E-IBL, Proof Scripts, and Identities: An Exploration of Theoretical Relationships

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The purpose of this theoretical report is to further current discussions of the relationships between Equity-Oriented Instruction (EOI) and Inquiry Based Learning (IBL) pedagogies. Specifically, it proposes a framing of Equity-Oriented Inquiry Based Learning (E-IBL) that foregrounds equitable practice, as opposed to viewing equitable practice as a gratuitous outcome of IBL pedagogies. Drawing on data from teaching experiments conducted in IBL-Introduction to Proof courses, the inter-relationships between knowledge, identity and practice (Boaler, 2002), Pickering’s ‘dance of agency,’ Gutiérrez’s dimensions of equity, and Bourdieu’s notion of habitus, this paper explores why intentional attention towards the critical axis of equity – that which links identity and power – is necessary, if IBL pedagogies are to promote equity.

Key words: Inquiry based learning, equity oriented instruction, identity, agency

Introduction

The purpose of this theoretical report is to further current discussions about the relationships between Equity-Oriented Instruction (EOI) and Inquiry Based Learning (IBL) pedagogies. Specifically, this report proposes a framing of Equity-Oriented Inquiry Based Learning (E-IBL) pedagogies that foregrounds issues of equity, as oppose to viewing equity as a gratuitous outcome of IBL. To understand this position, current framings of EOI and IBL are considered and used to explore rationales for viewing IBL as a pedagogy that promotes equity. Then, drawing on excerpts from teaching experiments in IBL courses, I examine why IBL pedagogies may not gratuitously promote EOI. The paper concludes with a framing of E-IBL.

A Framing of Equity-Oriented Instruction

Over the past two decades, researchers interested in student learning in school contexts have begun to reconceptualize equity in mathematics education. These researchers (Gutiérrez, 2008; Martin, 2009) have challenged our practice of “gap gazing” and argue for the de-essentialization of disparities in students’ academic achievement; i.e., against “the framing of mathematics achievement …(as) a kind of individualistic accomplishment” (Gutiérrez, 2008, p. 361) Indeed, drawing on Bourdieu’s notion of habitus2 (Bourdieu, 1984), researchers are illustrating the ways in which practices of structural exclusion enacted in students’ mathematics education function to marginalize working-class and culturally diverse students (Jorgensen, Gates, & Roper, 2014). This marginalization occurs through schooling practices that align with the habitus of some students but not others by requiring the linguistic capital and practices of particular classes. Working in ways that align with arguments both for de-essentialization and attention to habitus, Boaler (2002a) has sought to describe the situated nature of learning in schools and argued not only that students’ knowledge, practices and identity are inter-related (Figure 1) but that these inter-relationships “constitute the learning experience.” This model of the inter-relationship

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2 The term habitus refers to the informal knowledge and skills that are developed through one’s socialization “within the family, home, and immediate environment” so that one learns how to “act in and interpret their worlds” (Jorgensen, Gates, & Roper, 2014, p. 223). It is best thought of as the dispositions that position the individual to operationalize class status.
between identity, practice and knowledge emerged during Boaler’s studies of learning in diverse school settings. It posits that one’s knowledge is interactively constituted with one’s practices. In particular, Boaler found that, “practices such as working through textbook exercises, in one school, or discussing and using mathematical ideas, in the other, were not merely vehicles for the development of more or less knowledge, they shaped the forms of knowledge produced” (p. 43). Speaking to the different instructional practices employed in schools, Boaler notes that direct instruction places the student in a hierarchical relationship with the teacher, where the teacher is the authority and the students are “received knowers” (Boaler, 2002). In contrast, in discussion oriented classrooms students are called on to engage in acts of interpretation, expression, and agency. These practices do not promote students’ passive acceptance but rather called on them to “contribute to the judgment of validity, and to generate questions and ideas.” And is so doing they foster distinct relationships between students’ identities and the “knowledge to be taught.” Hence, as Boaler argues, the findings exemplify Wenger’s (1998) claim “learning transforms who we are and what we can do, it is an experience of identity” (Wenger, 1998, p. 215).

