Observable Manifestations of A Teacher's Actions to Understand and Act on Student Thinking

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This study produced a framework that describes different levels of teacher-student interactions during teaching. The framework characterizes observable teacher behaviors that are associated with each of the four levels of decentering that emerged from analyzing the teacher-student interactions of three teachers when teaching.

Keywords: Decentering, teacher-student interaction, teacher education, radical constructivism

Introduction

In recent decades, the importance of teachers' attending to and understanding their students' mathematical thinking, and building their instructional decisions on this understanding is highlighted in many research studies and publications in mathematics education (Ball & Cohen, 1999; Sowder, 2007). In *Principles and Standards for School Mathematics* (NCTM, 2000), one of the principles of effective teaching says "Effective teaching involves observing students, listening carefully to their ideas and explanations, having mathematical goals, and using the information to make instructional decisions" (p. 19). Therefore, it is crucial to characterize what is involved in attending to and understanding student thinking, and to illustrate how instructional decisions can be influenced by this understanding, especially in the context of teaching.

Studies have highlighted that understanding students' thinking and deciding how to act based on this understanding are not traits that are inherently possessed by teachers and they should be considered as types of expertise that need to be developed (Jacobs, Lamb, & Philipp, 2010). This position is supported by studies that have described teachers' difficulties in attending to, anticipating, and understanding students' mathematical thinking (Kazemi & Franke, 2004; Rodgers, 2002; Wallach & Even, 2005). In recent years, researchers have introduced and used theoretical constructs as a way to conceptualize the nature and development of this expertise. As one example, the construct of noticing has ben used to make inferences about teachers' ability to focus on students' mathematical thinking in a classroom environment (Sherin, Jacobs, & Philipp, 2011). Similarly, Jacobs et al. (2010) introduced the notion of professional noticing of children's mathematical thinking. They also recommend further research that "can connect teachers' professional noticing of children's mathematical thinking with the execution of their in-the-moment responses" (p. 197).

Piaget's (1955) construct of *decentering* has also been used as a theoretical lens to extend research on teachers' execution of in-the-moment responses based on student thinking (Carlson, Bowling, Moore, & Ortiz, 2007; Marfai, Moore, & Teuscher, 2011; Teuscher, Moore, & Carlson, 2016). In his work on child development, Piaget (1955) introduced the idea of decentering and described it as an action of adopting a perspective that is not one's own. More recently, Steffe and Thompson (2000) and Thompson (2000, 2013) extended Piaget's idea of decentering and conceptualized a meaningful human communication from the perspective of radical constructivism. Teuscher et al. (2016) state that, even though the construct of decentering has rarely been used to investigate interactions between a teacher and student(s), it has the potential to provide researchers with a framework for characterizing how a teacher's attention to (or lack of attention to) student thinking might impact the teacher's in-the-moment instructional decisions.

The purpose of this study was to characterize the degree to which a teacher attempts to make sense of and use student thinking when teaching. It was also our goal to describe the different levels of student-teacher interactions in terms of both the teacher's mental actions (i.e., decentering) and his or her observable behaviors. We have extended previous studies that described different levels of decentering (Carlson et al., 2007; Marfai et al., 2011) by presenting a framework of observable behaviors associated with teacher decentering. The framework has the potential to contribute to the research on effective teacher-student interactions by illustrating behaviors of a teacher that are associated with both non-decentered and decentered interactions between a teacher and her students.

Theoretical Framework: A Conceptualization of Human Communication in Radical Constructivism

According to Thompson (2000), people interact with others reflectively or unreflectively. In the case of teacher-student interactions, if the teacher acts reflectively, she then can act as an observer and be aware of the student's contributions to the interaction. Otherwise, the teacher acts as an actor, which prevents her from attempting to adopt the student's perspective (i.e., decentering).

