Relationships between Precalculus Students' Engagement and Shape Thinking

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This study examines relationships between community college precalculus students' understanding and engagement to link mathematical success to a malleable construct, and offer new insights for addressing consistently poor success rates in community college precalculus (Barnes, Cerrito, & Levi, 2004). Two-part interviews, consisting of a task and debriefing, were conducted with 8 students to investigate their shape thinking (Moore & Thompson, 2015), and engagement, conceptualized through flow theory (Csikszentmihalyi, 1975). Results suggest that students can be highly engaged in mathematics tasks regardless of understanding and that students exhibiting different ways of thinking about graph construction tended to experience different forms of engagement.

Keywords: Student engagement, Shape thinking, Precalculus, Community college

This study represents a portion of the author's dissertation in which community college precalculus students' engagement, understanding of precalculus concepts (e.g., covariation) and relationships between the two were investigated to address consistently poor success rates in community college precalculus (Barnes, Cerrito, & Levi, 2004). Others have demonstrated that student engagement is positively associated with academic achievement (e.g., Finn & Rock, 1997; Finn & Zimmer, 2012; Newmann, Wehlage, & Lamborn, 1992; Reschly & Christenson, 2012; Skinner & Belmont, 1993) and success in mathematics (e.g., Barkatsas, Kasimatis, & Gialamas, 2009; Lan et al., 2009; Martin, Way, Bobis, & Anderson, 2015; Rimm-Kaufman, Baroody, Larsen, Curby, & Abry, 2015; Robinson, 2013), where achievement and success are typically measured by students' performance on high-stakes assessments and student engagement is frequently reported by teachers or observers. Studying student engagement remains a focal point for educational research; this study contributes to that body of literature by investigating community college students' engagement and associations between engagement and understanding – as opposed to achievement on standardized tests. This study focuses on exploring relationships between student engagement and understanding of covariation. Information on such relationships would extend our understanding of the importance of fostering student engagement in community college precalculus classrooms.

Framework

Student Engagement

Student engagement is a metaconstruct consisting of emotional, behavioral, and cognitive components (Fredricks, Blumenfeld, & Paris, 2004). Flow theory (Csikszentmihalyi, 1975, 1990) is a valid framework for conceptualizing student engagement because both flow and engagement are comprised of similar components, both are described as states of intense concentration and investment in a task or activity, and both are intrinsically motivating (Steele & Fullagar, 2009). From the perspective of flow theory, student engagement is comprised of interest, enjoyment, and concentration (Shernoff, Csikszentmihalyi, Schneider, & Shernoff, 2003), where interest and enjoyment are elements of emotional engagement and concentration constitutes behavioral and cognitive engagement.

Covariation

Among other precalculus concepts (e.g., quantity/quantizing and function), covariational reasoning is paramount for success in future undergraduate mathematics courses (e.g., Carlson, Jacobs, Coe, Larsen, & Hsu, 2002). Moore and Thompson (2015) suggest that shape thinking provides perspective for describing students' covariational reasoning in the context of graphs. They describe students' shape thinking as static or emergent, where "static shape thinking involves operating on a graph as an object in and of itself" (p. 784). Monk (1992), defined iconic translations as a way of thinking in which perceptual features from a situation are associated with the shape of a graph for that situation. Also, Thompson (2015) explains that students may use thematic associations by regarding features of a situation as being necessary elements of the corresponding graph. Stevens and Moore (2016) elaborate that both iconic translations and thematic associations are examples of static shape thinking because both ways of thinking rely on perceptual features of an event for reasoning about a graphical representation; hence, operating on a graph as an object. On the other hand, "emergent shape thinking involves understanding a graph *simultaneously* as what is made (a trace) and how it is made (covariation)" (Moore & Thompson, 2015, p. 785, emphasis in original).

Methods

The purpose of this study is to explore and describe any relationships between community college precalculus students' engagement and shape thinking in the context of a task-based interview. The following research question is addressed. Is there a relationship between student engagement and shape thinking, and if so, what are characteristics of this relationship?

Setting and Participants

Data collection for the dissertation study took place during Fall 2016 at two Southeastern community colleges, where 15 precalculus instructors and 101 students participated. As part of the larger study, three instructors were selected for classroom observations, and their students were considered to take part in task-based interviews. Eight students from these three classrooms were selected based on their self-reported levels of engagement (i.e., interest, enjoyment, and concentration) during the first five weeks of the semester. These students were consistently more engaged during class time than their peers. Students with relatively high levels of engagement were selected to take part in interviews to increase the researcher's opportunity to "see" student engagement so any relationships between engagement and shape thinking could be explored.