Identity, however, is not influenced by practices alone. A key component of identity is one’s sense of agency. Moreover, as argued by Pickering (1995), working in mathematics requires a dance of agency: an interplay of human and disciplinary agency. Disciplinary agency refers to the ways that established practices and artifacts (e.g., proving practices, linguistic conventions, syntax, etc.) interact with and affect the work of mathematics. While individuals express human agency – generating ideas, symbols, terms, and practices – and impact the discipline, the products of human agency must also “surrender to the ‘agency of the discipline’” (Boaler, 2002, p. 49). In other words, human agency shapes and is shaped by one’s discipline.

![Figure 1. Adapted from Boaler (2002a).](image)

Taken together the works of Gutierrez (2008), Jorgensen, Gates, & Roper (2014), Bourdieu (1984), Boaler (2002) and Wenger (1998) collectively point to the key characteristics of Equity-Oriented Instruction (EOI). EOI necessarily disrupts the reproduction of the structural inequities that are shored up and replicated through students’ mathematics education. It intentionally attends to and broadens the forms of habitus that afford participation in schooling by valuing, among other things, the practices and “linguistic repertoires”— that is the capital (Bourdieu, 1984) – of those who are further marginalized by schooling (Jorgensen, Gates, & Roper, 2014). It affords the development of identities that enable rather than inhibit participation in the dance of agency and, therefore, students’ engagement in authentic mathematical practices. As practices are enacted in discourses (Gee, 2001), EOI requires students be afforded opportunities to engage in collaborative work that forestalls the impact of one’s social capital while also affording access to rich mathematics. It requires instructors avoid essentializing students while working to provide students with “opportunities to draw upon their cultural and linguistic resources (e.g., other languages and dialects, algorithms from other countries, different frames of reference) when doing mathematics, paying attention to the contexts of schooling and to whose perspectives and practices are ‘socially valorized’ (Abreu & Cline, 2007; Civil, 2006)” (Gutierrez, 2009, p. 5).
A Framing of Inquiry Based Learning Pedagogies

Inquiry Based Learning (IBL) pedagogies have been defined in a variety of ways. Often IBL pedagogies are defined as any form of instruction in which students actively pursue knowledge through activities and discussions (Rasmussen & Kwon, 2007). According to the Academy for inquiry based learning, IBL is a “big tent” term for, “Teaching methods in mathematics courses … where students are (a) deeply engaged in rich mathematical tasks, and (b) have ample opportunities to collaborate with peers (where collaboration is defined broadly).”

IBL pedagogies differ in (at least) two key ways from traditional, lecture-based mathematics instruction. First, curricular activities are often inverted. By this I mean that rather than introducing institutionalized knowledge and having students practice using that knowledge, IBL curricular tasks elicit students’ ways of understanding and then through task sequences provide opportunities for students to accommodate their understandings and develop disciplinary practices. The introduction of institutionalized knowledge is the final rather than first step in learning. Second, students are expected and encouraged to act with intellectual autonomy within collaborative settings. In other words, they are called on to demonstrate specific forms of human agency: (a) generating and proposing problem solving strategies; (b) comparing and contrasting approaches; and (c) engaging in acts of justification and validation.

Why researchers have argued IBL promotes EOI

The association between active learning and equity has a long and well warranted history. The results of the Treisman (1992) studies demonstrated to many in the mathematics community that opportunities to collaborate around rich mathematical tasks could change the outcomes of students who are disadvantaged by structural inequities. More recently, Freeman et al. (2014) conducted a meta-analysis of 225 studies that compared active learning pedagogies to lecture-based instruction. They found that active learning pedagogies significantly decreased failure rates and that “active learning confers disproportionate benefits for STEM students from disadvantaged backgrounds and for female students in male-dominated fields.” In a study that specifically focused on IBL pedagogies, Laursen et al. (2014) found not only did enrollment in IBL classes positively impact student success in subsequent courses but also that the IBL courses reduced the gender gap, with female students not only showing equal or greater learning gains but also higher levels of intention to persist than those in non-IBL courses.

Beyond these empirical studies, supports for IBL’s potential to promote equitable outcomes can be found in recent theoretical analyses. Tang, Savic, El Turkey, Karakok, Cilli-Turner, and Plaxco (2017) provided an analysis of IBL and its relationship to the dimensions of equity proposed by Gutierrez (2009). Specifically, Tang et al. argue that in collaborative learning environments, all students are invited to engage in the “doing, discussing, and presenting” of mathematics. The implication here is that IBL pedagogies increase access to rich mathematics, while also promoting achievement (Freeman et al., 2014; Laursen et al., 2014). Building on the findings of Hassi’s (2015) qualitative study, Tang et al. also discuss how collaborative learning environments in which students assert agency, foster growth in self-esteem and self-confidence and, therefore, students’ sense of power. Thus, according to Tang et al., IBL pedagogies act not only along the dominant axis of equity but also the critical axis.