A reflective interaction between two people is described as "the process of mutual interpretation and accommodation" (Thompson, 2013, p. 64). In this process, each participant attempts to understand what the other has in mind by building second-order models of the other's mental structures. Second-order models are "the hypothetical models an observer may construct of the subject's knowledge in order to explain their observations (i.e., their experience) of the subject's states and activities" (Steffe, von Glasersfeld, Richards, & Cobb, 1983, p. xvi). During the reflective interaction, each participant continuously adjusts his or her second-order models of the other's knowledge by comparing the other's responses with the responses that he or she anticipates (Thompson, 2013). Besides attempting to understand the other, each participant also makes an effort to have the other understand what he or she has in mind. In the case of teacher-student interaction, for example, the teacher considers how the student could interpret his or her utterances when attempting to convey his or her ways of thinking to the student based on a second-order model of the student's thinking. By continuously updating second-order models of the student's thinking through decentering, the teacher makes better decisions about how to convey his or her intended meaning to the student (Teuscher et al., 2016; Thompson, 2013).

If a teacher interacts with the student unreflectively, he or she is an actor rather than an observer of the student's thinking in this interaction. Thus the teacher is constrained to use his or her first-order model when making decisions about how to act (Teuscher et al., 2016). First-order models are "the models an individual constructs to organize, comprehend, and control his or her own experience, i.e., their own mathematical knowledge" (Steffe, et al., p. xvi).

Method

Subjects of the Study

The subjects of the study were three graduate teaching assistants (GTA) at a large public university in the United States. Two of the subjects were PhD students in mathematics and one was a PhD student in mathematics education. They were using the research-based and conceptually oriented Pathways curriculum (Carlson, Oehrtman & Moore, 2016). Prior to the beginning of the semester the subjects attended a 2-day workshop and during the semester when teaching they attended a weekly 1.5-hour seminar, both which focused on supporting the course instructors in developing deep meanings for and critical connections among the key concepts of the course.

Data Collection and Data Analysis

As the main sources of data, classroom observations were made during the spring semester 2017. Each subject's class was videotaped with a lapel microphone used to capture the teacher's explanations and conversations with students. Moreover, the first author of the study observed each lesson and took field notes.

We began our data analysis by identifying video excerpts in which the teacher was interacting with one of her students. We followed by transcribing these excerpts and began the process of studying the interactions carefully for the purpose of characterizing the degree to which the teacher exhibited decentering behaviors. We also took note of the mathematical meanings displayed by the teacher and the degree to which the teacher exhibited mathematical goals aligned with the mathematical goals of the Pathways curriculum. Our study of the videos led to our constructing codes to characterize the teacher's decentering actions, i.e., the degree to which they were constructing models of his/her students' thinking during interaction. In order to check the inter-coder reliability, two researchers independently coded randomly selected interactions (approximately 20% of the whole data set). We reached 85% agreement in our coding of these pieces of data. Discrepancies between the codes assigned by the two coders were discussed and a consensus on these codes was reached. The first researcher then coded the entire data set.

Following the coding process, we compared the collection of interactions and clustered interactions that were similar relative to the teacher's decentering actions. This analysis led to our identifying four types of student-teacher interactions. We then described observable behaviors and attempted to draw inferences about the mental actions (i.e., decentering) associated with each of the four levels of interactions. In the following paragraphs, we introduce the framework that emerged and then we illustrate how this framework can be used to characterize and describe teacher-student interactions.

A Framework for Analyzing Student-Teacher Interactions

The framework illustrates four different levels of student-teacher interactions. Level 1 and Level 2 are considered low-level interactions in terms of the teacher's decentering actions. We see that the teacher is acting from her/his mathematical meanings and is not considering how the student is thinking (Table 1). The primary difference between Level 1 and Level 2 interactions is that a teacher who is classified to be exhibiting Level 2 mental actions poses questions that probe students' thinking, while in a Level 1 interaction the teacher is only interested in students' answers and calculations, or getting students to echo the teacher's phrases. A Level 2 interaction is further characterized by the teacher posing questions and giving explanations aimed at moving students to his or her way of thinking.