Seven of eight students were enrolled full-time and all but two (Richard and Suzy, pseudonyms) were taking precalculus for the first time as college students.

Data Collection and Analysis

Data collection took place during interviews with students. Each interview consisted of two parts. The first part was task-based, where students worked through the *Taking a Ride* task, which has been used by Moore and colleagues to investigate students' shape thinking and other graphing activities (e.g., Stevens & Moore, 2016). The second part of each interview was a debriefing, where participants were asked to reflect on their interest, enjoyment, and concentration (i.e. engagement) while working the task. The data consist of video recordings, transcripts, and student artifacts.

Interview transcripts and artifacts produced during the task-based portion of interviews were coded for indicators of students' shape thinking based on definitions provided above. Transcripts from the debriefing portion of interviews were open coded to describe themes in participants' descriptions of their interest, enjoyment, and concentration while working the task. Finally, students' various ways of thinking coupled with themes emergent in their engagement are used to identify and characterize any relationships between student engagement and shape thinking.

Task description. In the Taking a Ride task, students are prompted to watch an animation of a Ferris wheel perpetually rotating clockwise and "graph the relationship between a rider's total distance traveled *around* the wheel and the rider's distance from the ground" (emphasis in original). Following this, participants view a second animation of a Ferris wheel; however, in this animation the ride stops periodically. Participants are then asked to discuss the relevance of their original graph to the new situation.

Results

This section is organized to first present results on students' shape thinking, and then describe themes in students' engagement.

Shape Thinking

Three students exhibited static shape thinking: Richard, Paula, and Sally. Though, all three students did not associate perceptual features of the animation and their graphs in the same way. Richard interpreted the image of the Ferris wheel as a coordinate system and explained how he envisioned such a coordinate system working. He explains the image/graph was structured with x's that were all zero "because the fact that every um lines over there [*pointing to the image in the animation*] kind of direct me straight to the middle [of the ride]." He used the image of the ride to establish radial coordinates, like those of a polar coordinate system, which he described as heights. Richard labeled his horizontal axis "time," which was constant at zero "because everything points at zero." Two visuals of Richard's coordinate system are provided in Figure 1.

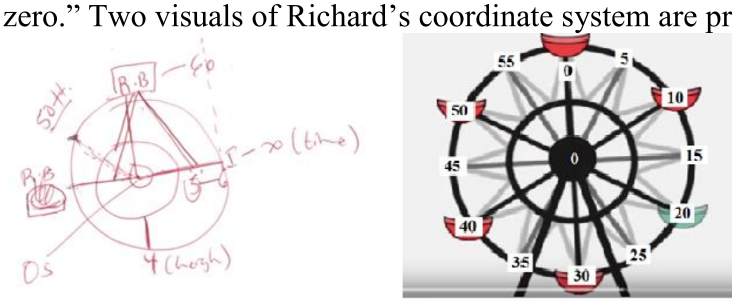

Figure 1. Richard's sketch (left) and a reconstruction (right) of his coordinate system.

Paula and Sally also demonstrated static shape thinking in their work on this task, by sketching circular graphs depicting the path of a rider on the Ferris wheel. Their justifications for circular graphs are exemplified by the following interview excerpt.

Paula: he's not going straight up or like going straight to the side, he's going in a circular motion so that is what I put it like that [a circular graph].

Interviewer: So, how would that change [*pointing to Paula's graph*] if the wheel were rotating the other way?

Paula: If it were rotating the other way, it would start here [*pointing to the right-most side of the wheel*] and then go around that way [*tracing around the wheel counterclockwise*]. So, it would go this way [*tracing the same path on her graph*].

Figure 2 shows these students' graphs.

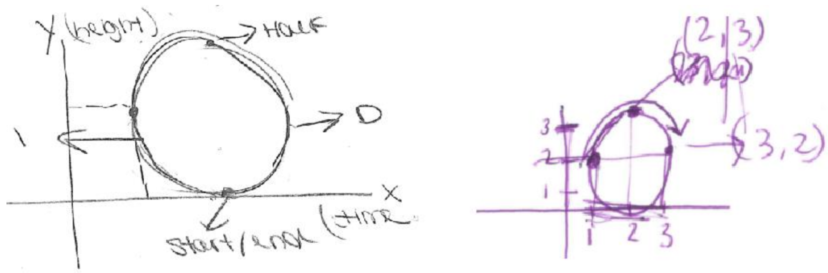

Figure 2. Paula (left) and Sally's (right) graphs of circular paths.

