Didactical Disciplinary Literacy

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Undergraduate mathematics students are routinely asked to learn from various "texts" such as textbooks, videos, and lectures. In order to understand how students read and learn from discipline-specific texts, literacy researchers in recent years (e.g. Shanahan & Shanahan, 2012) have begun to direct their attention to disciplinary literacy: the ways that disciplinary experts themselves interpret, create, and critique materials. In this vein, we set out to investigate the disciplinary literacy practices of calculus students and non-mathematics STEM faculty, but found that focusing on this form of literacy alone was insufficient to explain the differences between the students' and faculty members' practices. To address this, we propose a new construct of didactical literacy, provide examples, and discuss its details and ramifications.

Keywords: Disciplinary Literacy, Textbooks, Expert-Novice Studies

Background

Over the past several decades, numerous groups have advocated for learning mathematics by participating in mathematical practices, which includes mathematical communication (e.g., National Council of Teachers of Mathematics, 2000; National Governors Association, 2010; Schumacher, Siegel, & Zorn, 2015). Students have long been asked to use and learn from a variety of mathematical texts, such as textbooks, lectures, and videos; and the proliferation of new didactical formats such as "flipped" classrooms have led to increased interest in using these formats in mathematics classrooms (e.g., Maxson & Szaniszlo, 2015a, 2015b).

There has been relatively little research that has focused on literacy in the context of teaching, learning, and doing mathematics. Although there is a body of research focused on undergraduates' reading and comprehension of mathematical proof (e.g., Inglis & Alcock, 2012; Mejia-Ramos & Weber, 2014; Weber, 2015), there is scarce other research that describes how students interpret and learn from printed texts (e.g., Borasi, Siegel, Fonzi, & Smith, 1998; Draper & Siebert, 2004; Shepherd, Selden, & Selden, 2012; Shepherd & van de Sande, 2014) or lectures (e.g., Lew, Fukawa-Connelly, Mejía-Ramos, & Weber, 2016; Weinberg, Wiesner, & Fukawa-Connelly, 2014). As Doerr and Temple (2016) noted, the paucity of research combined with the increased need for students to interact productively with mathematical texts underscores the "need for greater attention to reading in mathematics instruction" (p. 6).

As Moje (2008) noted, as educators have come to view participation in discipline-specific discourses as an essential aspect of disciplinary learning, educators and researchers have begun to focus on literacy as an aspect of disciplinary practice. In this paper, we begin by discussing the concept of disciplinary literacy. Then, we describe selected results from a study we undertook to investigate the literacy practices of students and non-mathematics STEM faculty members. Based on these data, we propose a new form of literacy called *didactical disciplinary literacy*, which, we hypothesize, may play an important role in learning from didactical mathematics texts.

Literacy in Teaching and Learning Mathematics

The term "literacy" is used in a variety of contexts. In the mathematics education community, it appears most often as part of "quantitative literacy" (e.g., Steen, 2001), which focuses on the capacity of an individual to identify and work within quantitative situations and to use mathematical skills for citizenship. In contrast, the types of literacy that we focus on in this paper involve creating and learning from texts. We use the term "text" broadly, as defined by Draper and Siebert (2010): "a *text* is any representational resource or object that people intentionally imbue with meaning, in the way they either create or attend to the object, to achieve a particular purpose" (p. 28).

Literacy experts have emphasized the pervasive and complicated role that texts play in teaching and learning, and students' roles as active learners—and their interactions with new forms of media—have led researchers to propose a broad definition of literacy, such as: "the ability to negotiate (e.g., read, view, listen, taste, smell, critique) and create (e.g., write, produce, sing, act, speak) texts in discipline-appropriate ways or in ways that other members of the discipline (e.g., mathematicians, historians, artists) would recognize as 'correct' or 'viable'" (Draper & Siebert, 2010, p. 30).

Each discipline produces its own types of texts (Bass, 2011) and uses distinct, grammatical resources and agency (e.g., Coffin, 1997; Martin, 1993; Veel, 1997, Wingnell, 1994). For example, a history text might take the form of a historical argument (advocating for a particular interpretation of events) while a science text might take the form of a science explanation (describing how and why phenomena occur).

In addition to the disciplinary-specific aspects of texts, recent research has shown that experts in different disciplines engage in reading processes in different ways. In one of the few studies to examine aspects of mathematics disciplinary literacy, Shanahan, Shanahan and Misischia (2011) compared the reading practices of historians, chemists, and mathematicians and identified distinctive features of the ways they interacted with research texts from their respective discipline. For example:

- Historians used the time period in which a text was written and other contextual factors as interpretive tools; chemists attended to time period to determine the value of the text in the context of rapidly-evolving scientific theories; and mathematicians did not contextualize the text.
- Historians used intertextual connections to identify the effects of the author's perspective; chemists corroborated connections to explain the outcomes of various experiments; and mathematicians used corroboration to limit interpretive differences.