Kuster and Johnson (2016) proposed a four-component model of IBL that aligns with that proposed here. Cook, Murphy and Fukawa-Connelly (2016) have proposed a six-component model. Due to space limitations, these models are not discussed in this theoretical report.
Why IBL might not gratuitously promote EOI.

_Identity has as much to do with others as it does with self ... A large part of who we are is learned from how others interact and engage with us._ (Pierson Bishop, 2012, p. 38)

It is not the purpose of this section to argue that IBL pedagogies do not promote more equitable learning outcomes than traditional lecture-oriented pedagogies. Certainly, it would be a fool’s errand to do so given recent research (e.g., Freeman et al., 2014). Instead the purpose is to argue that IBL pedagogies are not necessarily EOI pedagogies and, consequently, do not produce equitable learning environments “for free.” Instead, intentional attention to equity is required.

To explore the ways in which IBL might fail to function as a form of EOI, I will discuss two data excerpts drawn from field notes and proof scripts collected during a series teaching experiments. These experiments occurred in IBL-Introduction to Proof courses taught at a designated Hispanic-serving university, where the majority are first generation college students eligible for need-based financial assistance. The classes were majority-minority classrooms: on average 67% were ethnic minorities and approximately one-third were students who identify as female. Students classified as Hispanic by institutional categories were the dominant minority group, with many preferring the terms Latino/Latina or Chicano/Chicana rather than Hispanic.5

**The first example.** The first data excerpt is drawn from field notes. It concerns an event of othering: viewing or treating an individual as distinct from or alien to oneself or one’s group (possibly without intent).

**The Vignette.** The class begins with a whole class discussion about the theorems the class will focus on proving that day and a target time for discussing their proofs. Students are asked to move into their small groups, which have been assigned by students counting off the numbers 1 through 7. Mariella6, a Latina, begins to move her desk towards her group. She stops a few feet short of her group because the other members of her group (three male students) have already moved their desks together and left no space for her desk. (The pre- and post-grouping of the desks is shown in Figure 2, with Mariella’s desk shown as a circle.) She quietly works on her own, occasionally looking at the male students who do not appear to notice her exclusion.

The instructor observes Mariella’s situation for approximately 20 minutes in an effort to provide adequate time for the male group members (or Mariella) to rectify the exclusionary situation. The instructor speaks with Mariella to confirm that the group of three male students is, in fact, her assigned group. Mariella requests of the instructor that she be allowed to work alone. The instructor respects her request, observing that she is uncomfortable. The classroom learning assistant (an advanced undergraduate) is asked by the instructor to check in with Mariella periodically. Several extended mathematical conversations are observed between them. After the class, the instructor asks two other female students from the class to speak with her individually outside of class. The instructor asks each student how she would prefer instructors respond in similar situations. Unprompted, both women share similar experiences where they were either physically excluded or “invisible” during group work. Both suggest moving Mariella to a group with another female. The next day Mariella is asked to change her group and, shortly thereafter,

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5 Following Gutierrez (2013), I use the terms Chicano and Chicana to refer to people with indigenous ancestry in the western United States. I recognize its use by students (and researchers) as intentional and political. Hence forth, I will use the gender neutral terms, Chican@ and Latin@.

6 All names are pseudonyms.
observed assisting the other female student. Instances of Mariella actively engaging with her new group while engaging in proving efforts are observed in several subsequent classes.