The remaining five students, Beverly, James, Suzy, Marianne, and Patricia exhibited emergent shape thinking while working the task. Beverly describes her graph as depicting the rider's position at a given time during the ride. She considers the horizontal axis to be "distance from the middle" or how she describes the rider's lateral displacement from a starting point. Her vertical axis is constructed similarly to reflect the rider's height above the ground. In this regard, she is coordinating simultaneous vertical and lateral displacements in a bounded space, both with respect to time, to produce her graph as the emergent path a rider travels around the ride. Her graph is shared in Figure 3.

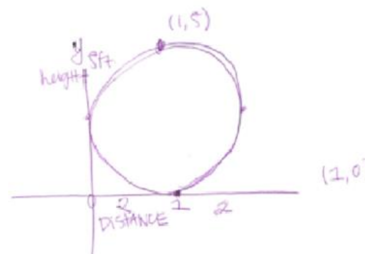

Figure 3. Beverly's sketch of the path a rider travels as an emergent trace.

Suzy and James both interpreted the prompt from the task to require two graphs: one for the relationship between a rider's distance traveled around the wheel over time, and a second for the rider's height over time. James' graph(s) in Figure 4 reflect how both students thought about this task. James sketches both relationships on the same plane, where the green graph depicts the rider's total distance traveled over time and the purple graph reflects the rider's height over time. James explains how he coordinates changes in time and height to construct his graph for that relationship, "at no time, you're at zero assuming it starts with the rider at the bottom…[then] halfway through that [height] would be halfway… so here's a graph." He continues, "for the total distance around it would be similar, but… it would extend forever."

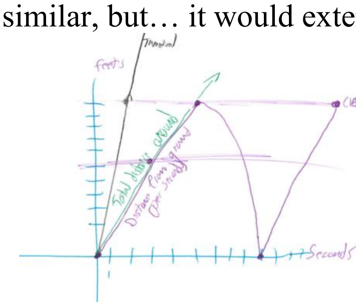

Figure 4. James' graph(s).

Marianne also interprets the prompt to be about a relationship between a rider's height and time, but she sketches a "sine or cosine curve" to represent the situation. Her emergent shape thinking is demonstrated in her explanation of how her graph changes for the second animation.

Marianne: So it stops about every like quarter of the way, so you would just have to like scrap your graph where it stops and draw a straight line. Um, but still have it like connect to the curve. So I guess it would stop like here [*sketching Figure 5*], so you would just straight line and it would resume. And then it would stop here, so straight line…

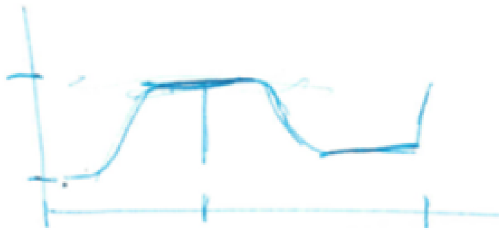

Figure 5. Marianne's graph of the second animation.

Lastly, Patricia coordinates changes in the rider's total distance traveled around the wheel with distance from the ground. In the following excerpt, Patricia demonstrates understanding her graph as an emergent trace created as the rider's distance traveled and distance from the ground covary by physically tracing her graph (Figure 6) as she explains its behavior associated with stopping in the second animation.

Patricia: …if we're traveling now [*tracing her graph with her pencil while watching the animation*] and I pause [*stops tracing*] I'm like, it doesn't affect [sic], like I'm not still going straight with my distance from the ground, and I'm not going down with my distance traveled because I am just standing there. Like I'm still, but then I keep going.

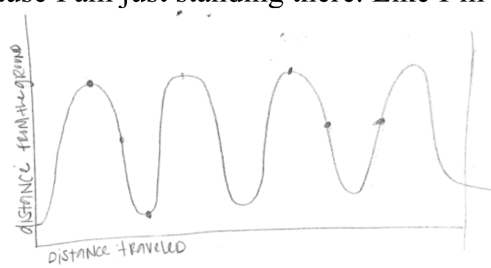

Figure 6. Patricia's graph.

The next section shares results from debriefings, to report on these students' engagement.