Level 3 and Level 4 of the framework are considered to be higher-level interactions in terms of the teacher's decentering actions. In these levels, the teacher attempts to understand the student's perspective and makes general instructional moves based on the student's current thinking when interacting with the student around the course's key ideas. The primary distinction between Level 3 and Level 4 interactions is that during a Level 4 interaction, the teacher exhibits behaviors that suggest that she both respects students' idiosyncratic ways of thinking and makes moves to support students in making connections.

Table 1. Framework for analyzing student-teacher interactions in terms of the teacher's mental actions and observable behaviors

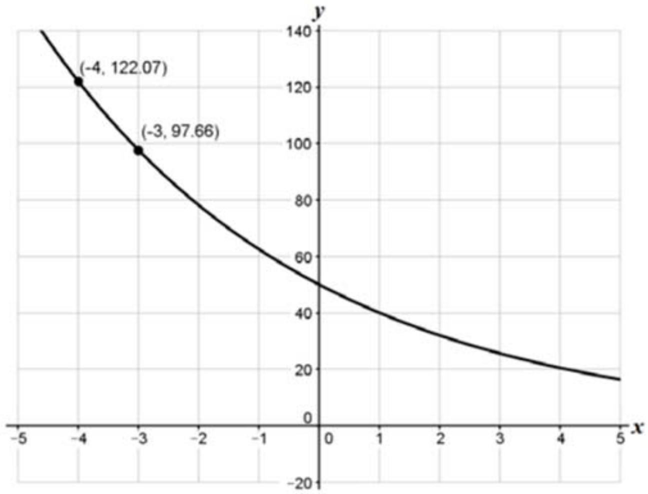
Mental actions	Levels	Description of the behaviors

• Not reflecting on aspects of the interaction that are contributed by students (i.e., interacting unreflectively)
• Creating first-order models of the interaction to organize, comprehend and control his/her own experience
• Operating entirely from first-order models (i.e., his/her own mathematical knowledge)
	Level 1: Shows no interest in students' thinking but shows interest in students' answers and takes actions to get students to say the correct the answer	• Asks questions to elicit students' answers
		• Listens to students' answers
		• Does not pose questions aimed at understanding students' thinking
		  ○ May pose questions focusing on procedures or calculations
		  ○ May evaluate how students' responses compare to his/her own way of thinking
		  ○ May pose questions to get students to echo key phrases and/or complete steps to get an answer

• Assuming that students' thinking is identical with him/her. In the case of recognizing students' thinking is different then him/her, not attempting to discern the students' mental actions driving the student's behaviors	*Level 2:* Appears interested in students' thinking, does not pose questions focused on students' thinking and attempts to move students to his/her thinking or perspective without trying to understand or build on the expressed thinking and/or perspectives of students	• Poses questions to reveal student thinking but does not attempt to understand students' thinking (e.g., Why? What does that mean? What does that term represent?)
		• Guides students toward his/her own way of thinking.
		  ○ Poses questions for the purpose of getting students to adopt his/her way of thinking
		  ○ Gives explanations aimed at getting students to adopt his/her way of thinking

• Reflecting on aspects of the interaction that are contributed by students (i.e., interacting reflectively) • Creating second-order models of students' thinking to explain his/her experience of students' states and activities	*Level 3*: Appears to make sense of students' thinking and/or perspectives, and makes general moves based on the expressions of the students	• Asks questions to reveal and understand students' thinking • Follows up on students' responses in order to perturb students in a way that extends their current ways of thinking • Attempts to move students to his/her thinking or perspective ○ Poses questions and gives explanations informed by students' current thinking and his/her understanding of the mathematical ideas