![Figure 2. Pre- and post-grouping desk arrangements](image)

**Vignette Discussion.** Why is this an instance of IBL not gratuitously promoting EOI? To be certain, some might argue that the students described in the vignette were not engaging in IBL because a central tenet of IBL is collaboration and the students weren’t collaborating. There are two issues with this response. First, the male students were collaborating. Second, Mariella had tried to join the group to collaborate but had been excluded. Another critique might center on the fact that the instructor could have remedied the situation by reminding the male students of the participation norms which were discussed extensively at the beginning of the course or that Mariella should have acted to end her exclusion, since participation is an expectation of all IBL students. Such responses, however, assume that the tenets of IBL should be privileged to such an extent that they are enacted in lieu of EOI practices. They ignore the costs marginalized students pay to participate when they are called on to enforce IBL practices and (potentially) act against their own identities, dispositions, or cultural practices. Moreover, privileging collaboration while ignoring these costs does little to mitigateitized marginalized students’ sense of exclusion or the potential for such practices to create the illusion of participation. And it is here that the problem lies. Even if all IBL students are expected to advocate for their own participation it is not the case that all students are called on to do so. More importantly, it is not the case that all will have identities, dispositions, or a cultural habitus that are at odds with such actions. Indeed, a post-class discussion with Mariella confirmed that she felt extremely uncomfortable “forcing” herself into the group, preferring instead to work alone after having been publicly othered.

**The Second Example**

*Mom, how do you say quesadilla in Spanish?*

-Sebastian, Age 7

As noted earlier in the paper, EOI requires students be afforded opportunities to engage in collaborative work that forestalls the impact of one’s social capital while simultaneously supporting and empowering students’ identities. The position taken in this paper is that one’s linguistic practices are not secondary to one’s identity but rather are an integral component (Bishop, 2012). The extent to which one’s language, culture and practices are valued in an environment determines the extent to which one’s identity is valued. Since 1998, Latin@ and Chican@ students have had to deal with the educational fallout of California Proposition 227. This proposition codified a stance towards bilingualism that views students’ use of non-English languages as a deficit rather than an asset to the students and their communities. It is one of the reasons Californian dialects that heavily integrate Spanish words are often practiced without users recognizing their use of another language – a point exemplified by Sebastian’s remarks.

Gee (2001, 2005) and Sfard and Prusak (2005) argue that identities are constructed through
discourse. Others, such as Bishop (2012), argue that discourses “play a critical role in enacting identities” (p. 44). Most who have taught university mathematics courses in environments where the majority of students are first generation urban students can readily attest to the varied and at times colorful slangs currently used. These languages stand in stark contrast to that employed with great continuity for thousands of years among the practitioners of the discipline of mathematics, especially when writing proofs. To illustrate this continuity, I ask the reader to consider the resemblance between the two proofs in Figure 3, the first from Euclid’s Elements (c. 350 BC, T.L. Heath’s 1909 translation) and the second from Mathematical Proofs: A Transition to Advanced Mathematics by Chartrand, Polinmi and Zhang (2008) (see pp. 145-6).

<table>
<thead>
<tr>
<th>1.6 If in a triangle two angles be equal to one another, the sides which subtend the equal angles will also be equal to one another.</th>
<th>6.17 For every nonnegative integer ( n ), ( 3(2^n-1) ).</th>
</tr>
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<td><strong>Proof:</strong> Let ( ABC ) be a triangle having the angle ( ABC ) equal to the angle ( ACB ); I say that the side ( AB ) is also equal to the side ( AC ). For if ( AB ) is unequal to ( AC ), one of them is greater. Let ( AB ) be greater; and from ( AB ) the greater let ( DB ) be cut off equal to ( AC ) the less; let ( DC ) be joined. Then since ( DB ) is equal to ( AC ), and ( BC ) is common, the two sides ( DB ), ( BC ) are equal to the two sides ( AC ), ( CB ) respectively; and the angle ( DBC ) is equal to the angle ( ACB ); therefore, the base ( DC ) is equal to the base ( AB ), and the triangle ( DBC ) is will be equal to the triangle ( ACB ), the less to the greater: which is absurd. Therefore, ( AB ) is not unequal to ( AC ); it is therefore equal to it.</td>
<td><strong>Proof:</strong> Assume, to the contrary, that there are nonnegative integers ( n ) for which ( 3(2^n-1) ). By Theorem 6.7, there is a smallest nonnegative integer ( n ) such that ( 3(2^n-1) ). Denote this integer by ( m ). Thus ( 3(2^{m-1}) ) and ( 3(2^n-1) ) for all integers ( n ) for which ( 0 \leq n &lt; m ). Since ( 3(2^{m-1}) ) when ( n = 0 ) it follows that ( m \geq 1 ). Hence, ( m = k + 1 ), where ( 0 \leq k &lt; m ). Thus ( 3(2^{2^n-1}) ) which implies that ( 2^n - 1 = 3x ) for some integer ( x ). Consequently, ( 2^n = 3x + 1 ). Observe that ( 2^{m-1} = 2^{2^n-1} = 2^2 \cdot 2^{2^n-2} = 4(3x + 1) - 1 = 12x + 3 = 3(4x + 1) ) Since ( 4x + 1 ) is an integer ( 3(2^{m-1}) ), which produces a contradiction.</td>
</tr>
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Figure 3. Proofs from Euclid (c. 350 BCE; 1909 translation) and Chartrand et al (2008).