This focus on the "particular norms for everyday practice, conventions for communicating and representing knowledge and ideas, and ways of interacting, defending ideas, and challenging the deeply held ideas of others in the discipline" (Moje, 2008, p. 100) has led some literacy researchers to move towards the construct of disciplinary literacy as a way to identify discipline-specific practices and to structure students' engagement with these practices. We define *disciplinary literacy* as the capacity to create, interpret, and critique texts using the practices, skills, tools, and ways of constructing, representing, using, and communicating knowledge that are specific to a particular discipline.

Investigating Literacy Practices for Mathematics Textbooks

The initial goal of our research study was to describe the literacy practices that are important for learning from reading mathematics textbooks. We expected that asking mathematicians to read a mathematics textbook wouldn't reveal such practices because they would likely already know most of the ideas presented in the textbook. Thus, we interviewed two groups of people. The first group consisted of five student volunteers from the third author's second-semester calculus class. The students regularly read their textbook outside of class and engaged in various online and in-class discussions of the reading.

Our goal for the second group was to find readers who might have similar disciplinary literacy practices as mathematicians but wouldn't have the same extensive background knowledge. Thus, we recruited five faculty members from our institution, one each from the physics, chemistry, biology, computer science, and economics departments. Although each of these faculty members had taken calculus courses as part of their coursework, none had actively engaged with ideas from introductory calculus for (at least) the past ten years.

For our text, we used two excerpts from a section entitled "Applications [of the integral] to Geometry" in the students' textbook (Hughes-Hallett et al., 2012): the section introduction and an excerpt on arc length. The students had already read about other applications of integration in their textbook, so we posited that they were likely to have constructed the necessary background knowledge for understanding the concepts in the chapter.

To collect data, each person participated in an hour-long interview, which was video-recorded to capture the interviewee's speech, gestures and writing. We asked each participant to read the excerpts for the purpose of learning the content and to stop at any places where they had questions or were confused to engage in discussion. After completing the reading, the participants were asked to describe the main ideas of the sections, and to explain the meaning of each graph and/or formula, how the terms in the graphs and formulas had been derived, and why the text's explanation of the connections and derivations made sense.

Results

Both the students and the faculty interviewees engaged in numerous interactions with the textbook that mirrored aspects of disciplinary literacy in mathematics—for example, they all engaged in close reading and rereading, understanding terminology and defining variables, interpreting graphics as part of the text, and not attempting to contextualize the text. The faculty members were not familiar with the mathematical concepts that were used in the sections—namely, Riemann Sums and limits—but the students were. However, the faculty members all were able to make sense of and construct mathematically-accurate descriptions of the concepts and, in many cases, to do so more successfully than the students.

In the process of identifying aspects of the students' and faculty members literacy skills, we noticed that the faculty members appeared to be identifying and using what we called "didactical aspects" of the textbook in order to make sense of the reading. We identified patterns in the faculty members' interactions and present them here with several examples from the faculty members' interviews. The italicized portions of the interview excerpts highlight particular language we identified as indicating a didactical focus. Although we also describe aspects of the students' work, space limitations prevent us from including excerpts from the students' interviews.

Framing the Text in Terms of the Authors' Intentions

All of the faculty members routinely framed their interpretation of the text in terms of the authors' intentions. For example, Professor E, when asked to describe "what's going on" at the beginning of the section on arclength, described the authors' motivation for using the hypotenuses of right triangles to approximate a curve:

> What's going on here is *they want to show you one of these really small lengths* and so along the horizontal axis, the distance is this amount right here, the change in x, but as x

> goes from this point to this point, we want to know how long that length is right there, and it's going to be longer because it's not a straight line.

Similarly, Professor K, when asked to describe why the textbook used a Riemann Sum before presenting the corresponding integral, described the authors' didactical intentions:

> I think *they're trying to provide steps for the reader* so there's no great leap in logic. That it makes sense that I have these small lengths. And then what am I gonna do with these small lengths? I'm going to add them up. So *I'm reminded at this step* that I'm doing a summation. That's essentially what an integral is.

Although most of the students also sometimes framed the text in terms of the authors' intentions and choices, their statements tended to focus more on mathematical rather than didactical aspects and ramifications of these choices.

