Factors Supporting (or Constraining) the Implementation of DNR-based Instruction in Mathematics

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DNR-based instruction in mathematics (Harel, 2008a, 2008b, 2008c) is a theoretical framework for the learning and teaching of mathematics. DNR-based professional development is a long-running program spanning seven years with multiple cohorts of in-service secondary mathematics teacher participants. This report investigates teacher change among five key variables: facilitating public debate, using holistic problems, attending to students' intellectual need, attending to the meaning of quantities and use of students' contributions. Commitment to change and perseverance, the nature of available curricular materials and teachers' view of the role of curriculum, a view of students as partners in knowledge construction, institutional context, collaboration and content knowledge were identified as factors that afford or constrain DNR implementation. This work has implications for the design of future professional development efforts and the development of a more robust theory of teacher change.

Keywords: Teacher change, DNR-based instruction, Intellectual Need, Professional Development, In-service

In their review of 106 articles reporting findings on mathematics professional development programs between 1985 and 2008, Goldsmith, Doerr and Lewis (2014, p. 21) point out that "existing research tends to focus on program effectiveness rather than on teachers' learning," while much less has been said about "*how* teachers develop knowledge, beliefs, or instructional practices". Understanding how teachers' development entails also understanding factors that facilitate or inhibit desired forms of development. In this report, we also strive to answer similar research questions: *How effectively are aspects of a professional development guided by a particular theoretical framework adopted by teachers and integrated into classroom teaching? And what factors challenge or support teachers in implementing this framework in the classroom?*

For the purposes of this report, the first question is seen as merely a starting point. The second questions, is in line with Goldsmith et al's (ibid) observed gap in the literature base. However, we argue that a crucial step in understanding how teachers develop is identifying factors that afford or constrain implementation of the targeted forms of knowledge, beliefs or practices. We are not alone in this goal. Indeed, Hill et al (2008) also sought to identify factors that support or hinder teachers' use of mathematical knowledge for teaching in practice as part of a larger research agenda. Similarly, Schoenfeld (2010), seeking to construct models of teachers' decision-making, found it "necessary and sufficient" to characterize teachers' knowledge, goals, orientations (i.e., belief, values, preferences, etc.) and decision making in order to construct their models. A natural extension to this work is understanding factors that afford and constrain teachers' ability to reach their goals or enact their stated beliefs.

Theoretical Perspective

DNR-based instruction in mathematics (*DNR,* for short; Harel, 1998, 2000, 2008a, 2008b, 2008c, 2013a, 2013b) is a theoretical framework for the learning and teaching of mathematics—

a framework that provides a language and tools to formulate and address critical curricular and instructional concerns. *DNR* can be thought of as a system consisting of three categories of constructs: *premises*—explicit assumptions underlying the *DNR* concepts and claims; *concepts*—constructs defined and oriented within these premises; and *claims*—statements formulated in terms of the *DNR* concepts, entailed from the *DNR* premises, and supported by empirical studies.

As the above list of references indicates, *DNR* has been discussed extensively elsewhere, and so in this paper we only reiterate briefly the definitions of the concepts pertaining to the concern of this study.

Assigning Holistic Problems: A holistic problem is one where a person must figure out, from the problem statement, the elements needed for its solution (Harel and Stevens, 2011). It does not contain hints or cues as to what is needed to solve it. In contrast, a non-holistic problem is broken down into small parts, each of which attends to one or two isolated elements. Often each of such parts is a one-step problem.

Intellectual need: Do students have a need for understanding the mathematics the teacher intends to teach? Does the teacher appeal to a problematic situation that puzzles students when introducing new mathematics?

Attention to meaning: When a problem has a context, unknown quantities have meaning with respect to that context (e.g. units related to quantities). Does the teacher attempt to attend to the meaning of quantities within the context of the problem?

Public debate: Is there evidence to believe that the whole class is following the discussion? Is the teacher making a successful effort to engage the whole class in debate through questioning and solicitation of contributions? Public debate also includes the need to evaluate mental images and their validity and efficiency.

Taking student contributions seriously: A student's contribution is considered to be taken seriously when it is allowed to live in the public space for discussion without immediate teacher evaluation. When taking contributions seriously, teachers solicit ideas and mental images from students, and facilitate public debate about these ideas to highlight and critique both underlying mathematics.