Now consider the following thought experiment: Imagine that humans had not invented mathematical proof well over 2000 years ago and that despite not having invented proofs, enough mathematics developed for some modern technologies (e.g., cell phones, twitter, and texting). What would our proving practices look like if they were invented by our culturally-diverse, economically-disadvantaged, urban youth? Would answers like that shown in Figure 4 be considered normative rather than examples of norm breaching (Herbst & Chazen, 2011)?

![Figure 4. Student Survey Response](Image)

Moreover, would students’ proof scripts, like that shown in Figure 5, be viewed as an instance of authentic mathematical discourse rather than as something written in another dialect? Would the pervasive code-switching that occurs in the dialog be seen as exemplifying a student’s masterful blending of two dialects – the urban and the mathematical – rather than as indicative of a lack of participation in unspoken, yet implicitly demanded, disciplinary dialectic practices?

The student’s script was drawn from a set of 43 proof scripts: written dialogs in which a student and a fictional peer discuss a proof so as to promote the peer’s understanding of any gaps or key points in the proof. It was chosen as an example of one of many instances of students describing deep mathematical issues using their normative discursive practices. Indeed, field notes indicate that throughout the IBL Introduction to Proof course, students had grown increasingly accustom to intensely discussing proofs in their everyday vernacular. It is included in the paper to demonstrate a tension between IBL and EOI. A key tenet of IBL is that students’ move towards institutionalized knowledge (and therefore, normative disciplinary practices) through their collaborative activities. It privileges rather than challenges normative practice by calling on instructors to enact discourse hierarchies in lieu of attending to the critical role discourses play in identity formation and student agency. Consequently, enacting IBL
pedagogies means working to curtail rather than recognize (or value) students’ discourses. In contrast, practitioners who privilege EOI practices over those central to IBL must attempt to navigate the tension between students’ means of expressing identity and disciplinary discursive practices. They must recognize that privileging EOI means rejecting discourse hierarchies while simultaneously providing opportunities for students to become knowledgeable of disciplinary discourses. In other words, drawing on Gutierrez (2009), this paper argues that privileging EOI when enacting IBL, means valuing instances in which students “change the game” (e.g., by seeing value in the student’s bridging of his own and disciplinary vernaculars) while also valuing the student’s success “playing the game” (e.g., by valuing the mathematical sophistication which underlies the detailed and precise mathematical refinements embedded in the student’s remarks).

**Figure 5. Joseph’s Proof Script Excerpt**

**A framing of E-IBL**

In this paper, I call into question the assumption that IBL pedagogies gratuitously promote EOI and argue E-IBL requires intentional attention to equity. I posit that intentional attention to equity calls on practitioners to employ EOI as a lens when viewing IBL learning environments. Applying such a lens necessarily entails foregrounding issues of structural exclusion and acting to disrupt the social mechanisms that result in their reproduction in institutional spaces (Jorgensen, Gates, & Roper, 2014; Battey & Leyva, 2016). It means privileging students’ identities and habitus when IBL practices call on students to act against either; e.g., by valuing varied forms of social capital (e.g., linguistic resources (Zahner & Moschkovich, 2011)) or addressing instances of *othering* by first attending to students’ identities and habitus; i.e., the costs some pay to participate. At its core, this framing posits E-IBL instructors must be willing to recognize that, as argued by Wenger (1998), learning is “an experience of identity” and that identity and power are negotiated in institutional contexts (Adiredja & Andrews-Larsen, 2017). Thus, privileging the demands of EOI over the tenets of IBL, requires instructors navigate the tensions present in spaces that support students not only “playing the game” but also “changing the game” (Gutierrez, 2009) as they develop expertise in mathematics.
References


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