated that he wanted to make sure students followed the arguments as they unfolded.

The instructor's comments	The student's comments
"Did some examples carefully and slowly, but told them that they have to get used to computing boundaries quickly and efficiently." "They still had difficulty going from an intuition to a formal proof, and I gave some	"I've so far enjoyed reading Dr. Brady's notes in the handout, which are rife with helpful commentary and ultimately very user-friendly. I've noticed how careful he is with building some of these mathematical concepts from the ground up."

handouts that sort of went through stuff fairly carefully."	"His pictures in the margins are abundant and very helpful, and there's always a nice subject line or topic sentence for each section along the lines of "here's what our ultimate goal is for the next few pages and here's how we're going to do it."

Table 3. Examples from the instructor's handouts.





Giving Students a Resource to Revisit

The instructor's orientation was to place a value on providing students with detailed notes that they could revisit outside of class time. The instructor viewed the textbook as wonderful in many ways, but was "a bit fast and loose" with the coverage of some topics, so he decided to create handouts to supplement the textbook.

The instructor's comments	The student's comments
"Even with the handout here, I have to say this is cool, but me telling you it is cool, or me going in there and writing very quickly on the board and showing this is not going to work. You need to go and figure out why it is cool yourself." "There are topics here that I think they should read, and they should read it carefully enough—meaning maybe a couple of paragraphs they need to spend several hours on teasing them out and then present it to their peers for 50 minutes." "We need to get our hands dirty to do this."	"Overall, I'm happy with the handout, as it works through several examples entirely, with plenty of marginal remarks by Dr. Brady as always. I always know that, given a handout, Dr. Brady has license to cruise through the material in that lecture even quicker than usual, but that I have it right there with me to review by myself later."

The Instructor Believed the Students Should Master the Material on their Own Time

The instructor noted that some of the handouts helped students draw connections across course content, and these were connections that took him quite some time to realize when he took the course himself. Therefore, the instructor's goal was to provide these connections for his students to facilitate their "a-ha" moments. This goal is aligned with Mason's (2002, p. 64) statement that students should "re-construct topics for themselves from lecture notes and text, and, increasingly, independently from text alone". Hence, the handouts were no substitute for the students putting in the time and effort outside of class to master the content. In the comment below, we noticed that the student did not realize the instructor's motivation for asking his students to complete ungraded assignments in the handouts.

The instructor's comments	The student's comments
"It had exercises for them to verify things. I am not grading it. It is up to them to make sure they understand it, and they come up and chat with me if they want."	"I can't remember exactly when our last homework assignment was, but it must have been weeks ago. Since then the course has consisted only of lectures and unofficial "I suggest you do these" exercises assigned by Dr. Brady. I have to admit that I haven't attempted all of them, partly because there are so many and partly because the lack of incentive (that they won't be graded)."

Concluding Remarks

This qualitative narrative study investigated a mathematician's ROGs through his handouts, teaching journals, and conversations that occurred during weekly research team meetings. Additionally, we analyzed one of his students' journals to investigate whether the student understood the instructor's goals for creating the handouts.

Analysis of the instructor's teaching journals and transcripts of the weekly team meetings revealed that the Geometer noted a myriad of reasons why he created handouts for his students. We focused on three goals: (1) helping students to build intuition, (2) giving students a resource to revisit, and (3) assisting students with mastery of the course content outside of class time, and we also provided quotes linking the instructor's and students' thoughts about the handouts.

The student appreciated the detailed handouts, particularly the hand drawn images that helped students understand the gist of the topics before the formal proofs were introduced. In one of his journals, he mentioned: "The immense amount of effort Dr. Brady must put into class preparation shone through again with this handout." However, the student was keenly aware that the handouts allowed the instructor to "cruise" through the material faster than he might otherwise. Further, the student may not have recognized the value in completing the ungraded exercises that the instructor suggested in the handouts. To put this into the instructor's words, the students had to grapple with the handout. It was not enough just to read it over once. They had to "get their hands dirty" and work on it on their own time.

We suspect that many other mathematics instructors share similar pedagogical goals. Creating handouts can be time-intensive, but once the handouts are created, they can be adapted from semester-to-semester. We would like to make some pedagogical recommendations for the inclusion of handouts in advanced courses. First, instructors may consider alerting their students to their goals for creating the handouts. For instance, instructors could let their students know that they will be covering the material more quickly than if they did not have a handout prepared, but the students can revisit the handouts outside of class to more fully grasp the content. Second, handouts can provide an avenue that helps instructors show how concepts unfold step-by-step to help their students grasp an initial intuition about the content. The instructor that we studied also noted that when he created handouts, it helped him to feel like he had a plan and was not scattered for his lecture. Third, if instructors recommend that students complete ungraded exercises, they could explicitly state why they believe these exercises will benefit the students' learning, even though they are not incentivized with course points.

As members of the mathematics community, we are constantly faced with challenges of finding the best ways to maximize our students' understanding. Making the right pedagogical decisions that are aligned with our goals and beliefs are not always trivial. Although, we recognize that we only examined one student's journals in this study, it is rewarding to know that some of our instructional decisions and efforts are effective and appreciated by our students.

References

Aguirre, J., & Speer, N. M. (2000). Examining the relationship between beliefs and goals in teacher practice. *Journal of Mathematical Behavior*, *18*(3), 327–356.

Creswell, J. W. (2013). *Qualitative inquiry and research design: Choosing among five approaches*, (3rd ed). SAGE.

Hannah, J., Stewart, S., & Thomas, M. O. J. (2011). Analysing lecturer practice: The role of orientations and goals. *International Journal of Mathematical Education in Science and Technology*, *42*(7), 975-984.

Hatcher, A. (2001). *Algebraic Topology*. From: www.math.cornell.edu/~hatcher/AT/AT.pdf

Mason, J. (2002). *Mathematics teaching practice: A guidebook for university and college lecturers*. Chichester: Horwood Publishing.

Paterson, J., Thomas, M. O. J., & Taylor, S. (2011). Decisions, decisions, decisions: What determines the path taken in lectures? *International Journal of Mathematical Education in Science and Technology*, 42(7), 985-996.

Schoenfeld, A. H. (2010). *How we think. A theory of goal-oriented decision making and its educational applications.* Routledge: New York.

Schoenfeld A.H. (2015) How We Think: A Theory of Human Decision-Making, with a Focus on Teaching. In: Cho S. (Ed) *The Proceedings of the 12th International Congress on Mathematical Education.* Springer, Cham.

Stewart, S., Thompson, C., & Brady, N. (2017). Navigating through the mathematical world: Uncovering a Geometer's thought processes through his handouts and teaching journals. *10th Congress of European Research in Mathematics Education* (CERME 10).

Thomas, M. O. J., & Yoon, C. (2011). Resolving conflict between competing goals in mathematics teaching decisions. In B. Ubuz (Ed.), *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 241-248), Ankara, Turkey.

Törner, G., Rolke, K., Rösken, B., & Sririman. B. (2010). Understanding a teacher's actions in the classroom by applying Schoenfeld's theory teaching-in-context: Reflecting on goals and beliefs. In B. Sriraman, & L. English (Eds.), *Theories of mathematics education, Advances in Mathematics Education* (pp. 401-420). Berlin: Springer-Verlag.