DNR-based professional development (DBPD) is a long-running program spanning seven years with multiple cohorts of in-service secondary mathematics teacher participants. This report investigates teacher change among five key variables: facilitating public debate, using holistic problems, attending to students' intellectual need, attending to meaning, and use of students' contributions. Collectively, these key variables have previously been identified as crucial teaching practices in student-centered classrooms targeting the development of students' mathematical content knowledge compatible with the Common Core's standards for mathematical practice (e.g., University of Michigan, 2006; Harel, Fuller and Soto, 2014; Shoenfeld, 2013). In his attempt to articulate the complexities constructing the TRU Math Framework and presenting the framework itself, Schoenfeld (2013, p. 613)) identified similar teaching actions, identifying them as, "known in the literature to be important" (ibid, p. 610).

Methods

DBPD consisted of two related support structures: (1) summer institutes and mid-year follow-up sessions and (2) on-site professional development. Both efforts targeted teachers' knowledge of mathematics, knowledge of student learning, and knowledge of pedagogy. This report examines DNR implementation for 33 teachers in a major urban area of the southwestern United States. DBPD included a focus on the five teaching practices: *public debate, holistic* *problems, intellectual need, attention to meaning,* and *taking contributions seriously* as defined above.

Repeated classroom observations of teacher participants were conducted and used to evaluate participants' implementation of DNR and to chart changes in participants' teaching over time. Two forms of data were generated using these observations. First, researchers examined whether or not a particular teaching practice was demonstrated in each participant's classroom during an entire classroom observation across two later years of the program's existence. Rather than relying exclusively on quantitative results for groupings, the authors triangulated available data sources (including their own experiences with the participants as well as formal debrief interviews) in order to maximize reliability associated with the classifications, thus reflecting the change in each of the participating teachers over time. Second, researchers looked more closely at interview data with participants conducted after classroom observations that could be used to give insight into factors that afford or constrain implementation. A summary of findings follows.

Results

How effectively are aspects of DBPD adopted by teachers?

Teachers were observed at least twice over the course of a four-year period, yielding data that describe change over time for each participant. In order to explore overall changes in teaching over time, the evaluator created a factor score that aggregately considered implementation of each of the five DNR parameters (public debate, use of holistic problems, attention to intellectual need, taking student contributions seriously, and attention to meaning) as one normally distributed score. This factor score, calculated using the principal factors method across all five DNR parameters, yielded a clear one factor structure, and thus served as an estimate of DNR implementation. Observed levels of implementation of each of the five DNR parameters were coded as absent or present during observations, and coded in duration by seconds, yielding a data structure that was capable of examining proportion of class periods where various levels of DNR implementation were observed. Across the sample for this study, the mean level of DNR implementation was calculated at 1.51, with a pooled standard deviation of 0.34. Results of a panel regression analysis examining whether or not average implementation of DNR increased over time for the entire group yielded a non-significant p value for the variable "time", indicating that average level of DNR implementation did not increase. However, this lack of aggregate results masks important variation within the population.

Analysis of change within subjects/by teacher revealed five distinct groups:

Evolvers: These teachers exhibited noteworthy increases in their DNR implementation score over time, an increase of one standard deviation or more over time (specifically, from first recorded observation to last observation).

Decliners: These teachers exhibited a decreased DNR implementation score of one standard deviation or more over time.

Consistent high implementers: These teachers exhibited DNR implementation score changes of less than one standard deviation, but were consistent in their high DNR implementation score, scoring at least one standard deviation above the mean at some time.

Consistent moderate implementers: These teachers exhibited DNR implementation score changes of less than one standard deviation, but were consistent in their moderate DNR implementation score, scoring within one standard deviation of the mean at both times.

Consistent low implementers: These teachers exhibited DNR implementation score changes of less than one standard deviation, but were consistent in their low DNR implementation score, scoring at least one standard deviation below the mean at some time.

Triangulated classifications are used for this publication, and are presented below as a table of frequencies across the five categories.

Table 1. Frequency table for categories of DNR implementation by DBPD participants.

Categorization	Number of Participants
Consistently Low	6
Decline	8
Consistently Medium	4
Evolvers	9
Consistently High	6

Closer examination of these groups (and specifically, of the stories these teachers provided through interviews) revealed important details about factors that both facilitate and inhibit DNR implementation. Finally, we noticed that affordances were simply the opposite of the constraints. Therefore, we report findings thematically, describing how a particular side of each theme (such as presence of absence of collegiality in the work environment) related to DNR implementation. In this same vein, we look at conditions that facilitated successful teachers, and conversely, those that facilitated disengagement with the DNR mission and a failure to implement in the classroom.