Thinking about Didactical Motivation

The faculty members identified didactical motivation as the basis for the ways the authors presented the concepts. For example, when Professor K was asked to explain why the book's method for developing arclength made sense, she stated a didactical motivation and sequence, rather than a mathematical/logical description of the concepts:

> *They're starting with things that are easier to understand*. So delta x and delta y are easy to understand. We can really see those. They're physical lengths in my picture. So I can see them, and I can see how if I [gestures with fingers as if squishing lengths] made these lengths smaller and smaller, I could approximate a curve. *So they've shown me how to get* delta x and delta y, and then *using tools I've learned before*, we can rewrite these in terms of these [indicating a small quantity with her thumb and forefinger] very small steps, these derivatives. And then turning that into an integral, so *they're taking me through their process and explaining why they're doing it. Which is really important*. *So I'm not just facing an equation with no understanding of where it came from*. But with the understanding, I can apply this now with greater confidence.

When other faculty members were asked why the book first used a Riemann Sum before introducing the integral, they framed their description in terms of a didactical motivation that the Riemann Sum would be easier to understand than the integral. For example, Professor D noted that the Riemann Sum was based on intuition:

> I think *they're just reestablishing that intuition* that like, really an integral is just a sum of just an infinitely small pieces. And it's just a natural—and they're saying also the arc length is approximately equal to—that's what the squiggly lines are saying—but in the limit, it's actually equal to.

The students were less likely than the faculty members to frame aspects of the text in terms of didactical motivation. When they did so, they often framed their explanations partly in terms of mathematical necessity rather than solely in terms of pedagogy.

Thinking about Didactical Structure

In addition to identifying didactical motivation for various aspects of the text, several of the instructors also identified the didactical *structure* of the text and used this to inform their learning and to wrestle with uncertainty. Professor E summarized an example of such a structure:

> I've taken a lot of math courses and I think that this is the standard pedagogy that I see in math textbooks. So they kind of try to say in words what they're doing, those words get translated into notation and then there's examples.

Another didactical structure is to outline a general procedure and then use that procedure to develop specific applications. In our study, several faculty members described the structure of the chapter introduction, identified how this structure was mirrored in the presentation of the arclength derivation, and used this knowledge to develop a conception of arc length. For example, when asked to describe the book's method for arc length, Professor K framed the entire derivation in terms of the outline in the chapter introduction:

> I'm not quite sure how to describe it, except that *they're following the same steps that they suggested in their box* [in the chapter outline]. So the first step that they have here is that they show how you can find the length by breaking this up into small pieces. So in delta x it's a two-dimensional function, and so delta x and delta y. So they show how you could approximate it. And then they take that into a summation over very small pieces. So you go from these delta y's to derivatives. So that gives me very small pieces. And then they take that sum, and they turn it into an integral. *So they're following their own steps and laying out their procedure.*

There were several faculty members who, at the start of the interview, couldn't describe what Riemann Sums, limits, or integrals were, but identified and used this didactical structure to assist them in constructing a new understanding of what each of these terms meant. In contrast, only one student identified the structure of the chapter introduction and related it to the arc length derivation, but she didn't appear to actively draw parallels between the two structures

Didactical Authority

The faculty members also engaged with the text from a position of didactical authority by offering suggestions to change the presentation of ideas to make concepts clearer. For example, Professor M. suggested adding several diagrams to the arclength derivation to illustrate the steps of the derivation:

> The order in which things are presented, I think they could've, you know—it would have taken more paper, but I think they could've done a little bit better job of starting with this curve [points to diagram], just to illustrate what they're talking about, you know. 'Cause like I go straight from here [points to arc length introduction] and I look at this [points to diagram], and I'm like, "What are these?" [points to algebraic expressions in diagram] You know? And they start introducing stuff I haven't read about yet [points to "Length" calculation]. So you know, if I were presenting this in a class, I would sort of say these words [arc length introduction], show this picture [diagram] without these things [blocks off the text in the diagram with his hands]. And then I'd say these words [text following diagram], and I'd start to pop in the new things.

Recognizing Didactical Conventions/Necessity

The faculty members also recognized didactical convention of assuming that the readers possess all of the background knowledge that has been previously addressed. For example, when asked whether there were any parts of the textbook that might be difficult for students, Professor I said:

> Hopefully by the time you've got to this point in your calculus book, you know what the relationship is between delta-x and dx, even though that's a little bit fuzzy to me. Or

> was—still is a little fuzzy. *So you can't explain what a derivative is every time in the book uses a derivative. You have to remember that's the slope, blah blah blah blah*.

In contrast, several students commented that they wanted the book to re-explain concepts as they were (re-)used.

Didactical Literacy

As Shepherd and van de Sande (2014) noted, students—particularly at the undergraduate level—are asked to use their mathematics textbooks as a learning tool. Thus, it is important to understand how students read and learn from textbooks. At the same time, we argue that textbooks are a special type of mathematical text. We propose that the literacy most relevant to engaging with mathematics textbooks may also be specialized and merits focused attention.

