Supporting Prospective Teachers' Understanding of Triangle Congruence Criteria

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This poster describes an instructional sequence for supporting college geometry students' justifying Euclidean triangle congruence criteria using properties of isometries. We hypothesized that investigating transformations in the taxi-cab metric would perturb students' understandings of the relationships between triangle congruence criteria and isometries, so they would more explicitly identify the properties of transformations as a necessary part of their justifications of triangle congruence criteria. We report on the results of pre-post written assessments of our college geometry students' justifying SAS to a hypothetical 10th grade student.

Keywords: Geometry, Congruence, Mathematical Knowledge for Teaching

Research has demonstrated that supporting college geometry students' understandings of transformational geometry remains a challenge (e.g., Hegg & Fukawa-Connelly, 2017). Because many of our college geometry students are prospective secondary teachers, supporting their mathematical knowledge for teaching secondary geometry is an important course goal. The United States' Common Core State Standards state that high school geometry students should be able to justify triangle congruence criteria (ASA, SAS, SSS) as a consequence of properties of rigid motions (NGACBP/CCSSO, 2010). Hegg and Fukawa-Connelly (2017) found that college geometry students struggle with explicitly using relevant properties of transformations in such justifications, and they suggest that "asking for the kinds of explanations of ideas that [college geometry students] would give [secondary geometry] students has value in both giving researchers insight into their understanding of the content and giving policy-makers a better understanding of what additional supports will be needed going forward" (p. 8).

We designed a written task prompting our college geometry students to justify SAS to a hypothetical 10th grade student, and we administered the task before and after an instructional sequence investigating transformations in taxi-cab geometry in our Spring 2017 courses. With consideration of Harel's (2013) notion of intellectual need, we theorized that after experiencing perturbation that some Euclidean isometries are *not* isometries in a different metric (the taxi-cab metric, $d_t((x_1, y_1), (x_2, y_2)) = |x_1 - x_2| + |y_1 - y_2|$), our students would understand triangle congruence criteria in Euclidean geometry as depending on the *properties* of the isometries and thus be better-prepared to support future secondary students' in constructing that way of thinking about triangle congruence criteria. Using a constant comparative method (Glaser, 1965), we each independently coded students' written justifications of the SAS congruence criteria, considering aspects such as their (a) understanding of the premises and conclusions of SAS, (b) use of other Euclidean axioms or theorems to justify SAS, and (c) use of transformations to justify SAS. We then rated the written justifications on validity, substantiveness, and appropriate use of transformations. Our analysis suggests that students' responses were complicated by their beliefs about (and conceptions of) what it means to justify in secondary school mathematics. In our poster we will present the task, explicate findings, and future directions.

References

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