• Operating from second-order models of student thinking
• Assuming that students have idiosyncratic thinking and attempting to discern the students' mental actions driving the students' behaviors
	Level 4: Takes action to understand students' thinking, appears to understand the expressed thinking and/or perspective of students and takes actions that build on and respect the rationality of these expressions	• Prompts students to explain their idiosyncratic ways of thinking
		  ○ Poses questions to gain insights into students' thinking
		• Draws on students' idiosyncratic ways of thinking to advance students' understanding of key ideas in the lesson
		  ○ Poses questions and/or gives explanations that are attentive to students' thinking and/or aimed at advancing students' understanding of an idea
		  ○ Poses questions and/or gives explanations to support students in making connections between different viable ways of thinking of a mathematical idea



1. Find the ratio of output values that correspond to increases of 1 in the input value in order to determine the growth or decay factor.
2. Determine the 1-unit percent change by comparing the change in the output values to the function value at the beginning of a 1-unit interval for x.
3. Identify or determine the value of the function when x=0.
4.Use the information from parts (a) through (c) to define a function formula for the relationship.

Figure 1. The task used in the classes where the excerpts in the illustration1 and illustration 2 come from (Carlson et al., 2016)

Illustration 1:

The task in Figure 1 requires that students understand that the growth factor in an exponential function represents the relative size of two output values in terms of both multiplicative and percent comparisons (Carlson et al., 2016).

Before the conversation in Excerpt 1 began, the teacher discussed the task by describing how he expected students to think when they see this type of question. He stated, "I know it is a decay factor because it looks like as I move my input up my outputs are going down. I want you to look at it and be thinking these kinds of thoughts". He followed by describing how he determines a 1-unit decay factor. The teacher appeared to be focused on getting the students to imitate how he approaches this type of question, in contrast to showing any interest in the students' thinking. He then turned his attention to finding the function's initial value (Excerpt 1, Line 1).

Excerpt 1

[Line1] *Teacher:* I need to find my initial value. How might I find it? However, I do know a way to find it because I know that every time to get my new output at -2, what do I multiply 97.66 by?
[Line2] *Student1:* .8
[Line3] *Teacher:* .8; the decay factor. So at -2, I have .8 times 97.66. Ok, to get my value at -1 what do I multiply this number by?
[Line4] *Student2:* .8
[Line5] *Teacher:* .8 again, right? So I'm just a kind of walking my way down the graph to figure out what my value is at 0. So I know that I'm gonna have to multiply by .8 once to get the -2, twice to get the -1, and three times to get the zero, right?

We classified this interaction between the teacher and students in Excerpt 1 at Level 1 since there is no evidence that the teacher was interested in the students' ways of thinking about exponential growth or the idea of growth factor. He posed questions and listened to students' responses. However, the teacher's questions were directed at getting students to express the computation to get the correct answer (Lines 1, 3). After one student suggested a factor for multiplying (Line 4), the teacher failed to acknowledge her response; instead he proceeded to explain how the decay factor could be used to find the initial value of the exponential function. This explanation was a presentation of the teacher's way of thinking with no regard for whether his explanations were relevant to the student (Line 5). During this exchange the teacher did not attempt to reveal and understand students' meaning of a 1-unit growth/decay factor, nor did he build a second-order model of his students' thinking. The teacher's questions and explanations were based on his first-order model (his understanding), instead of models he built of students' thinking/meanings.

Illustration 2:

Before the conversation in Excerpt 2 began, the teacher asked students to express their ways of thinking about how they could determine the initial value of the function. One of the students expressed that the initial value could be determined by finding the 1-unit growth factor first. The teacher followed by asking the student to express how she determined the 1-unit growth factor. This response suggests that the teacher was interested in understanding the student's meaning of a 1-unit growth factor. The student then explained that she determined the 1-unit growth factor by dividing 122.07 by 97.66. The teacher probed the student about the fraction ($\frac{122.07}{97.66} \cong 1.25$) by saying, "Take a second and looked at this fraction. Is there anything standing out about this fraction?" The teacher's decision to focus students' attention on the value of the growth factor appeared to be for the purpose of getting students to see that a growth factor of 1.25 is not reasonable. The teacher's question led the students to realize that the ratio ($\frac{97.66}{122.07} \cong 0.8$) would produce a reasonable growth for scaling. During this interchange the teacher's interest in supporting students' thinking resulted in him helping the student confront her weak meaning for growth factor. His questions appeared to be based on his understanding of the student's problematic way of thinking. Then the teacher prompted the student to consider how to approach finding the function's initial value (Excerpt 2).