What factors challenge or support teachers in implementing DNR in the classroom?

Participant interview data identified commitment to change and perseverance, the nature of available curricular materials and teachers' view of the role of curriculum, a view of students as partners in knowledge construction, institutional context, collaboration and content knowledge as factors that afford or constrain DNR implementation. However, given the current space constraints and the qualitative nature of the data analysis, we report here one example of an observed factor, commitment to change and perseverance.

Commitment to Change and Perseverance

Across groups, risk-taking and willingness to take initiative emerged as common traits among consistently high implementers and evolvers. Similarly, the absence thereof characterized consistently low implementers and decliners. Three participants describe sharply contrasting points of view which explain their rationale for choosing to implement (or not) two different aspects of DNR-based instruction, holistic problems and public debate.

P1: They're trying to get us all to do the Euclid unit, and I'm just not on board - I was going to do it, but my school was not into it! Then I decided against it. There is a lot of construction and proofs, and that's what our students struggle with the most. The problems in there are geared for teachers, so you'd have to re-write everything to be more appropriate for students.

DBPD participants experienced the unit P1 refers to as “The Euclid Unit” (see Harel, 2014 for a description) over two summers. P1 described an initial willingness to implement this curriculum. Ultimately, she decided against implementing the unit for the following reasons: (a) lack of support at her “school” – where “school” may refer to departmental colleagues, students, parents or administrators, (b) a belief that construction and proving problems are beyond what her students’ are capable of appreciating and productively struggling with, and (c) a need to lower the level of the questions, generating work beyond the scope of her capacity. We save constraints (a) and (b) for discussion in subsequent sections while focusing on constraint (c) here.

In contrast to P1, P2 and P3 illustrated how a commitment to change supported implementation, regardless of their experience with the same kinds of constraints faced by P1. That is, these participants knowingly chose to search for ways to overcome existing constraints because of their belief systems.

P2: The students were not buying into public debate – they always just want to know the steps. That’s a struggle, especially with the advanced classes. Same story over and over. I believe in what I’m doing, but the resistance is always there. Most students are buying in, doing great, will publicly debate and challenge themselves. They understand that the methods and strategies we’re applying [are] good for them… A lot of times the students work in groups of four where they talk to one another a lot already. I don’t do direct instruction as much as any other teachers around here. That takes a lot of coaching every time – basically the months of September and October where I really have to drill the process into them, including training the kids and talking to the parents about why I do it like this.

P2 discussed how her belief in the role of public debate drove her to persevere in the face of student and parental pressure. P1 also cited a constraint regarding lack of support among important stakeholders (i.e., her “school”). Also in contrast to P1, P2 describes an actual, rather than a perceived, form of resistance. However, her belief in the benefits of public debate in the learning process led her to persist in her implementation.

P2 also describes a time frame in which she knows she will have to endure this resistance along with an understanding that communication with parents and powerful student learning experiences can combat this resistance despite the amount of effort it will take. We note that at one point in time P2 confronted and overcame these constraints before developing confidence that resistance eventually fades, and usually after the first two months of class. In the end P2 developed a sense for how long resistance will last and the benefits of persistence.

Also in contrast to P1, P3 describes below how a valuable experience at the DBPD summer institutes led to implementation of a particular holistic problem.

P3: Anytime I get a holistic problem, I try and spot some of the techniques they've used…like, the teacher-researcher gave us a problem where a sweeping line was covering some area…and I used that type of thing with my students. I thought, I can make that

> simpler and thus appropriate for 9th graders… You have to have faith that the kids will learn something valuable.

We first note potential differences in beliefs between P1 and P3. P1 reported feeling pressure to implement an entire DNR-based curriculum. P3 described a desire to bring a particular problem to his students, an approach emphasized during the summer institutes. P3 also described a desire to find and implement a general principle. Both participants felt a need to translate problems introduced at the DBPD to their students' level. P3's choice was supported by a belief in students as partners in knowledge construction saying, "You have to have faith that kids will learn something valuable." Consequently, P3's commitment to change surpassed constraints of the amount of work needed to translate a DNR concept (holistic problems) in his classroom.