Richards (2002) noted that "school mathematics" is a different domain of discourse than the domains used by research mathematicians, mathematically literate adults, and academic journals. Love and Pimm (1996) argued that the mathematical writing in textbooks is not just "a special version of mathematics written for a learner" (p. 375), but rather its own type of mathematical text, written specifically for mathematics students; it "provide[s] an authoritative pedagogic version" of mathematics (Stray, 1994, p. 2), is written from the position of a teacher (Kang & Kilpatrick, 1992), employs its own type of rhetorical forms (Fauvel, 1998), and attempts to provoke the development of specific cognitive structures in the reader (Van Dormolen, 1986). Moreover, textbooks employ structural literary devices, including exposition, explanation, questioning, exercises, examples, and formatting, and each of these structures typically has a specific didactical function (Love & Pimm, 1996).

Members of a discipline interact with disciplinary texts in ways that are guided both by disciplinary practices and discipline-specific features of the texts. Moreover, among disciplinary texts, there are *didactical texts*, created for the purpose of teaching. Didactical texts are usually created using particular norms of teaching within the discipline, and these norms are instantiated as conventions in the structure and discourse of the textbook, such as "introduce big ideas to frame later examples" or "connect theory to [what is viewed as] ideas that will be intuitive to students."

To capture the specialized nature of students' and teachers' interactions with didactical texts, we introduce the notion of *didactical literacy*: the capacity to create, interpret, and critique didactical texts using the practices, skills, tools, and ways of constructing, representing, using, and communicating knowledge that are specific to the didactical practices of a particular discipline. Didactical literacy encompasses the knowledge and skills to recognize, interpret, and use these conventions to effectively read the didactical texts—that is, to use, and critique, them as learning tools.

We hypothesize that, like disciplinary literacy, didactical literacy might be specific to particular disciplines. For example, didactical conventions in the teaching of history might be quite different than the conventions in mathematics. In the same way that disciplinary literacy in other STEM fields is similar to mathematics disciplinary literacy, the didactical conventions in other STEM fields might be close enough to those in mathematics to have enabled the faculty participants in our study to recognize and use the didactical structures in the mathematics textbook. However, this is an open question that could be empirically answered in future studies.

Neither disciplinary nor didactical literacy is subordinate to the other. For example, both students and faculty members in this study engaged in various disciplinary literacy practices associated with reading mathematics. However, simply having the capacity to engage in mathematics disciplinary literacy practices was not enough for the students to engage in the didactical literacy practices in the same ways as the faculty members.

Although a certain amount of disciplinary content knowledge is likely necessary to be didactically literate within a discipline, one may engage in didactically literate practices when missing background knowledge relevant to the text. For example, several faculty participants in this study lacked (what we viewed as) essential knowledge for understanding the concepts presented in the textbook, but were able to use the didactical structure in the textbook to construct mathematically accurate meanings for the related terms.

We hypothesize that didactical literacy is distinct from didactical knowledge (i.e., general knowledge about how students learn) and from specialized knowledge about how students learn within the discipline (e.g., mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008)). Didactical literacy is identified with the didactical conventions within a discipline, which may or may not be based on knowledge of how students learn.

Connections

We believe that the construct of didactical literacy has connections to other research areas in undergraduate mathematics education. It could be used as a way to understand the types of decisions teachers make when preparing materials for their classes. For example, Lai, Mejia-Ramos and Weber (2012) found that mathematicians have various conventions for creating and modifying proofs for the purpose of teaching students; specifically, they advocated for adding introductory and concluding sentences, using formatting to emphasize main ideas, and eliminating redundant information. Although such conventions for constructing or modifying proofs may be accepted by the mathematics didactical community, students may not be in a position to utilize this structure to interpret, construct, and critique mathematical proofs. That is, the ability to engage with these conventions productively may be part of mathematics didactical literacy that students do not generally possess. Didactical (and disciplinary) literacy also has the potential to be used as a theoretical lens to enable researchers to construct alternative explanations for phenomena. For example, Lew, Fukawa-Connelly, Mejia-Ramos and Weber (2016) analyzed students' interpretation of a lecture-based proof and identified both content- and communication-based barriers for the students in comprehending the lecturer's intended main points. Recasting the results in terms of literacy could enable researchers to understand other dimensions of both the structure of the lecture and the students' interpretation, and enable educators to find alternative methods for creating and helping students interpret these texts.

The idea of didactical literacy likely also has implications for teaching. As Fang and Coatoam (2013) noted, teaching disciplinary literacy involves engaging students in the content, literate practices, discourse patterns, and ways of reasoning within the discipline. Enacting disciplinary and didactical literacy practices involves both intellectual and social engagement with the discourse community in the discipline. Thus, helping students use didactical texts effectively may involve making didactical practices explicit and helping students develop the related literacy practices by supporting their participation in the practices of teaching mathematics.

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