Excerpt 2

[Line1] *Teacher*: How do we go from having a 1-year growth factor to confirming that our initial value is 50?
[Line2] *Student1*: What I did was I just divided .8 the 97.66 so then I kept going down three, three downs until my input is 0.
[Line3] *Teacher*: Ok. So you said you multiplied or divided by a 0.8?
[Line4] *Student1*: Divided
[Line5] *Teacher*: So you did like 97.66/0.8. What are you computing with that?
[Line6] *Student1*: The initial value when the input is -2.
[Line7] *Teacher*: So, to find f(-2) we take f(-3), which is 97.66 and divide by 0.8. What do you think?
[Line8] *Student2*: I don't know why this happened but you plug this in you get to 122.07
[Line9] *Teacher*: So you're saying that if you compute this value, you get 122.07?
[Line10] *Student2*: Yeah.

[Line11] *Teacher*: Ok. So, if we put that in your calculator we should all get this 122.07. Why is that happening? You should recognize that number, because it's f(-4), right? Why we're getting f(-4) back when we do this computation? Student3, what do you think?

[Line12] *Student3*: When you divide the output by the growth … bigger…so instead we need to multiply.

We classified this interaction between the teacher and students at Level 3. We observed that the teacher initially prompted students to explain how to find the initial value of the exponential function using the 1-unit growth factor (Line 1). The teacher prompted one student to explain his approach; he then asked the student to provide a rationale for his approach (Line 5), demonstrating that he was interested in understanding how the student was thinking. When the student replied by saying that he was finding the initial value when the input is -2, the teacher followed by re-expressing the student's explanation; his explanation (Line 7) suggests that he understood how the student was thinking. He continued by posing questions to reveal how the student was thinking (e.g., Why are we getting f(-4) back when we do this computation?, [Line 11]).

Discussion and Conclusion

Prior research has characterized teachers' attempts to understand students' thinking, including how they respond when students express their thinking and whether they take student thinking into consideration during teaching. Researchers have used the idea of decentering as a theoretical lens to make inferences about a teacher's ability to make sense of student thinking (Teuscher et al., 2016). Piaget's construct of decentering has been considered as a powerful lens for researchers when focusing on how teachers build models of students' thinking and to what degree they use student thinking to make instructional decisions (Teuscher et al., 2016). Moreover, Thompson's (2013) conceptualization of a productive interaction between two people (i.e., the interaction where "each participant is oriented to understand what others have in mind and is oriented to have others understand what he or she intends" (p. 63)) extends the idea of decentering. There are also studies in which different levels of a teacher's decentering actions during his or her interaction with students are characterized based on this theoretical perspective (Carlson et al., 2007; Marfai et al., 2011; Teuscher et al., 2016). This study extends these research efforts by introducing a framework that provides a fine-grained characterization of teacher-student interactions. The framework describes two levels of a teacher's mental actions (i.e., non-decentered and decentered) and four levels of the teacher's observable behaviors that are associated with both non-decentered and decentered actions. The levels in the framework will be useful for both researchers and teacher professional developers by illuminating subtle and productive ways in which a teacher can leverage student thinking when interacting with students.

Studies also point out that all teaching actions are strongly related to the teachers' mathematical meanings for teaching (Thompson, 2013; Thompson, Carlson & Silverman, 2007). In Thompson's (2015) view, teachers' mathematical meanings for teaching are the main sources of their instructional decisions and actions. While this study does not examine how a teacher's meanings for a mathematical idea influence the quality of his or her decentering actions, we will investigate the relationship between teachers' mathematical meanings and their decentering actions in future research.

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