Regardless of implementation level, participants reported that generating holistic problems was difficult and time consuming. On many occasions we observed, and participants reported, that they needed to try several versions of these problems before finding one that elicited productive student images useful in advancing the teacher's mathematical agenda. This was expressed nicely by one high implementer who noted after several years of implementation, "Almost all the problems I use I make up…". Another high implementer reported, "I'm always writing the material on my own. The book that we have doesn't support DNR-type instruction at all … I have very limited time. So, curriculum is a big need." While both of these comments also point other factors to be discussed later, we emphasize that this factor, commitment to change and perseverance, was a necessary ingredient in these participants' *eventual* shift toward the development of a curriculum consisting of a large number of holistic problems.

Assigning holistic problems, facilitating public debate and attempting to attend to students' intellectual need are high risk behaviors with the potential for high rewards or catastrophic failure. A common theme among low implementers and decliners was the observation that that assigning holistic problems could lead to high student frustration, especially from students who had little experience with them. This was often cited as a reason they did not implement key aspects of DNR, while high implementers and evolvers demonstrated a commitment to change and perseverance in implementation. Nearly all participants expressed fear of complaints from parents (and students) who might not understand the intention of a problem, why teachers were not providing algorithmic approaches in advance or why teachers might record an incorrect solution on the board. As one participant said, "A challenge is the resistance from parents and students when DNR or Common Core Standards are implemented since they are used to being told what to do."

In addition to targeted DBPD, change in teaching practice or sustaining the forms of teaching practice compatible to DNR-based instruction requires a concerted effort. Successful implementation of DNR requires teachers to see teaching as a mission – a mission of implementing and evangelizing innovative mathematics instruction – rather than a job.

Discussion/Summary

We summarize what we have learned, what remains open and what implications for instruction/future programs.

What have we learned?

Echoing the findings of other mathematics educators (e.g., Schoenfeld, 2010; Harel, Fuller & Soto, 2014; Ball, Hill & Bass, 2005), we are still far from understanding the inner workings of how teacher change their practice, especially "*how* teachers develop [and change existing] knowledge, beliefs, or instructional practices". It takes a professional with an exceptional commitment to change and perseverance in the face of the many obstacles that constrain teaching practice. Choosing a holistic problem for students is a non-trivial task, especially in light of the current culture of textbook school mathematics where the decisive majority of problems are non-holistic. We have found that successful implementers of DNR believe that students are partners in knowledge construction. Even when teachers do hold this belief, they may still struggle to solicit student thinking, anticipate student difficulties and bring the whole class conversation to a meaningful closure in a reasonable amount of time. There are many institutional pressures that constrain DNR-compatible teaching.

A constant theme among DBPD participants, regardless of implementation level, was a desire for a DNR-based curriculum. This large task requires a large set of resources, both human and fiscal. Nevertheless, DBPD participants repeatedly pointed out that curriculum would be a practical way to make inroads with colleagues, administrators, parents and students.

Another lesson learned is the importance of collaboration among teachers involved in DBPD. In order for DNR-based professional developers to influence instructional practice, participants must have access to first-hand experiences in which DNR-based instruction is demonstrated first-hand with actual students at the level of the participants. One important finding was that DNR-implementation was actually independent of the level of student being taught. While this result seems to contradict our finding that teachers needed to experience DNR-based instruction with students, in actuality it is more of a statement about teacher perception rather than fact.

Finally, we note that our participants enjoyed doing mathematics together. Successful implementers found ways to parlay their content knowledge into the selection or refinement of holistic problems for their students, the goals they set for public debate, the ability to make better sense of what students were saying and meaning, and the ability to make something out of those statements. Thus, giving their students a sense of ownership over mathematical ideas in the classroom.

Questions for further research

These findings point out that high implementers demonstrated a set of beliefs about learning and teaching coupled with particular dispositional traits (e.g., sees teaching as a mission, demonstrates perseverance/adherence to the belief that learning can only be accomplished through problem-solving, views content knowledge as central to good teaching). Were these participants selected for these traits or did DBPD influence them in some ways? If so, how?

We cited many institutionally related constraints here (and there are certainly others). How can DBPD providers attend to these in the future? For example, two participants at the same site, with the same preparation period, taught the same content and they seemed to benefit most from the DBPD. Another question concerns curriculum. How much will teachers feel is sufficient to support DNR implementation? An entire year? A particular grade band? All grades?

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