Student Engagement

Concentration. To begin, the average amount of time spent working the task was about 48 minutes, ranging from about 26.5 minutes (Patricia) to 76 minutes (Sally). This persistence with working and explaining their thinking evidences high levels of concentration in all students. Further, when asked about their concentration during debriefings, two themes emerged in responses regardless of shape thinking. First, students explained their focus on elements of the task they found confusing. For example, Patricia (emergent shape thinking) was "throw[n] off that time's not in there." Second, students discussed needing to concentrate on their explanations while working the task. Paula (static shape thinking) exemplifies this theme by reflecting on how evaluating her thoughts inhibited her work on the task, "just like the fact that maybe I was just wrong, so I would think about something and then I'd be like, no that is wrong, don't say that." Students concentrating on their own thinking is possibly due to the task-based interview setting, but does reflect high levels of concentration while working a mathematical task.

Interest and Enjoyment. All eight students did not describe similar feelings towards interest and enjoyment on the task. In fact, there appear to be differences in the self-reported levels of interest and enjoyment among groups of students whose shape thinking was categorized differently. Specifically, those who exhibited emergent shape thinking tended to enjoy the task because it was challenging, promoted problem-solving and thinking, and the context was relatable; these students also tended to find the task interesting because it was challenging, promoted problem-solving, and allowed for autonomy. For instance, Suzy discussed that she found the task interesting because "it makes a person think… this is very important to try to think logically and solve problems in real life."

Though, not all students demonstrating emergent shape think expressed high levels of enjoyment and interest. For example, Marianne indicated that she enjoyed the task because she "enjoyed thinking things out, like trying to make sense of the wheel and drawing it on paper." However, she mentioned that the open-ended nature of the task "put[ting] me back in that place where like I was unsure of myself." Further, Patricia found the task uninteresting and unenjoyable, both of which she attributed to the open-ended nature of the task prompt and not knowing what to do. Thus, Marianne and Patricia associated low confidence while working on the task to lower levels of enjoyment (and interest in Patricia's case).

On the other hand, students who demonstrated static shape thinking while working on this task did not enjoy the task but tended to find it interesting. They associated their lack of enjoyment with finding the task challenging, confusing, and allowing for too much autonomy. Paula and Sally also discussed low confidence being associated with their low level of enjoyment. For example, when asked about her enjoyment, Paula stated, "pretty bad… because I think it is just all wrong… Especially with this one [*referring to the first animation*] because I never saw this before, like the whole circle in just one little section." However, these students did report that working the task was interesting because it was challenging and open-ended. Sally explicitly states this apparently contradictory result "so interesting because difficult; not enjoyable because difficult."

Table 1 presents a matrix of themes associated with student engagement (i.e. concentration, enjoyment, and interest) discussed by students during debriefings based on shape thinking.

Table 1. Themes associated with engagement by shape thinking

<table>
<tr><th></th><th>Concentration</th><th>Low Enjoyment</th><th>Low Interest</th><th>Enjoyment</th><th>Interest</th></tr>
<tr><td>Static Shape Thinking</td><td rowspan="2">Persistence

Focus on confusing elements

Concentrate on explaining</td><td>Challenging

Confusing

Autonomy

Low confidence</td><td></td><td></td><td>Challenging

Open-ended</td></tr>
<tr><td>Emergent Shape Thinking</td><td>Low confidence (Patrician & Marianne)</td><td>Autonomy (Patricia)</td><td>Challenging

Promotes problem solving

Context</td><td>Challenging

Promotes problem solving

Autonomy</td></tr>
</table>

Conclusion

There appears to be a relationship between community college precalculus students' engagement and shape thinking. Specifically, students exhibiting differences in shape thinking described differences in their interest and enjoyment, such that those exhibiting static shape thinking tended to be interested while working the task but experienced low enjoyment. On the other hand, students exhibiting emergent shape thinking tended to find the task both interesting and enjoyable, except for Patricia, who expressed struggling with confidence. Regardless of students' understanding of covariation and shape thinking, these students demonstrated and discussed high levels of concentration.

This study has shed light on factors community college precalculus students associate with their levels of engagement while working through a challenging mathematical task. Researchers have showed that student engagement is positively associated with academic achievement (e.g., Finn & Rock, 1997; Finn & Zimmer, 2012; Newmann et al., 1992; Reschly & Christenson, 2012; Skinner & Belmont, 1993) and success in mathematics (e.g., Barkatsas et al., 2009; Lan et al., 2009; Martin et al., 2015; Rimm-Kaufman et al., 2015; Robinson, 2013). Results presented in this study suggest that community college precalculus students can be highly engaged in mathematics tasks regardless of understanding and that students exhibiting different ways of thinking about graph construction tended to experience different forms of engagement. These results demonstrate the importance for establishing learning environments that foster student engagement.

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